Orbit response matrix correction based on exploration enhanced evolutionary algorithm

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Abstract

For a large ring, the response matrix has tens of thousands of data points which can fully include the linear optics of the ring. For 4 th generation diffraction limitation ring which uses strong sextupoles and octupoles, the response matrix derived from closed orbit tracking will influence by the nonlinearity and make it difficult for LOCO (Linear Optics from Closed Orbit) to match lattice parameters and correct lattice error. In this study, we propose to use an evolutionary algorithm that integrates multiple methods to enhance exploration capabilities to find the global optimal solution and demonstrate this algorithm can better ensure the response matrix correction and larger dynamic aperture than linear LOCO.

Introduction

LOCO has been a powerful beam-based diagnostics and optics control method for storage rings. The core idea of LOCO is using SVD to invert the Jacobian matrix. However, it is a linear method which is valid only when the starting solution is not far from the real minimum. When dealing with nonlinear least square problem, it has to adopt an iterative approach and often falls into local minimum.

SSMB(Stable State Micro Bunching) is proposed by Pro. Zhao in 2010. The lattice for SSMB scheme is characterized by an extremely small momentum compression factor($\alpha_c \sim 10^{-6}$) and second-order momentum compression factor($\alpha_{c2} \sim 10^{-5}$). Therefore, such a lattice with strong nonlinearity results in LOCO being easily trapped by local minimum.

Evolution-based algorithms such as PSO(Particle Swarm Optimization) and NSGA-II(Non-dominated Sorting Genetic Algorithm) do not rely on gradients so they have the possiblity to do a global search and find the true minimum. PSO ensures faster convergence capabilities by approximating the current global optimum(G_best), and NSGA obtains stronger global search capabilities by increasing the mutation rate. Combining the two algorithms can make full use of their advantages. We use the two algorithms in a parallel way and exchange data between the two algorithms when updating the current global optimum(G_best) in PSO and updating the next population in NSGA-II.

In our case, the global search capability is particularly important. In order to further enhance the exploration capability, we additionally introduced the opposition based learning and Levy flight methods to modify some of the offsprings.

Methodology

This algorithm consists of four parts to to ensure global exploration and convergence.

Particle Swarm Optimization

$$V_i^{t+1} = \omega V_i^t + c_1 r_1 (p_{best,i} - X_i^t) + c_2 r_2 (g_{best,i} - X_i^t)$$
$$X_i^{t+1} = X_i^t + V_i^{t+1}$$

Opposition Based Learning

$$x \in [a, b]$$
$$x_o = a + b - x$$

Levy Flight

$$X_i^{t+1} = X_i^t + S$$
$$S = \alpha \frac{u}{|v|^{1/\beta}}$$
$$u \sim N(0, \sigma_u^2), \quad v \sim N(0, \sigma_v^2)$$
$$\sigma_u = \left(\frac{\Gamma(1+\beta)\sin(\pi\beta/2)}{\Gamma(1+\beta/2)\beta 2^{(\beta-1)/2}}\right)^{1/\beta}, \quad \sigma_v = 1$$

NSGA-II

Simulated Binary Crossover(SBX)

$$\begin{cases} c_i^1 = 0.5 \times [(1+\beta)x_i^1 + (1-\beta)x_i^2] \\ c_i^2 = 0.5 \times [(1-\beta)x_i^1 + (1+\beta)x_i^2] \end{cases}$$

$$\beta = \begin{cases} (2 \times rand)^{1/(1+\eta_c)}, & rand \le 0.5 \\ (1/(2-2\times rand))^{1/(1+\eta_c)}, & otherwise \end{cases}$$

Polynomial Mutation(PM)

$$x_i = x_i + \Delta_i$$

$$\Delta_i = \begin{cases} (2 \times rand)^{1/(1+\eta_m)} - 1, & rand \le 0.5 \\ 1 - (2 - 2\times rand)^{1/(1+\eta_m)}, & otherwise \end{cases}$$

The algorithm is as follows:

First initialize the parameters (p_0,v_0,pbest,gbest) for PSO and p_1 for NSGA-II.

Then use the initial population to obtain a new population(p_0,p_2) through PSO and NSGA-II(p_2>p_1).

Keep the original population size(p_0+p_1) through non-dominated sorting.

Do opposition based learning on the remaining offspring and levy flight on proportion of all population(p_0+p_2).

Do non-dominated sorting on (p_0,p_2,p_o,p_l) and keep the original population size.

Update parameters(pbest,gbest)

Algorithm: exploration enhanced evolutionary algorithm

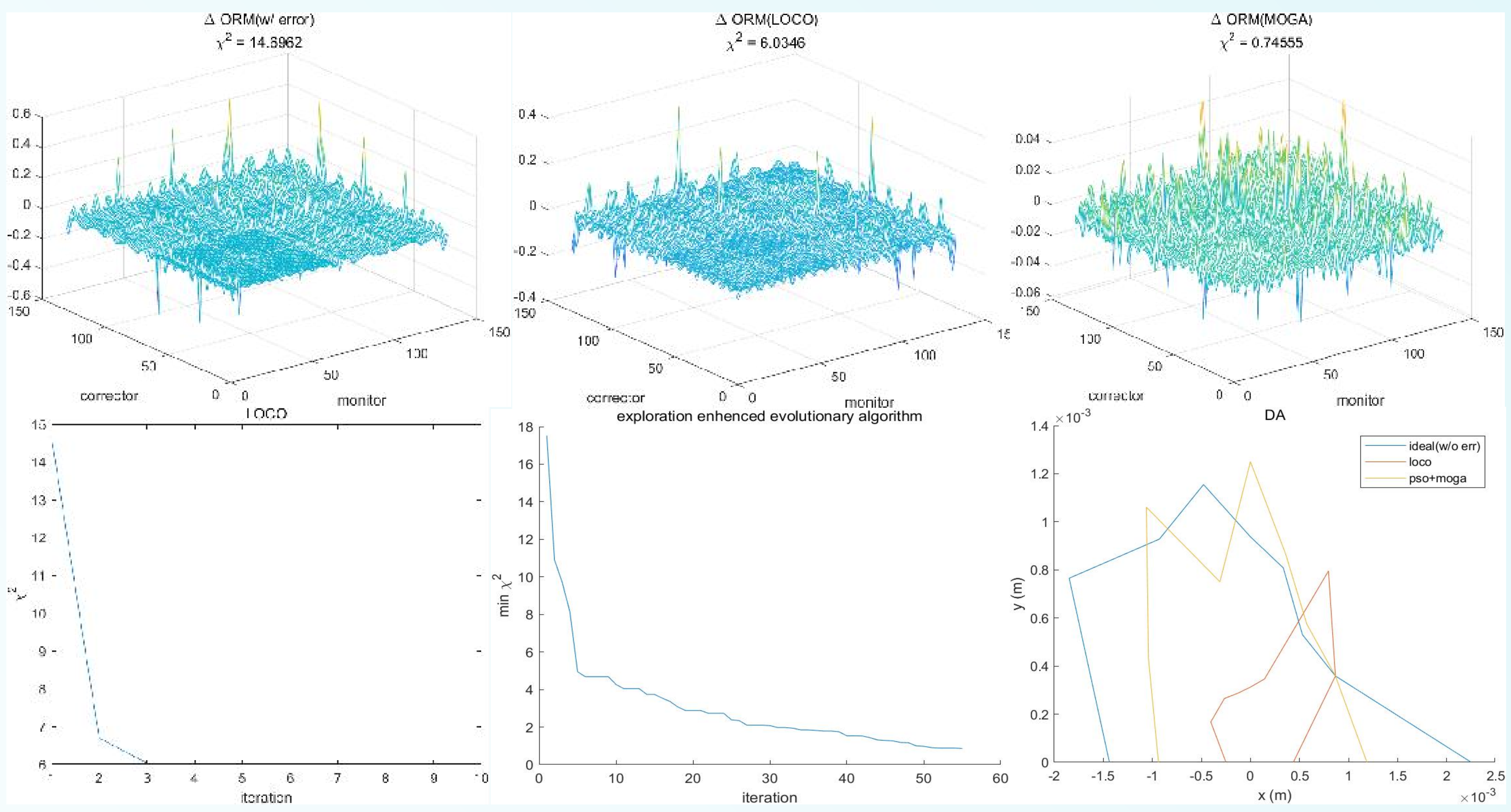
```
input: max_iteration, pop_size
initialization: swarm : p0, v0    pop : p1    archive : pbest, gbest
evaluate: p0,1 ← evaluate(p0,1)
while n < max_iteration do
    n ← n + 1
    PSO : p0, v0 ← PSO(p0, pbest, gbest, v0)
    update_pbest: pbest ← NDS([pbest, p0])
    NSGA − II : p2 ← NSGA − II(p1)
    p ← [p0, p2]
    evaluate: p ← evaluate(p)
    non-dominated sorting: pgood, pbad ← NDS(p)
    opposition based learning: po ← OBL(pbad)
    levy flight: pl ← LF(p)
    non-dominated sorting: p, _ ← NDS([p, po, pl])
    update_gbest: gbest ← NDS([gbest, p])
end while
output: p
```

Results

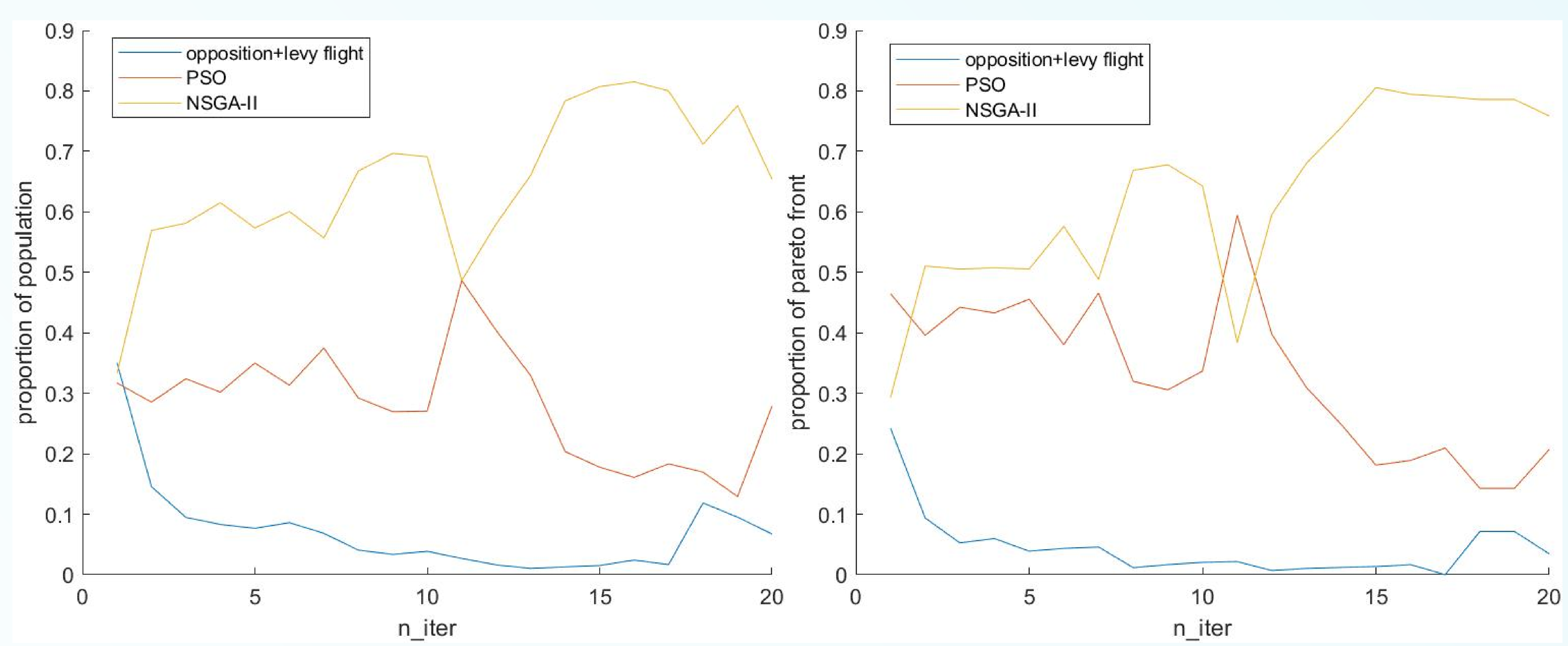
The SSMB storage ring has 32 dipoles, 108 quadrupoles, and 84 sextupoles. It initially uses 66 dipole correctors and 66 BPMs for closed orbit correction, as well as 36 normal quadrupoles, 14 skew quadrupoles, 28 dipoles to correct response matrix and dispersion. The error setting is shown in the table below.

parameter	value
dipole/quadrupole fractional strength error/roll	$1 \times 10^{-4}/0.1mrad$
quadrupole misalignment x/y	$10\mu m/10\mu m$

For such a lattice with strong nonlinearity, LOCO quickly converges to a local minimum and cannot compensate for response matrix error well. While exploration enhanced evolutionary algorithm can jump out of local minima and find a better solution. Through the comparison of dynamic apertures, it can be clearly seen that the solution obtained by the latter is significantly better than that of LOCO.



As can be seen from the figure below, in the early stages of iteration,4 parts of algorithm effectively contributed to the offspring and current pareto front, which demonstrates the effectiveness of exploration enhanced evolutionary algorithm.



Conclusion

In this paper, we propose an orbit response matrix correction method based on exploration enhanced evolutionary algorithm, opening up possibilities for orbit response matrix correction of lattice with strong nonlinearity. An instance implies that combining PSO and MOGA and introducing opposition based learning and Levy flight, can greatly improves the global exploration capability of the algorithm, which can significantly improve the effect of response matrix correction($\chi^2 = 0.75$ compared with LOCO which has $\chi^2 = 6.03$) and obtain better dynamic aperture.

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