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Beam Dynamics Study for RF Tolerance Estimation in Korea-4GSR

Eunkyoung Shin, Ji-Gwang Hwang
Gangneung-Wonju National University

Bonghyuk Choi, Hyunchang Jin, Seunghwan Shin
Korea Basic Science Institute

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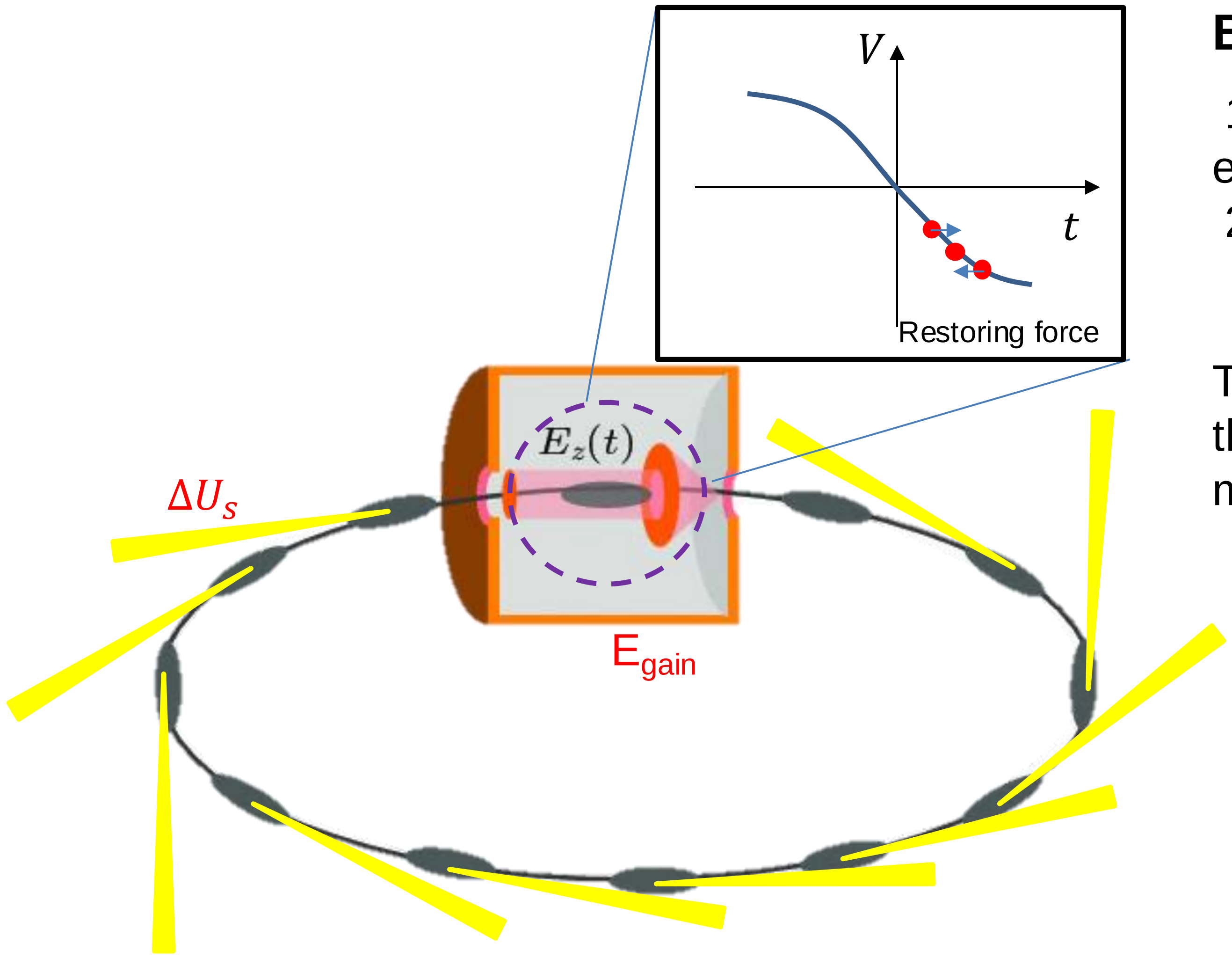


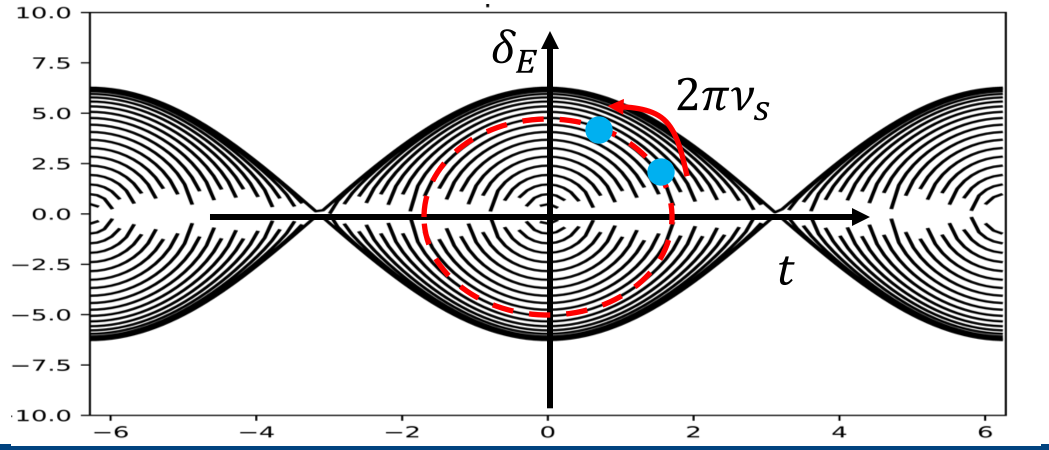
Beam dynamics in a storage ring:

1. electrons emits synchrotron radiation, resulting in energy loss, E_{loss}.
2. an RF cavity recovers the energy loss

$$E_{gain} = eV_{eff} \sin \phi_s = -\Delta U_s$$

Therefore, the particle dynamics can be approximated to the equilibrium state. In a linear system, the transfer matrix can be interpreted as

$$\begin{pmatrix} z \\ \delta_E \end{pmatrix}_n = \begin{pmatrix} \cos 2\pi\nu_s & \frac{\eta c}{\omega_s} \sin 2\pi\nu_s \\ \frac{\omega_s}{\eta c} \sin 2\pi\nu_s & \cos 2\pi\nu_s \end{pmatrix} \begin{pmatrix} z \\ \delta_E \end{pmatrix}_{n-1}$$



Longitudinal dynamics in a storage ring

- Like the surfer-board moves well with the proper phase of waves in the water, there are adequate moments to add energy to electrons.

- In a storage, the stored beam is naturally settled to the phase that defined by the balance between the energy loss and gain

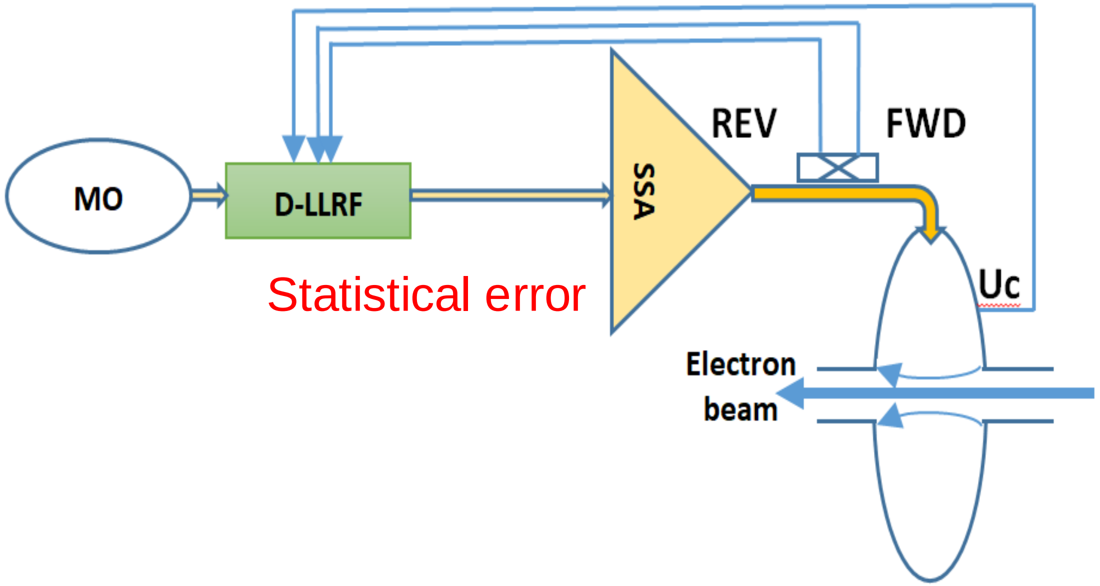
$$\Delta U_s = qV_{max} \sin \phi_s$$

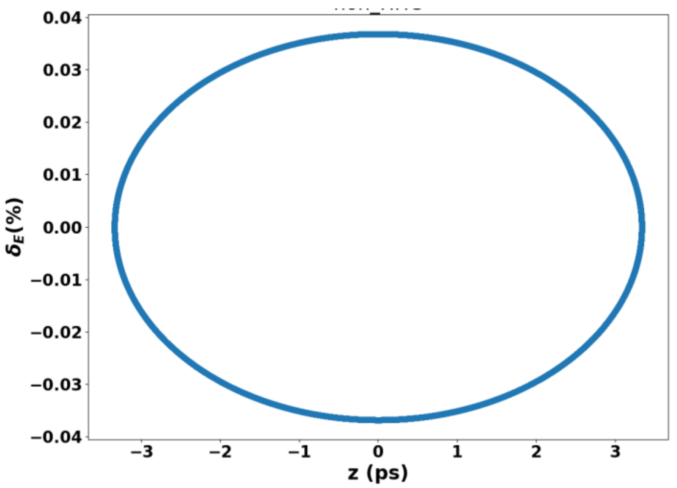
$$\phi_s = \sin^{-1} \frac{\Delta U_s}{qV_{max}}$$

- For the Korea-4GSR storage with $V_{max} = 3.5\ MV$ and $\Delta U_s = 1.45\ MeV$, the phase is about 155.53 deg
  (* 0 deg refers the maximum acceleration phase
  = on-crest phase)

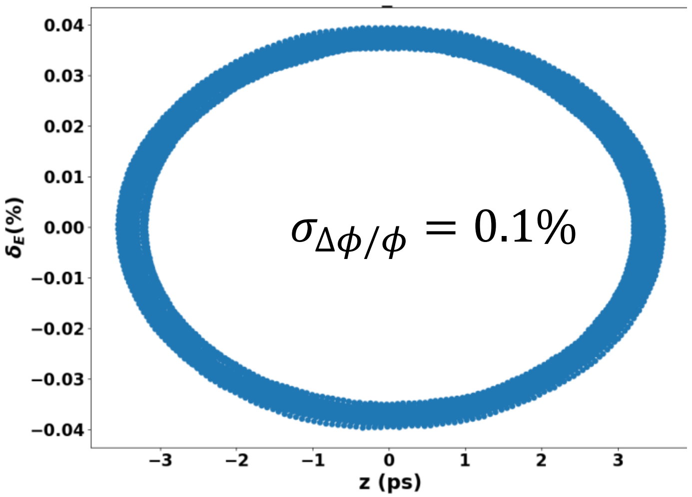
RF system and jitter

The main role of the RF system in a storage ring is to compensate the energy lost by the electrons in terms of synchrotron radiation. For ensuring the stability of the RF system, it has a dedicated feedback loop with a Digital Low-Level RF (D-LLRF) system that monitors the phase and amplitude at consequent places.





Effect of the error in RF system



The phase and amplitude errors modify the energy offset:

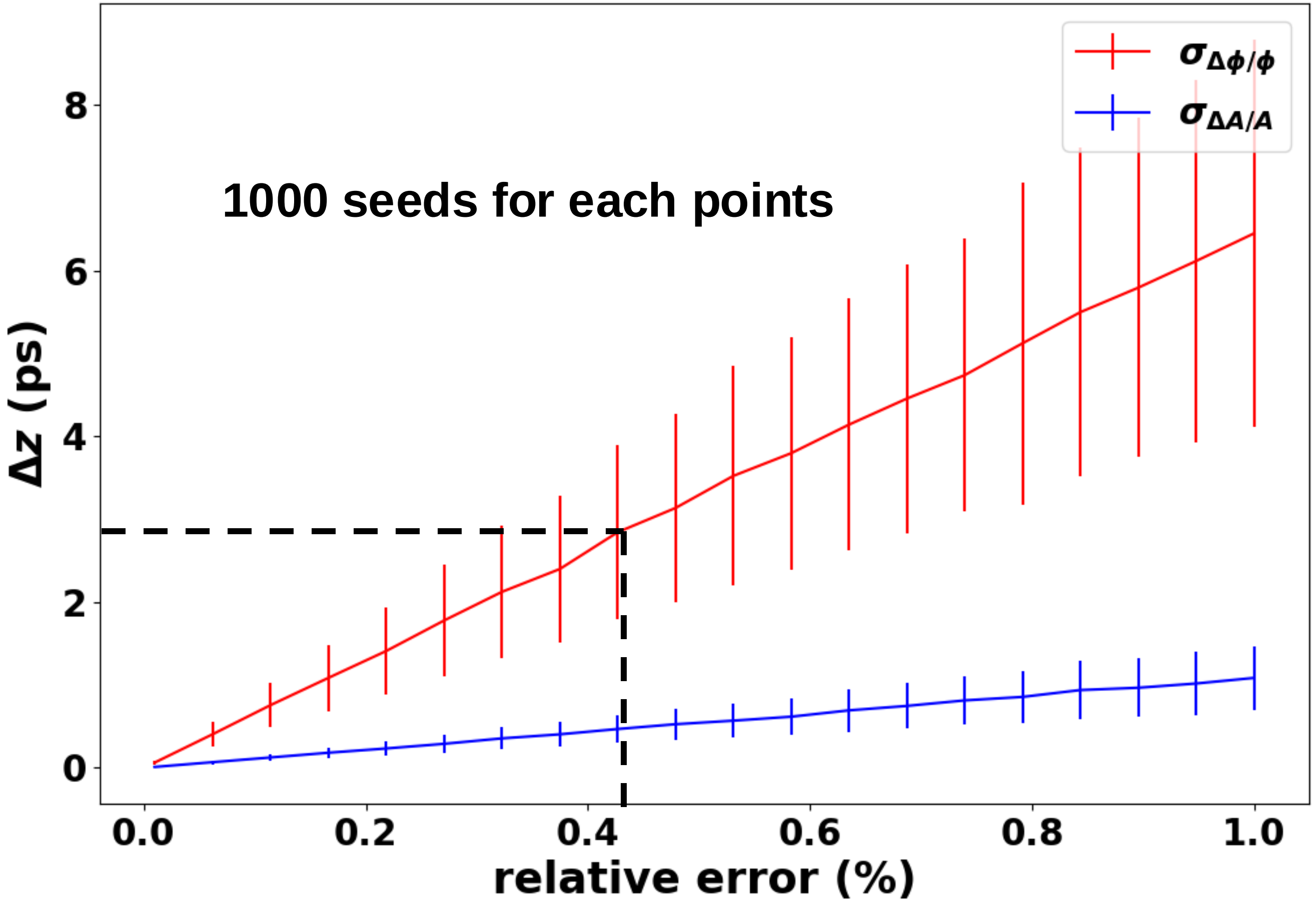
$$E_{gain} = e(V_{eff} + \Delta V)\sin(\phi_s + \Delta\phi) \neq -E_{loss}$$

The energy offset will introduce the motion of the center in the longitudinal direction.

$$\Delta E \neq 0 \rightarrow \ \delta = \delta_0 + \Delta\delta$$

Evaluation of the effect of the jitters

In a linear model with the main RF cavities, the phase error is the main contributor to the longitudinal oscillation.



Considering the temporal resolution in experiments, the RF tolerance can be determined by the acceptable region (about 20 % of σ_z).

* Bunch length σ_z in 4GSR : ~ 12 ps

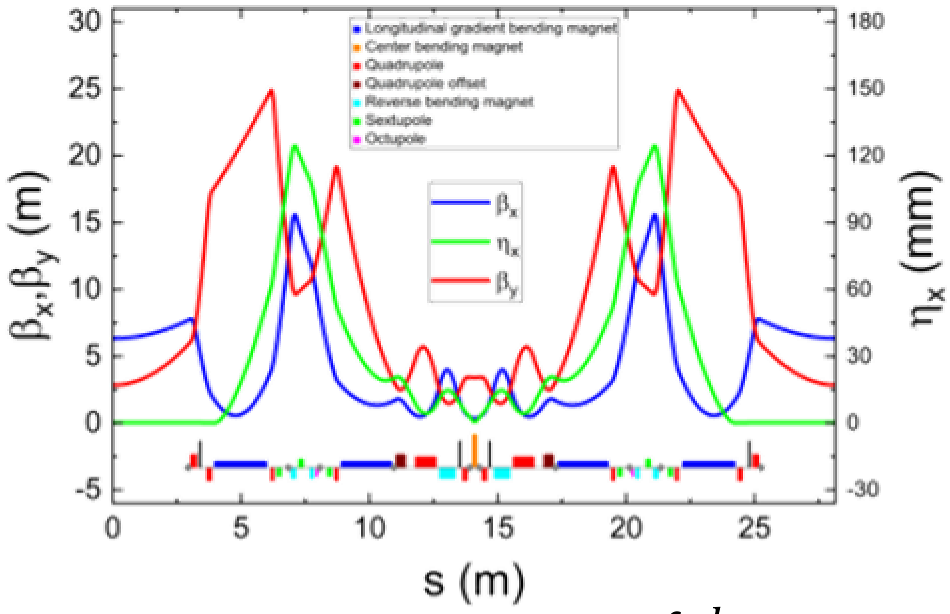
Acceptable error range for the RF parameters

$\sigma_{\Delta\phi/\phi}$: greater than 0.4 %

$\sigma_{\Delta A/A}$: greater than 1 %

The state-of-the-art for LLRF is on

$$\sigma_{\Delta\phi/\phi}, \sigma_{\Delta A/A} \lesssim 10^{-4}$$



$$\varepsilon_{x0} = C_q \gamma^2 \frac{I_5}{j_x I_2}$$

$$I_2 = \oint \frac{ds}{\rho^2} \propto \Delta U_s \rightarrow \text{Const.}$$

$$I_5 = \oint \frac{\mathcal{H}_x ds}{|\rho|^3}$$

$$\rightarrow \quad \mathcal{H}_x = \gamma_x {\eta_x}^2 + 2\alpha_x \eta_x \eta_{px} + \beta_x {\eta_{px}}^2$$

$$\rightarrow \quad \varepsilon_{x0} \downarrow \; \rightarrow \; \alpha_p = \frac{1}{C} \oint \frac{\eta_x}{\rho} ds \downarrow$$

$$\rightarrow \quad \downarrow \sigma_z = \delta_0 \sqrt{\frac{E}{e f_{ref}} \frac{\alpha_p}{V'_{RF}}} \rightarrow \tau_{life} \downarrow$$

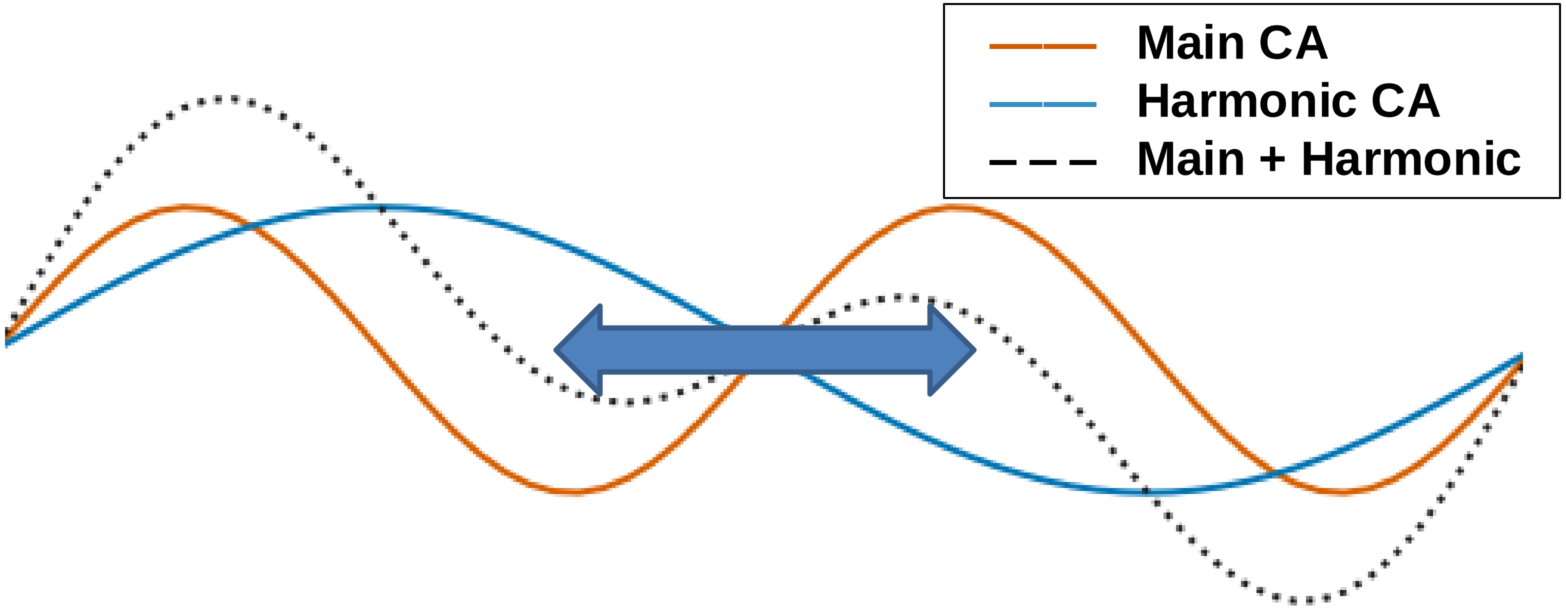
Specifications

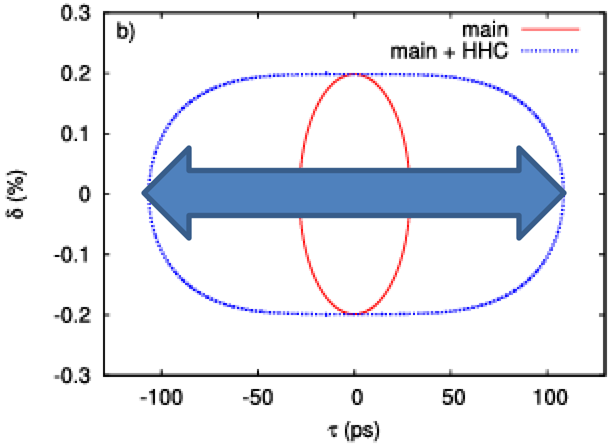
- Beam Energy: 4 GeV
- Beam Emittance: less than 100 pm·rad (TDR: 62 pm·rad)
- Circumference: 800m
- Beamlines : more than 40
- Accelerator: Gun, Injector LINAC, 4 GeV Booster
- Lattice: Hybrid 7 Bend Achromat (H7BA)

Higher Harmonic Cavity (Bunch lengthening)

BESSY II 500 MHz cavity

BESSY II 1.5 GHz cavity





Narrow and long wave

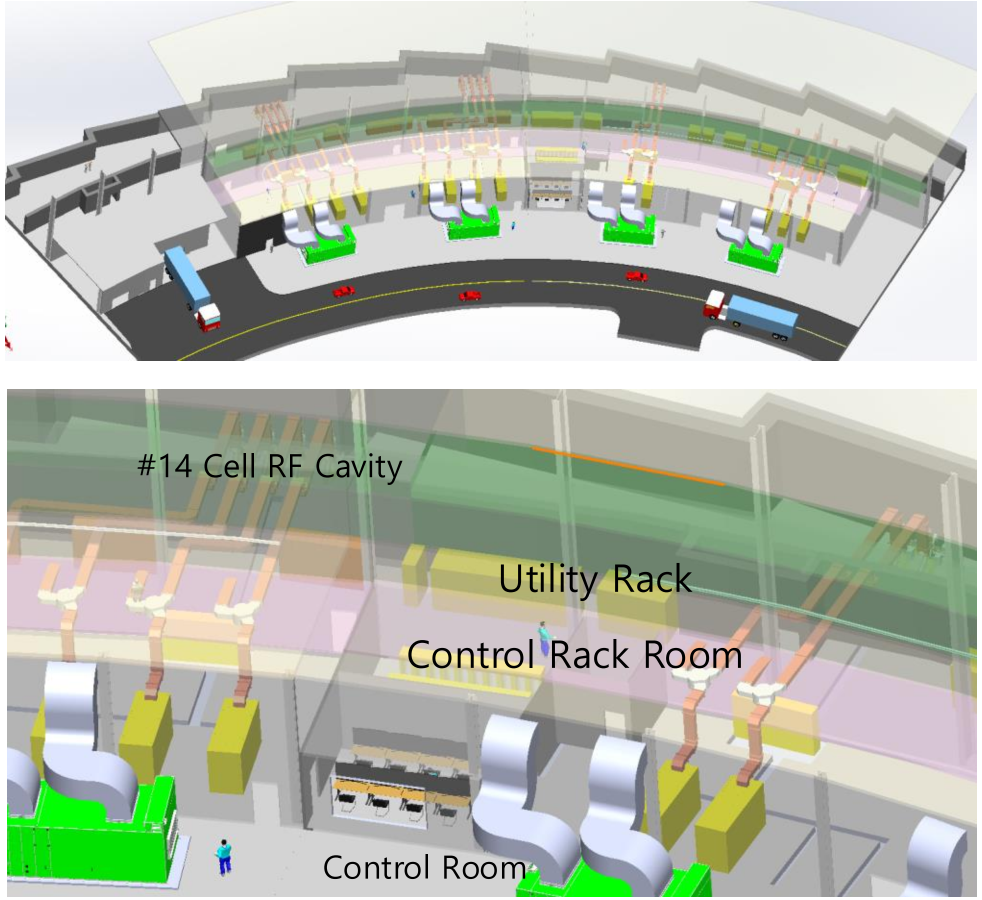
Voltage profile for 500 MHz cavity

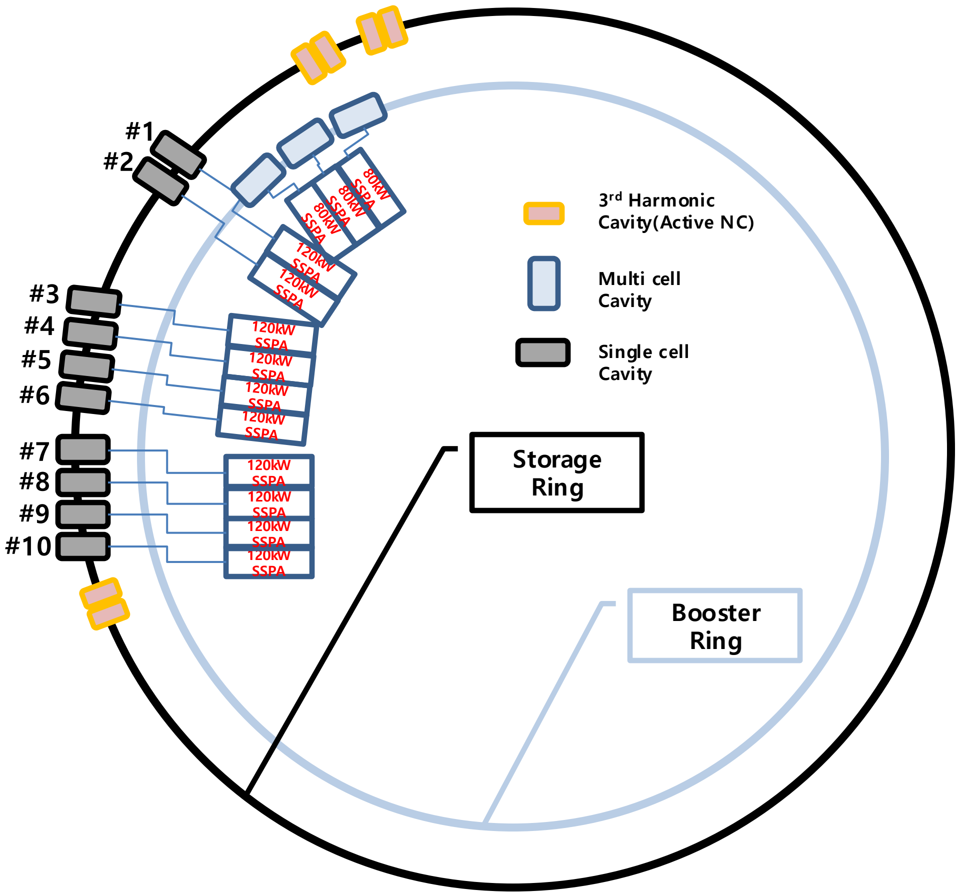
$$V(\tau) = V_{rf}\sin(\omega_{rf}\tau + \phi_s) - \frac{U_0}{e}$$

with harmonic cavity

$$V(\tau) = V_{rf}[\sin(\omega_{rf}\tau + \phi_s) - r\sin(m\omega_{rf}\tau + \phi_m)] - \frac{U_0}{e}$$

Higher Harmonic Cavity (Bunch lengthening)





500 MHz RF cavities

# of cavity: 10

V_{main} = 3.5 MV

HOM-damped cavity

10 x 120 kW SSPA

1.5 GHz cavities

# of cavity: 6

V_{3rd} = 0.9 MV

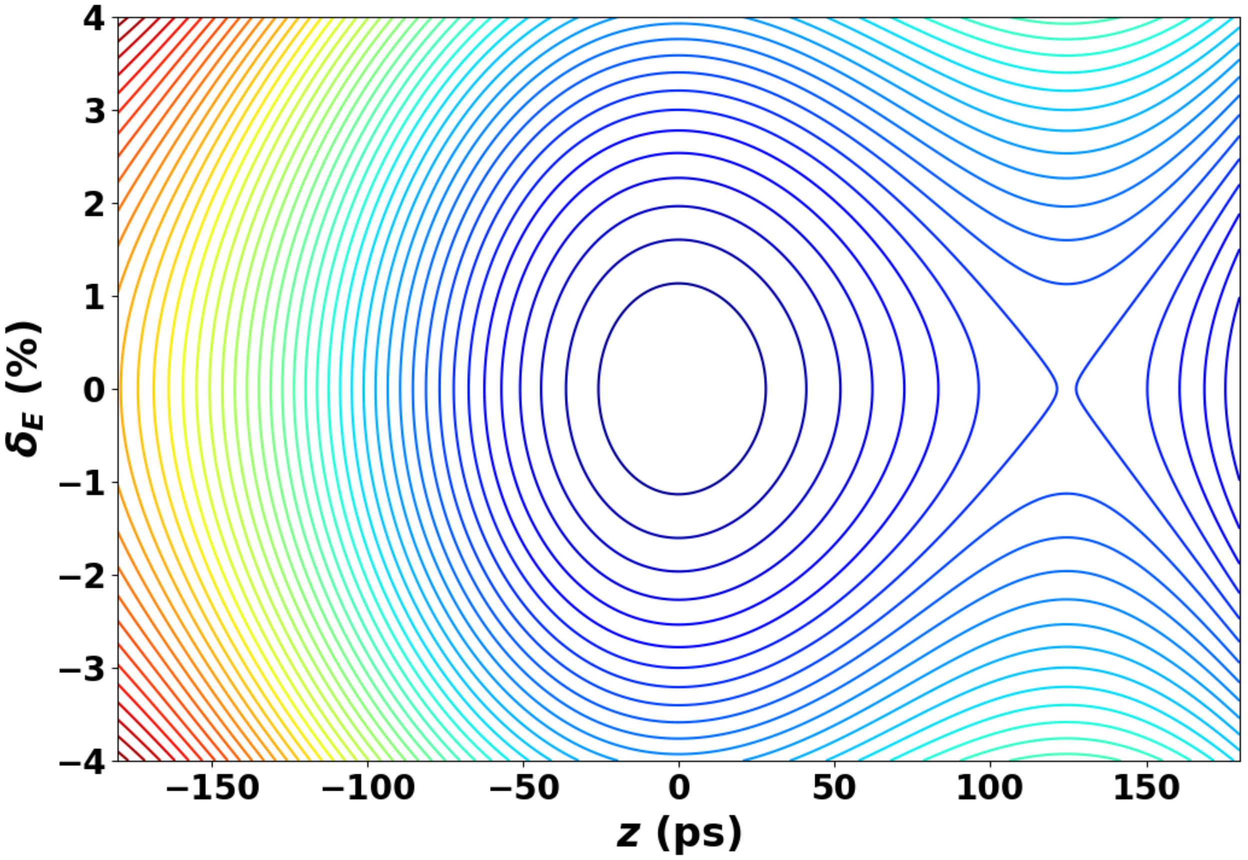
(max 1.0 MV)

V_{3rd}/V_{max} ~ 26 %

Lengthening factor ~ 3

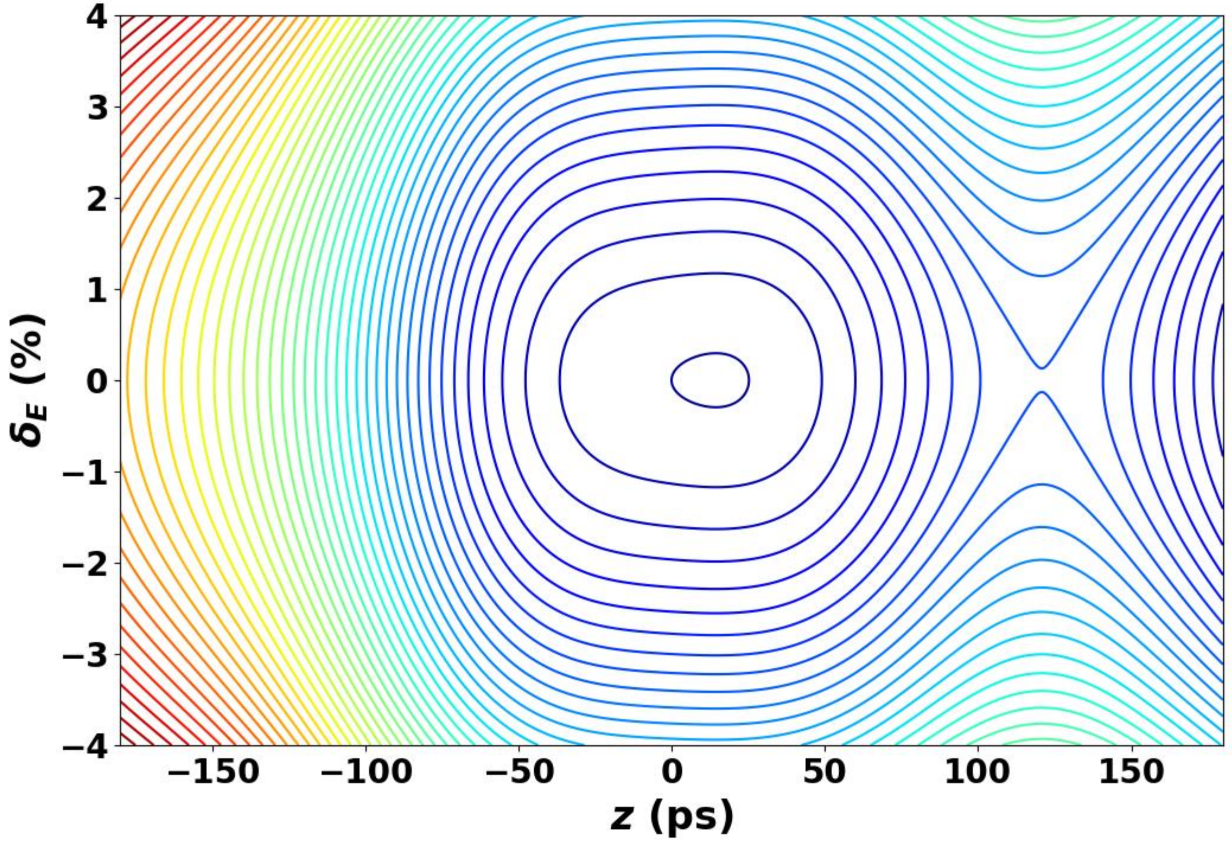
Hamiltonian dynamics (main cavity only)

$$H(\tau,\delta) = \frac{\eta}{2}\delta^2 + \frac{eV_{rf}}{E_0 T_0 \omega_{rf}}\left[\cos(\omega_{rf}\tau + \phi_s) - \cos\phi_s + \omega_{rf}\tau\sin\phi_{s0}\right]$$

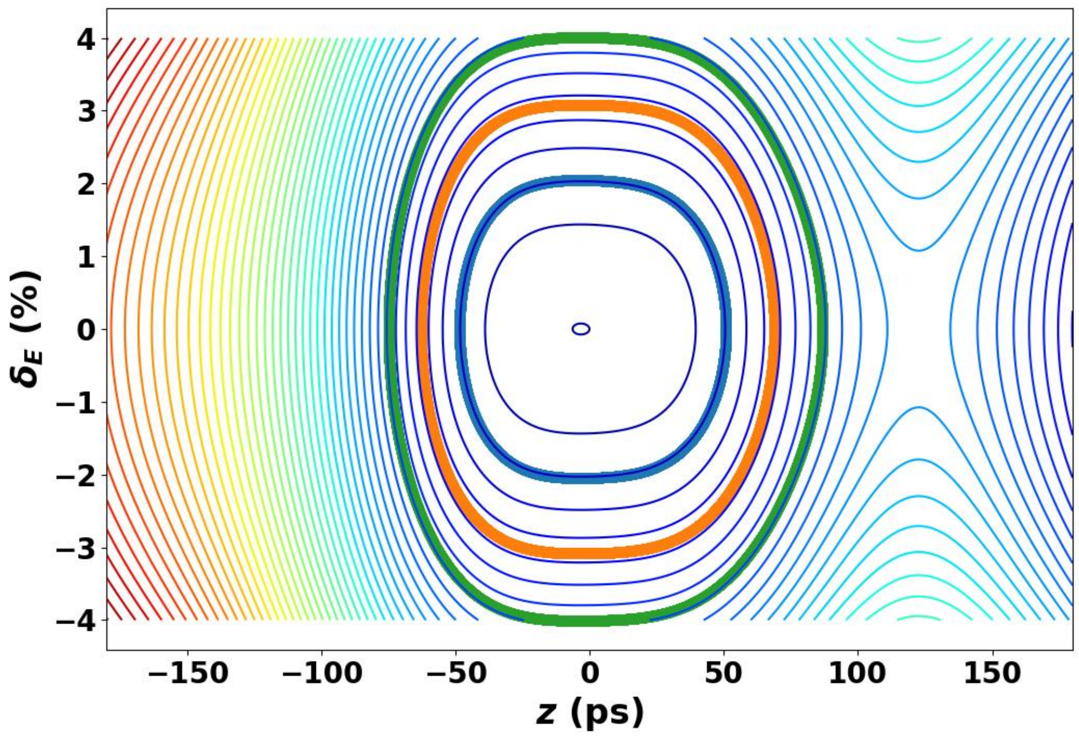


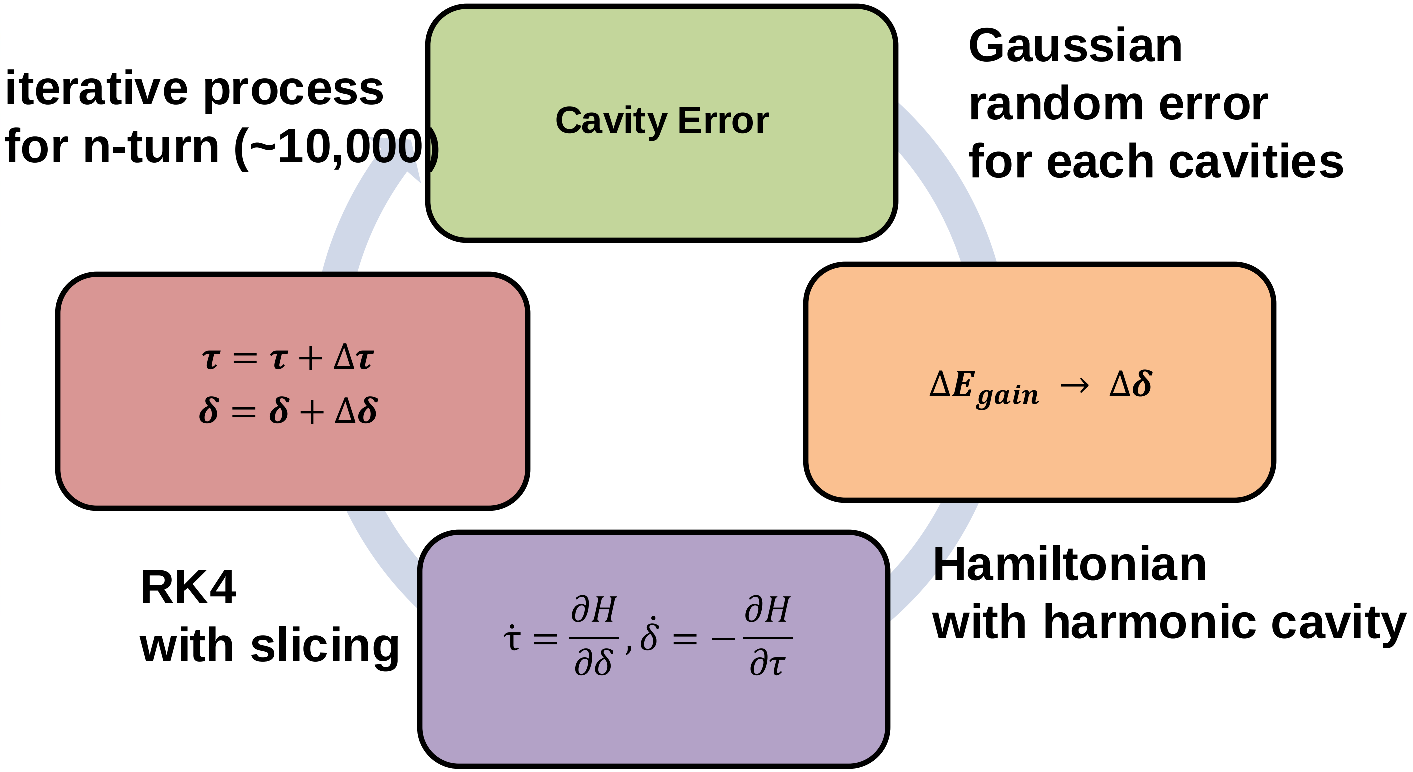
Hamiltonian dynamics (with harmonic cavity)

$$H(\tau,\delta) = \frac{\eta}{2}\delta^2 + \frac{eV_{rf}}{E_0 T_0 \omega_{rf}}\left[\cos(\omega_{rf}\tau + \phi_s) - \cos\phi_s + \frac{r}{m}\cos\phi_m - \frac{r}{m}\cos(m\omega_{rf}\tau + \phi_m) + \omega_{rf}\tau\sin\phi_{s0}\right]$$



Hamiltonian dynamics (with harmonic cavity)





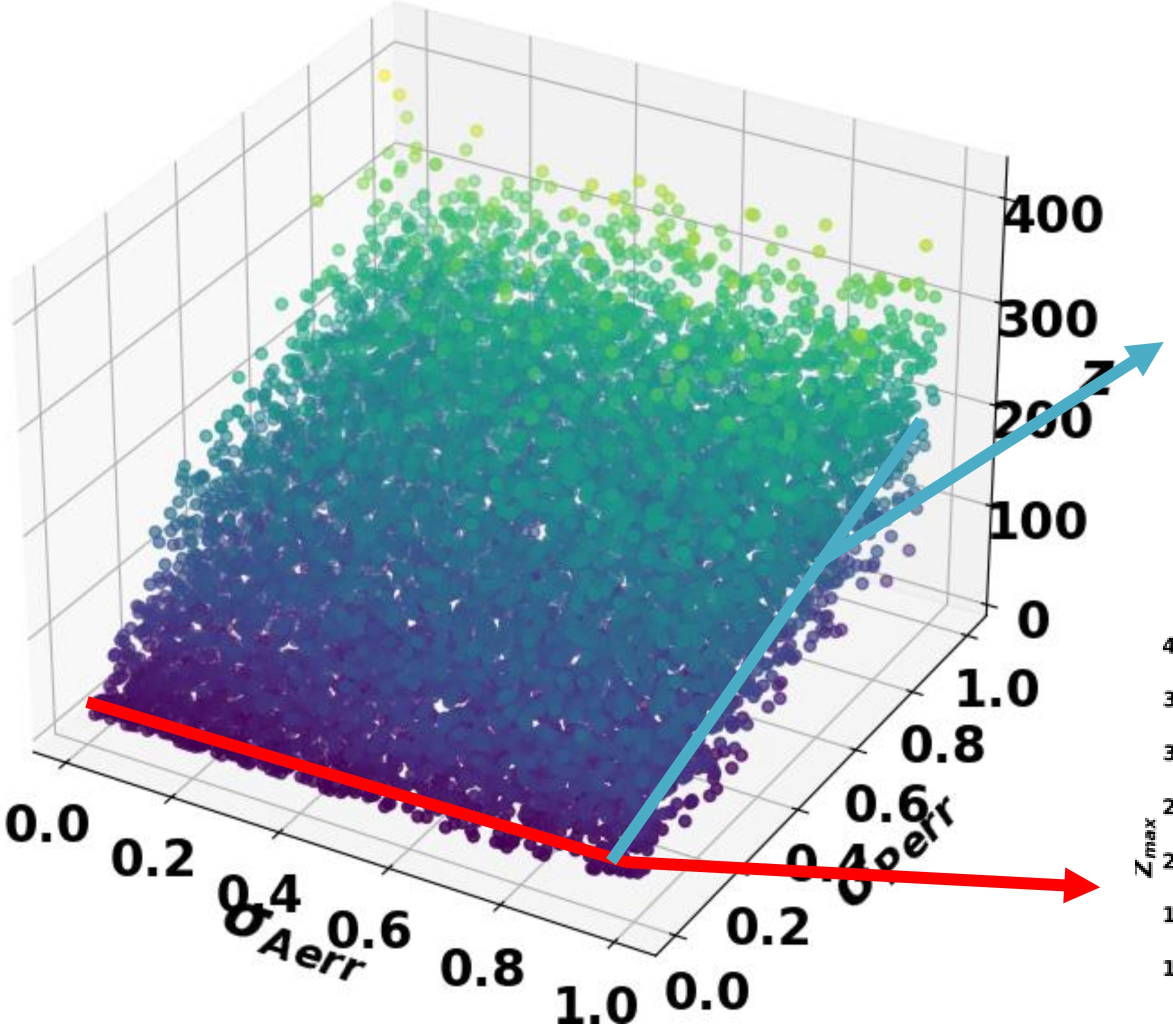
Hamiltonian EOM

$$H(\tau,\delta) = \frac{\eta}{2}\delta^2 + \frac{eV_{rf}}{E_0 T_0 \omega_{rf}}\left[\cos(\omega_{rf}\tau + \phi_s) - \cos\phi_s + \frac{r}{m}\cos\phi_m - \frac{r}{m}\cos(m\omega_{rf}\tau + \phi_m) + \omega_{rf}\tau\sin\phi_{s0}\right]$$

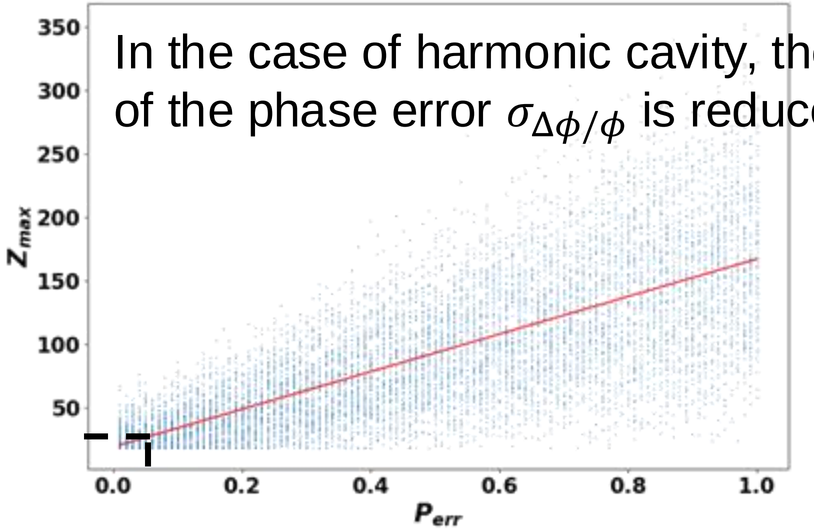
$$\dot{\delta} = -\frac{\partial H}{\partial \tau} = \frac{eV_{rf}}{E_0 T_0}\left[\sin(\omega_{rf}\tau + \phi_s) - r\sin(m\omega_{rf}\tau + \phi_m) - \sin\phi_{s0}\right]$$

$$\dot{\tau} = \frac{\partial H}{\partial \delta} = \eta\delta$$

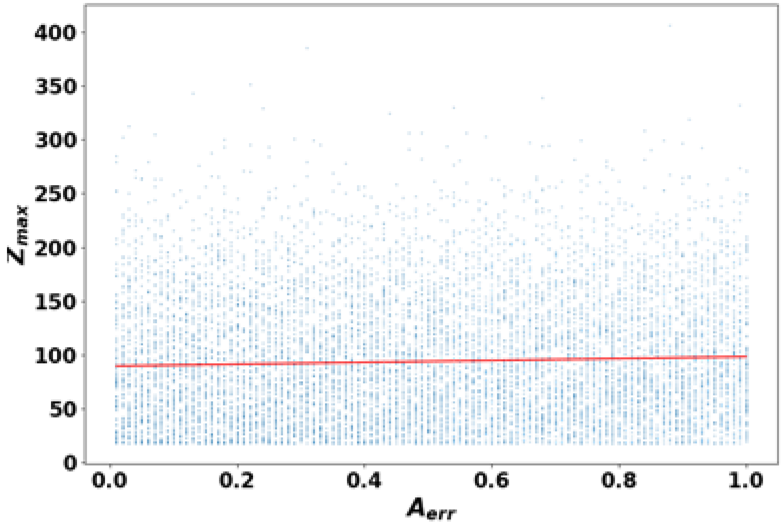
Beam jitter with HHC cavities



In the case of harmonic cavity, the acceptable range of the phase error $\sigma_{\Delta\phi/\phi}$ is reduced to < 0.08%.



Bunch length σ_z in 4GSR with harmonic cavity: ~ 35 ps



Summary

- The inevitable statistical error in the LLRF system that performs the feedback for rectifying the variation of the electromagnetic field in the RF cavity stimulates the perturbation of the central position in the longitudinal phase space.

- The tolerance of the phase and amplitude errors in the LLRF system should be determined from the detailed evaluation of the longitudinal dynamics. With HHCs which are often mandatory for the 4GSRs to increase lifetime, the particle motion can only be estimated by Hamiltonian mechanics.

- The study confirms that the longitudinal perturbation is more sensitive to the phase error than the amplitude error, and it also evidences that the tolerance of RF phase jitter was greatly limited to less than 0.08% with the HHCs.