Modelling control loops in beam dynamics simulations

Helga Timko
CERN, SY-RF Group

LLRF2023 Workshop, 경주시, 대한민국, 26th October 2023

With many thanks to B. Karlsen-Baeck and the entire BLonD community

Content

Introduction

- BLonD and its use
- Code structure

Longitudinal tracking

- Reference frame
- Equations of motion
- RF manipulations

Collective effects

- Induced voltage
- Multi-turn wake
- Synchrotron radiation

Global control loops

- Embedding them
- CERN models
- RF noise & modulation

Local control loops

- Embedding them
- SPS and LHC models
- Use cases

Closing remarks

- Coupling loops
- Video tutorial

Content

Introduction

- BLonD and its use
- Code structure

Longitudinal tracking

- Reference frame
- Equations of motion
- RF manipulations

Collective effects

- Induced voltage
- Multi-turn wake
- Synchrotron radiation

Global control loops

- Embedding them
- CERN models
- RF noise & modulation

Local control loops

- Embedding them
- SPS and LHC models
- Use cases

Closing remarks

- Coupling loops
- Video tutorial

Introduction

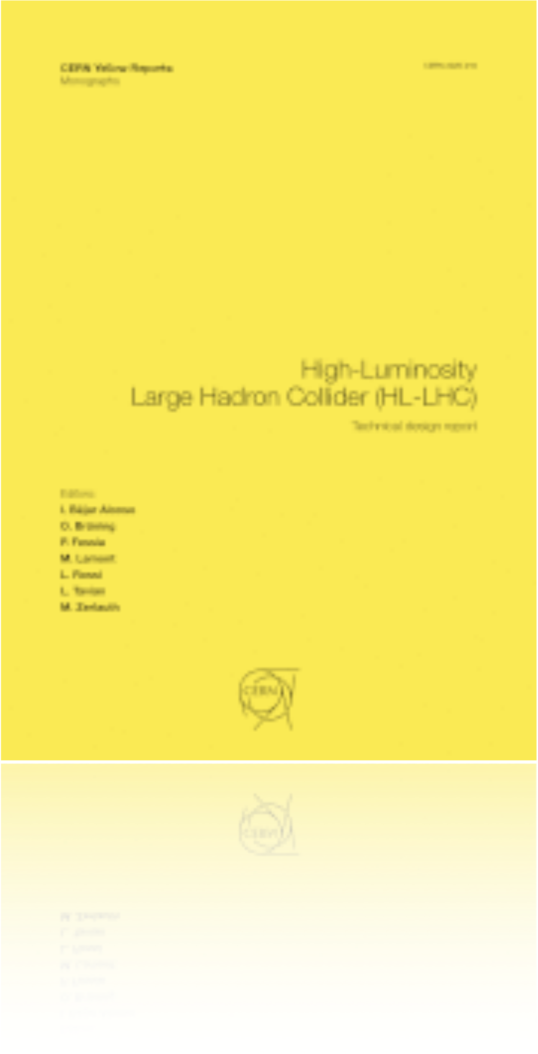
What do we use beam dynamics simulations for?

- Understand unexpected beam behaviour in machine operation
- Design new machines or upgrades to push limitations e.g. in bunch intensity
- Commission modified or new systems

When did we use such simulations?

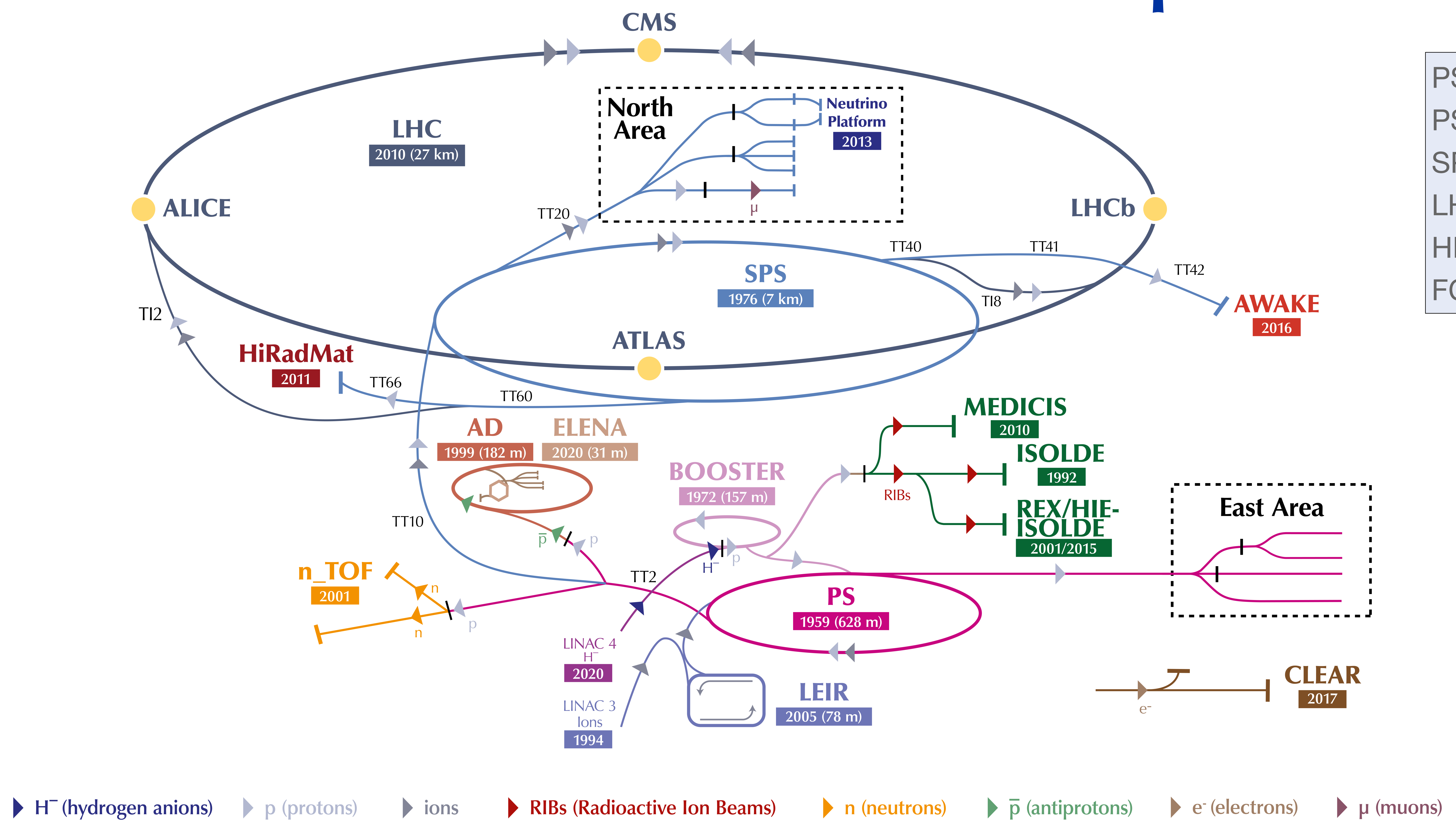
- Make sure we understand our machine
  - E.g. reproduce the measured beam stability threshold in simulations
- LHC Injectors Upgrade (LIU)
  - E.g. SPS: impedance reduction & momentum slip stacking design
- High-Luminosity LHC (HL-LHC)
  - E.g. RF power limitations at injection
- Future Circular Collider (FCC) and Muon Collider studies
  - E.g. Design of RF system, beam parameters, and beam stability margin


LHC INJECTORS UPGRADE
Technical Design Report - Volume I: Protons


CERN Yellow Reports
High-Luminosity
Large Hadron Collider (HL-LHC)
Technical design report


THE EUROPEAN PHYSICAL JOURNAL SPECIAL TOPICS
Regular Article
FCC-hh: The Hadron Collider
Future Circular Collider Conceptual Design Report Volume 3

The CERN accelerator complex



PSB = Proton Synchrotron Booster
PS = Proton Synchrotron
SPS = Super Proton Synchrotron
LHC = Large Hadron Collider
HL-LHC = High-Luminosity LHC
FCC = Future Circular Collider

The BLonD simulation suite

Beam Longitudinal Dynamics (BLonD) simulator [1]

- Models longitudinal beam motion in synchrotrons
- Includes RF specific items
  - Longitudinal machine impedance
  - RF manipulations
  - RF noise/modulation
  - LLRF loops
- Development mainly driven by CERN needs
  - Pushing the machines to their design limit
  - Effort for understand beam motion with the present LLRF systems
  - In the future, **we can design LLRF systems based on beam motion**

There is a growing user community outside CERN

- GSI, KIT, KEK, J-PARC, HIAF, Fermilab, Jefferson Lab, …

Getting started with BLonD simulations

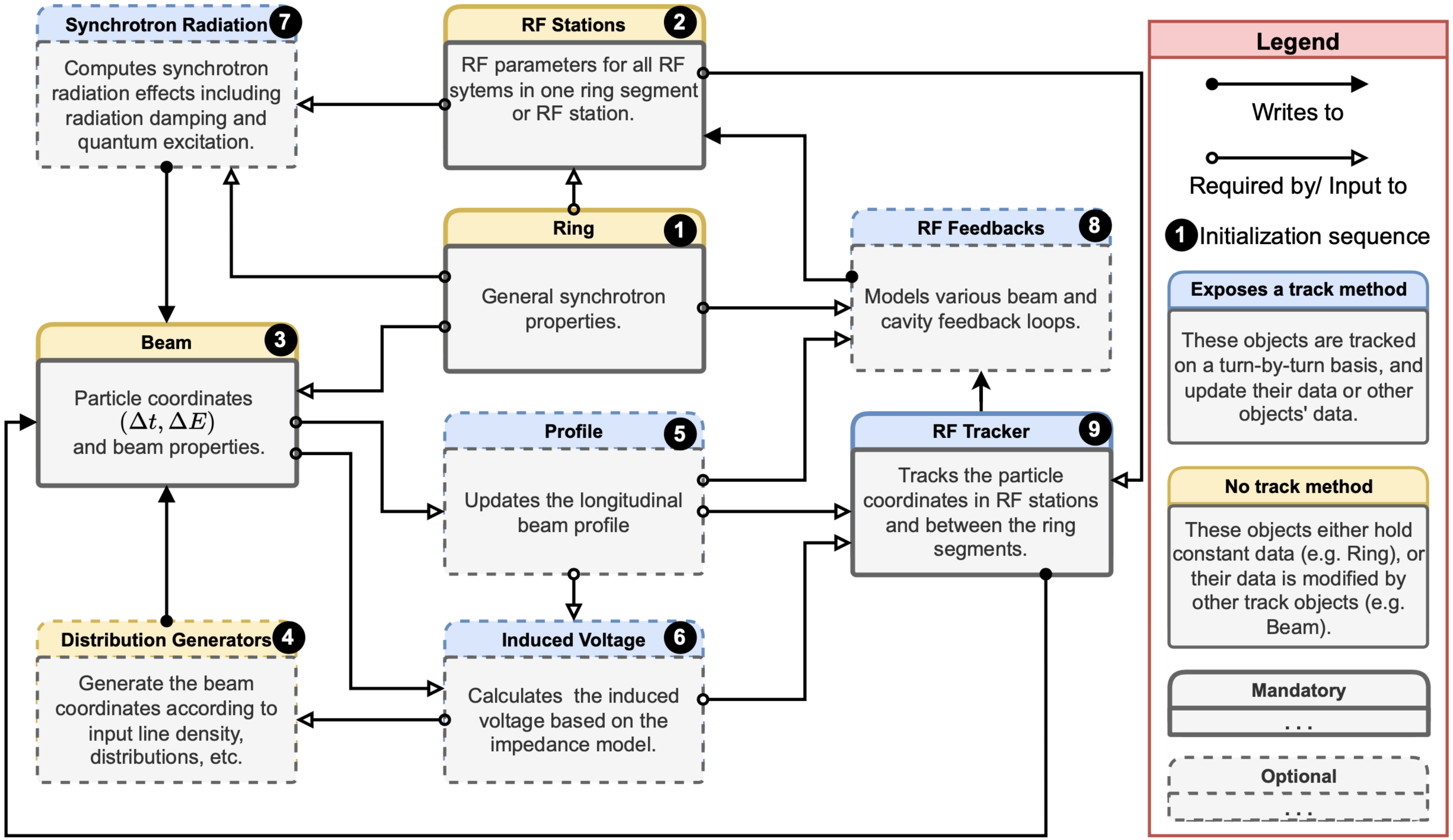
- Code repositories:
  - Inside CERN: https://gitlab.cern.ch/blond/BLonD
  - Outside CERN: https://github.com/blond-admin/BLonD
- Documentation:
  - Implementation and variables https://blond-code.docs.cern.ch/

[1] H. Timko *et al.*: *'Beam Longitudinal Dynamics Simulation Studies'*, Phys. Rev. Accel. Beams, to be published, 2023.

Code structure

Modular, flexible structure

- Object-oriented Python code with C++/CUDA computational kernels
  - Optional usage of most modules
  - Defined by the user in their input file



BLonD code structure with mandatory (solid line) and optional (dashed line) objects

Courtesy of K. Iliakis

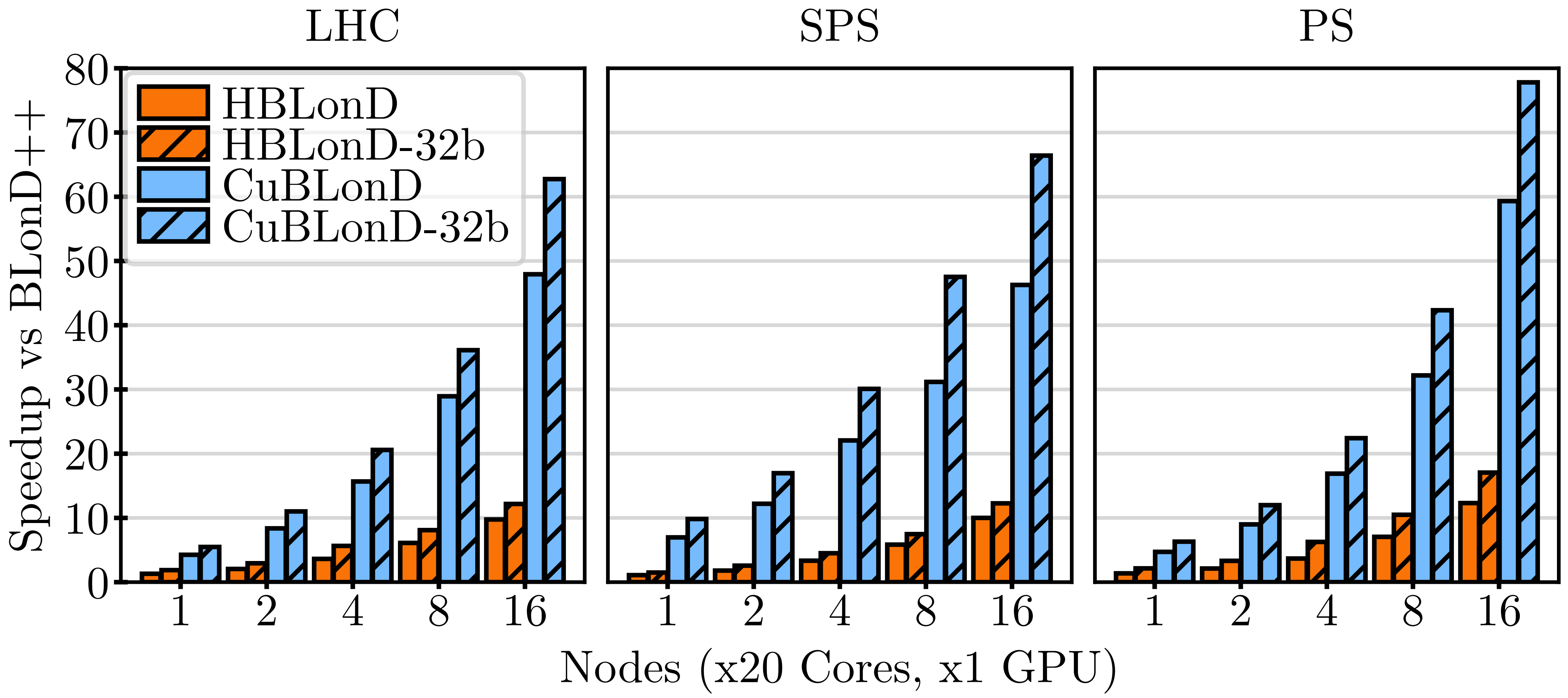
Optimisation & benchmarks

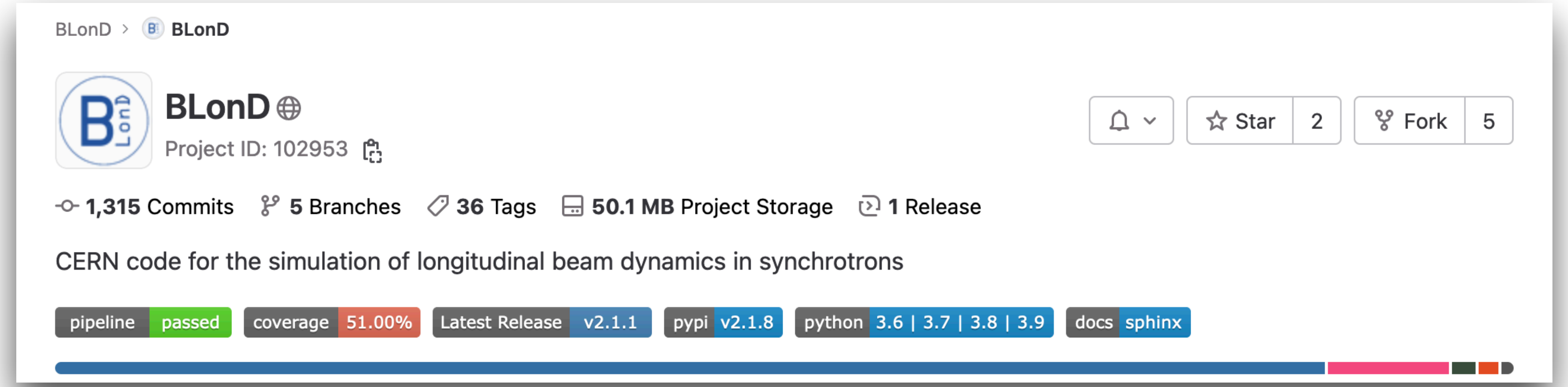
BLonD applies high-performance computing methods [2]

- **BLonD++** optimised single-core version
  - Python & C++
- **HBLonD** distributed MPI version
  - x10 speed-up
- **CuBLonD** GPU-accelerated version
  - x10-17 speed-up, CUDA and CuPy

Trust in the simulator based on its testing history

- Unittests, benchmarks, code-to-code comparison
  - Only deploy versions that developers had time to test
  - Use continuous integration tools





Top: speed-up of HBLonD and CuBLonD vs BLonD++

Bottom: continuous integration tool active for BLonD

Courtesy of K. Iliakis

[2] K. Iliakis: ‘Large-scale software optimization and micro-architectural specialization for accelerated high-performance computing’, PhD Thesis, 2022.

Content

Introduction

- BLonD and its use
- Code structure

Longitudinal tracking

- Reference frame
- Equations of motion
- RF manipulations

Collective effects

- Induced voltage
- Multi-turn wake
- Synchrotron radiation

Global control loops

- Embedding them
- CERN models
- RF noise & modulation

Local control loops

- Embedding them
- SPS and LHC models
- Use cases

Closing remarks

- Coupling loops
- Video tutorial

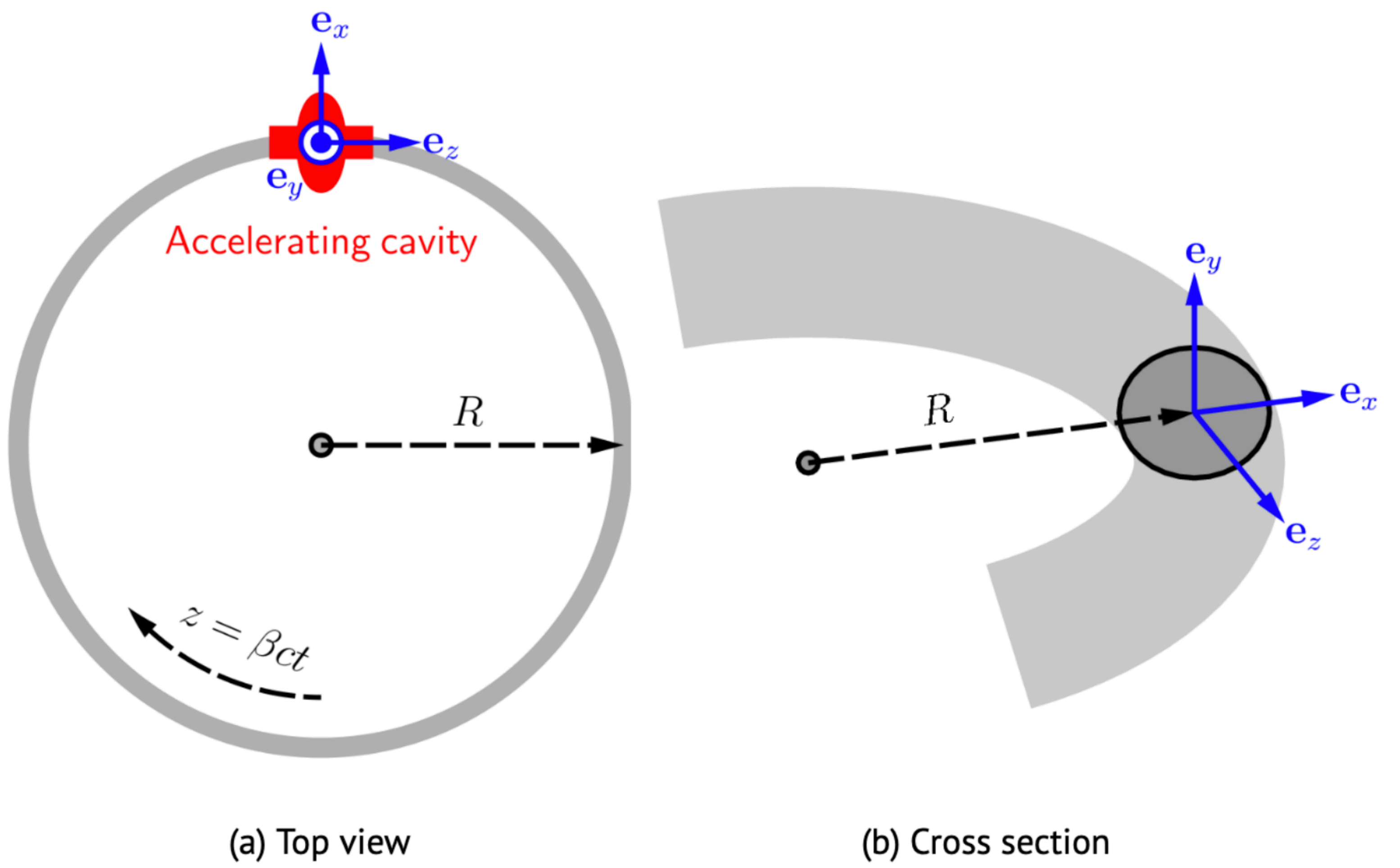
Reference frame

- Longitudinal modelling only
  - Slippage factor represents transverse lattice
  - User input: orbit & energy/momentum/B-field $p_{d,(n)} = |q|\rho B_{d,(n)}$
- Turn-by-turn mapping
  - RF station: discrete energy kick from passage through the (multi-harmonic) cavitie(s)
  - Arc segments: drift and other interactions (optional)
  - Sub-cycling of one turn possible; use case e.g. FCC
- Snapshot of the system at first RF station
  - Reference or design "clock" is

$$t_{d,(0)} \equiv 0 \quad \text{and} \quad t_{d,(n)} \equiv \sum_{k=1}^{n} T_{\mathrm{rev},(k)} \text{ for } n \geq 1$$

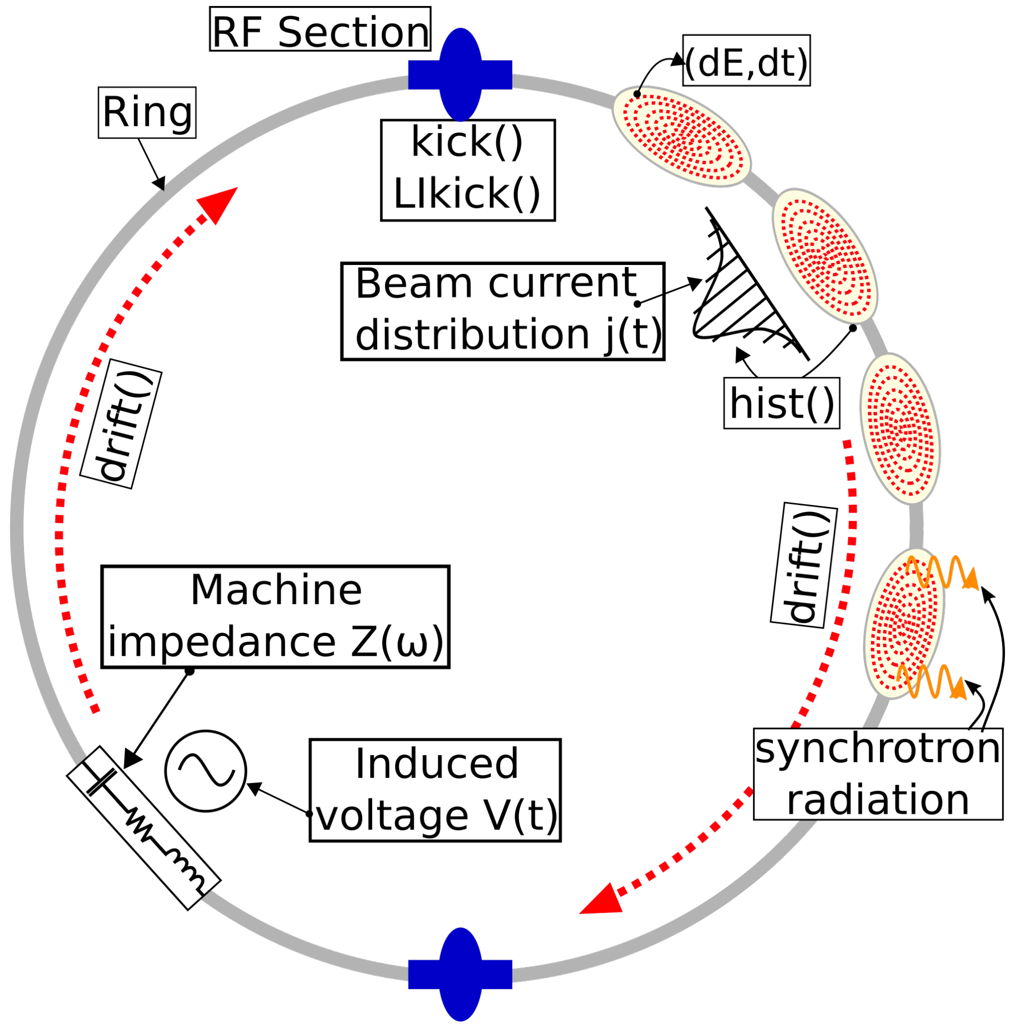
  - Defined by

$$T_{\mathrm{rev},(n)} = \frac{2\pi R_d}{\beta_{d,(n)} c}$$



Longitudinal and transverse coordinates in a synchrotron

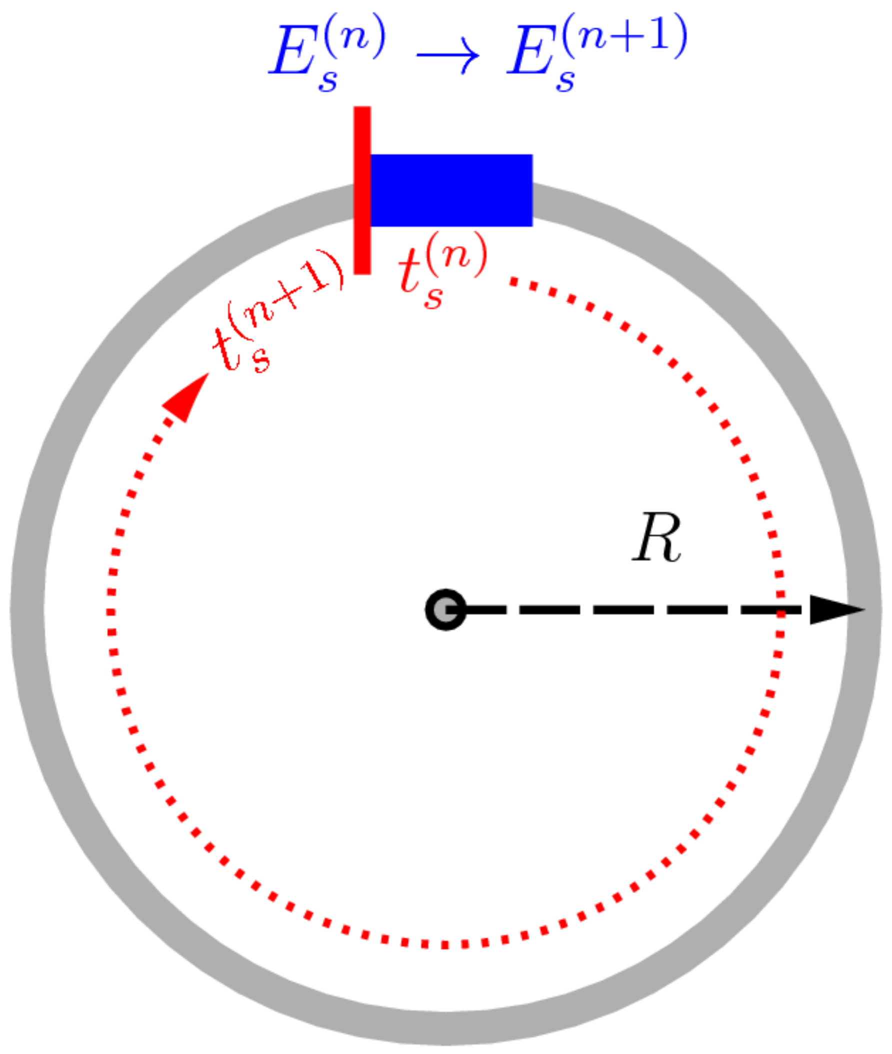
β_d	relativistic beta from design momentum
ρ	bending radius of magnets
B_d	design dipole magnetic field
c	speed of light
R_d	design orbit
q	particle charge
T_{rev}	revolution period



Example of a synchrotron model

Courtesy of K. Iliakis

Equations of motion



Convention used in BLonD: first kick, then drift

- Time and energy deviations from reference particle: $\Delta t_{(n)} \equiv t_{(n)} - t_{d,(n)}$ and $\Delta E_{(n)} \equiv E_{(n)} - E_{d,(n)}$
- Kick equation

$$\Delta E_{(n+1)} = \Delta E_{(n)} + \sum_{k=1}^{n_{\mathrm{rf}}} qV_{k,(n)} \sin(\omega_{\mathrm{rf},k,(n)}\Delta t_{(n)} + \varphi_{\mathrm{rf},k,(n)}) - (E_{d,(n+1)} - E_{d,(n)}) + E_{\mathrm{other},(n)}$$

Multiple RF systems in one location

Change of design energy (magnetic field)

All other effects on energy (e.g. induced voltage)

- Drift equation

$$\Delta t_{(n+1)} = \Delta t_{(n)} + T_{\mathrm{rev},(n+1)} \left[\left(1 + \alpha_{0,(n+1)}\delta_{(n+1)} + \alpha_{1,(n+1)}\delta_{(n+1)}^2 + \alpha_{2,(n+1)}\delta_{(n+1)}^3 \right) \frac{1 + \frac{\Delta E_{(n+1)}}{E_{d,(n+1)}}}{1 + \delta_{(n+1)}} - 1 \right]$$

Symplecticity

- EOMs are **symplectic** and therefore **area preserving**

$$\left.\begin{aligned} \Delta t_{(n+1)} &= \Delta t_{(n)} + f\left(\Delta E_{(n+1)}(\Delta t_{(n)}, \Delta E_{(n)})\right) \\ \Delta E_{(n+1)} &= \Delta E_{(n)} + g\left(\Delta t_{(n)}\right) \end{aligned}\right\} \Rightarrow \mathscr{M}\mathscr{S}\mathscr{M}^{\mathrm{T}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \mathscr{S}$$

$$\mathscr{M}_{ik} \equiv \frac{\partial y_i}{\partial x_k}$$

$\delta = \dfrac{\Delta p}{p_d} \simeq \dfrac{\Delta E}{\beta_d^2 E_d}$ relative momentum offset

$\eta = \eta(\delta) = \eta_0 + \eta_1\delta + \eta_2\delta^2 + \ldots$ slippage factor

φ_{rf} RF voltage phase at particle arrival

ω_{rf} RF frequency

V RF voltage amplitude

Momentum-compaction vs slippage factor:

$$\eta_0 = \alpha_0 - \frac{1}{\gamma_d^2} = \frac{1}{\gamma_T^2} - \frac{1}{\gamma_d^2}$$

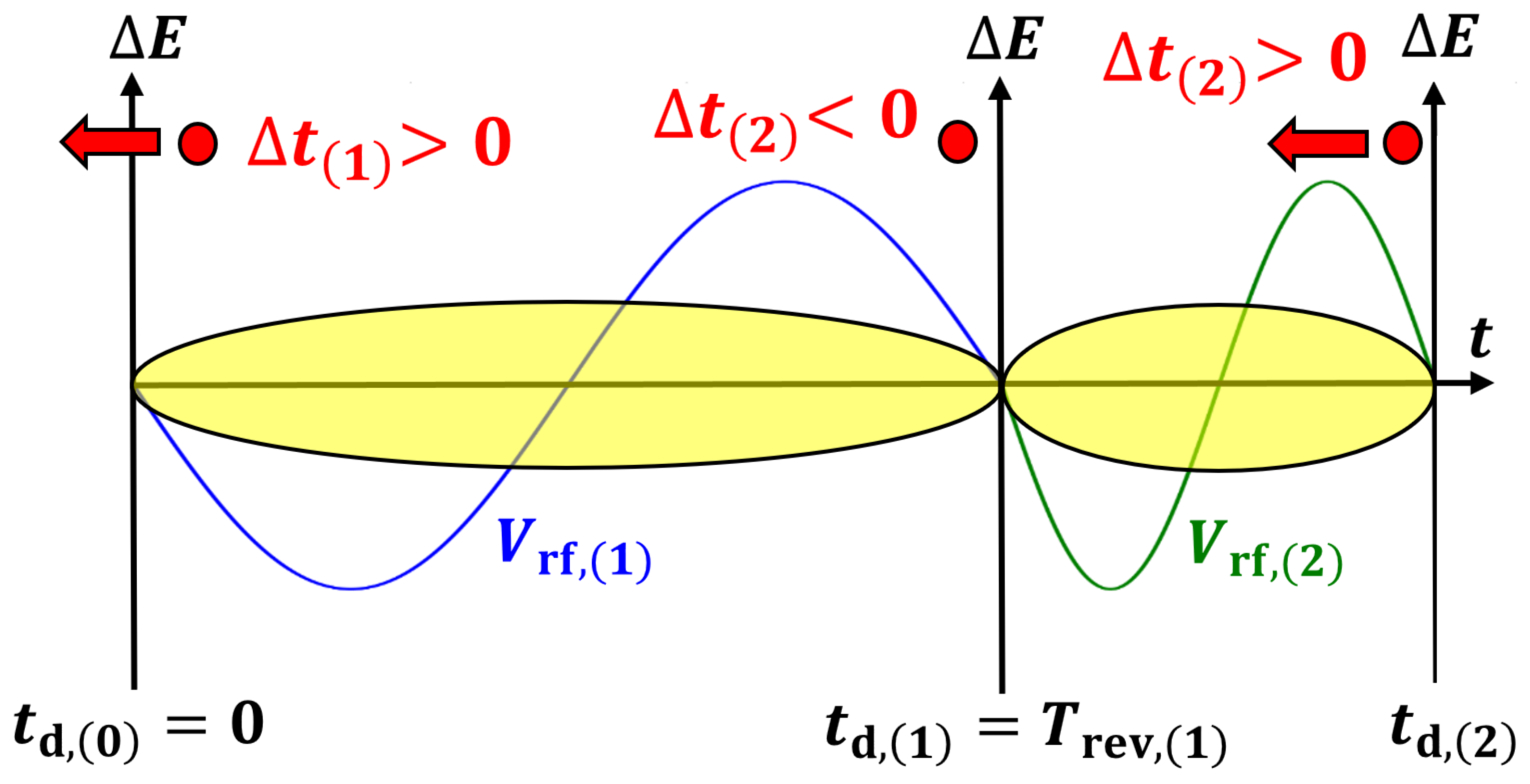
$$\eta_1 = \frac{3\beta_d^2}{2\gamma_d^2} + \alpha_1 - \alpha_0\eta_0$$

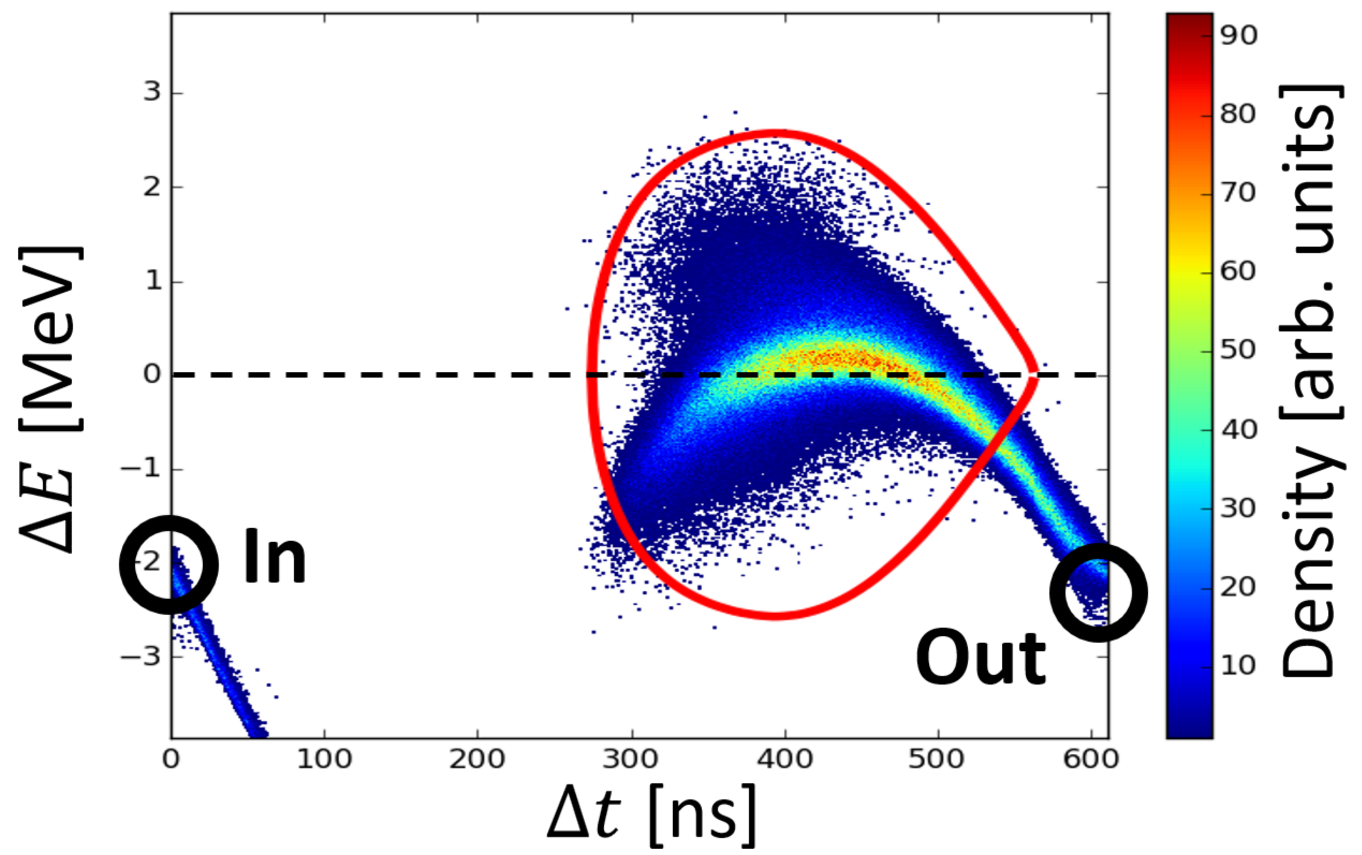
$$\eta_2 = -\frac{\beta_d^2(5\beta_d^2 - 1)}{2\gamma_d^2} + \alpha_2 - 2\alpha_0\alpha_1 + \frac{\alpha_1}{\gamma_d^2} + \alpha_0^2\eta_0 - \frac{3\beta_d^2\alpha_0}{2\gamma_d^2}$$

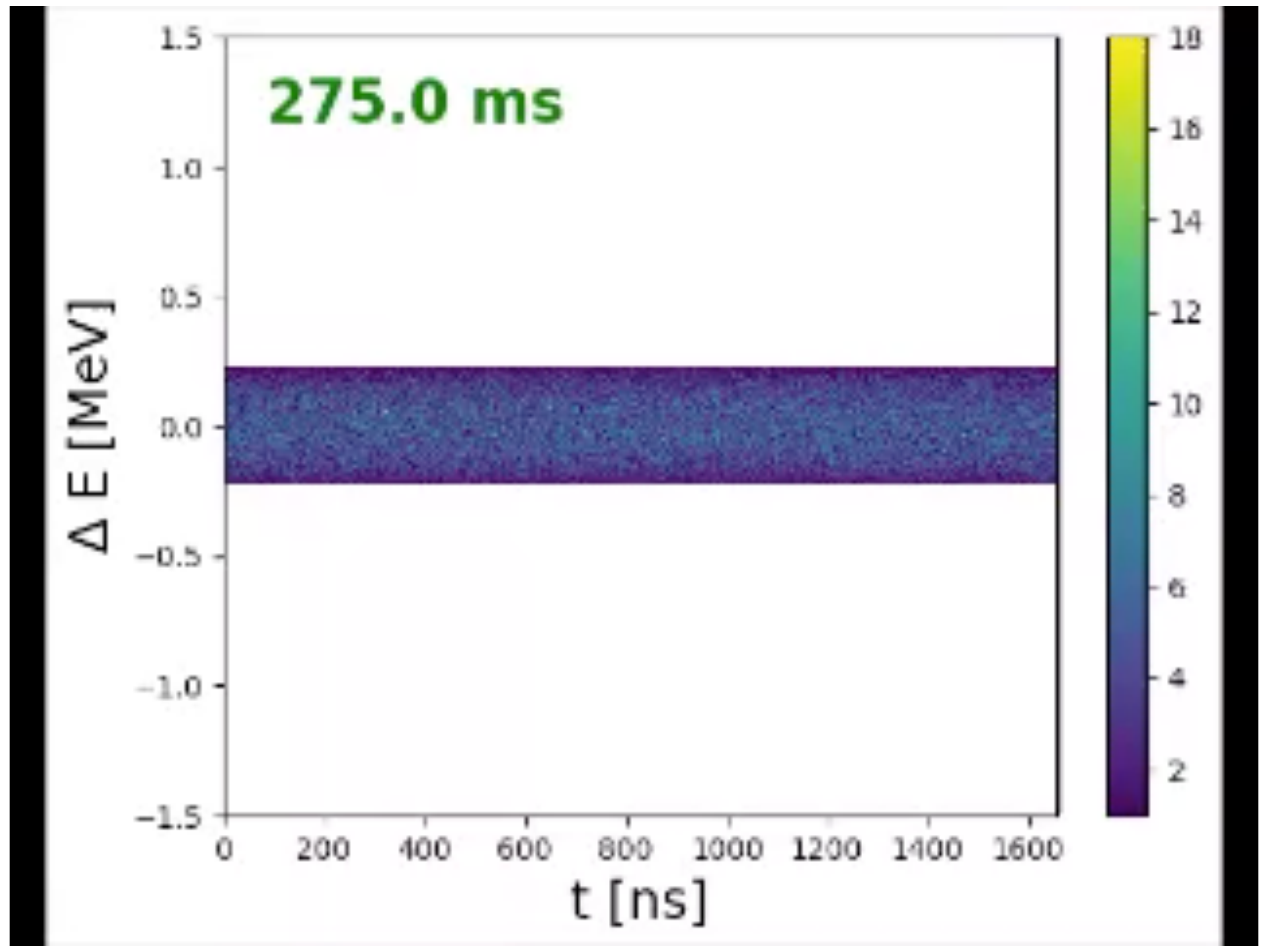
Periodicity

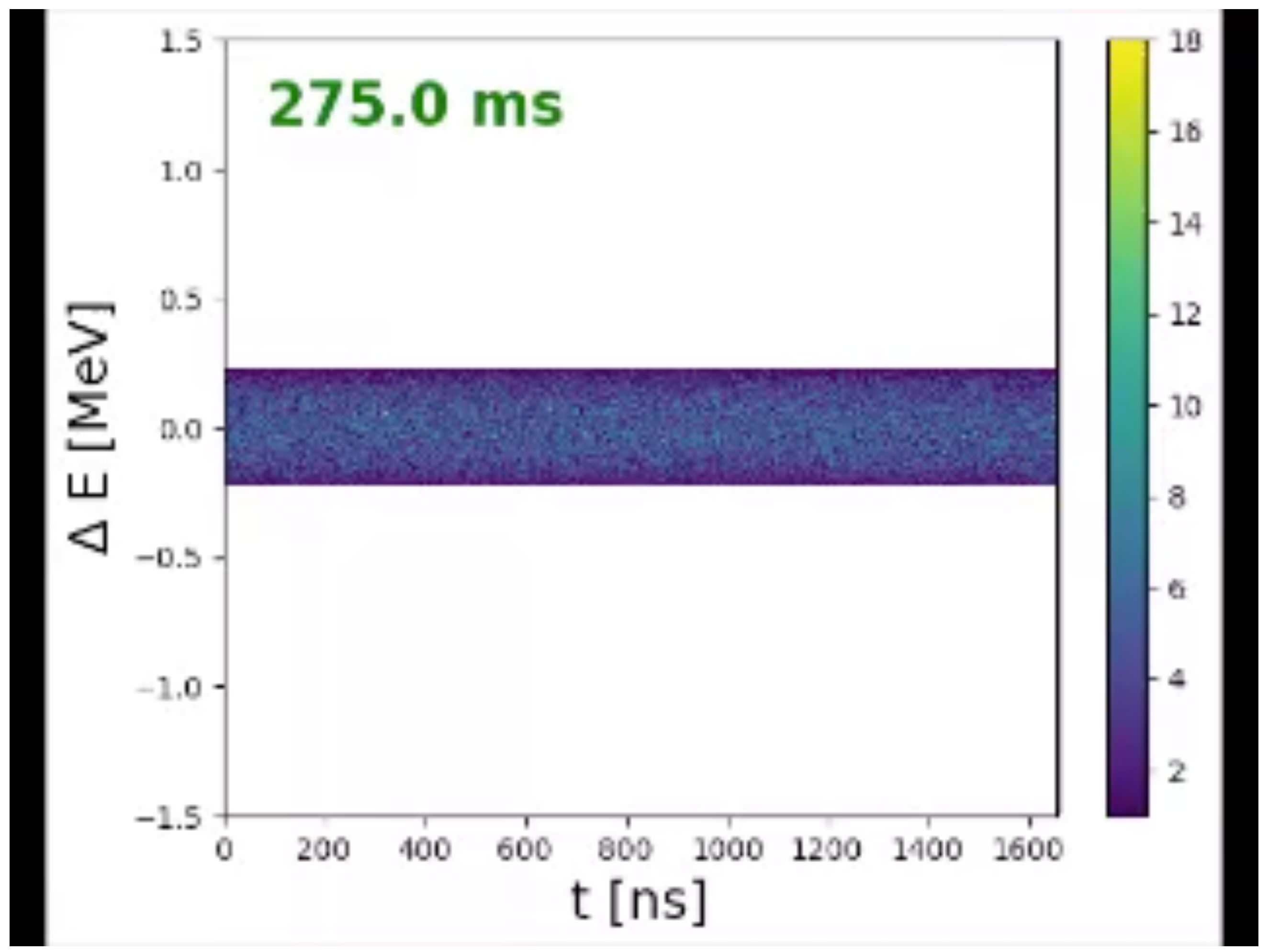
According to the turn-by-turn discretisation, the time coordinate of any particle should be $(0, T_{\mathrm{rev},(n)})$

- In some cases, the particles might cross these time boundaries
  - E.g. small h machines, full machine, debunching beam, etc.
- Optional periodic boundary conditions to the coordinate frame:
  - Particles that have $\Delta t_{(n)} < 0$ have to be tracked twice
  - Particles that have $\Delta t_{(n)} > T_{\mathrm{rev},(n)}$ have to be put on hold for a turn









PSB injection on acceleration ramp

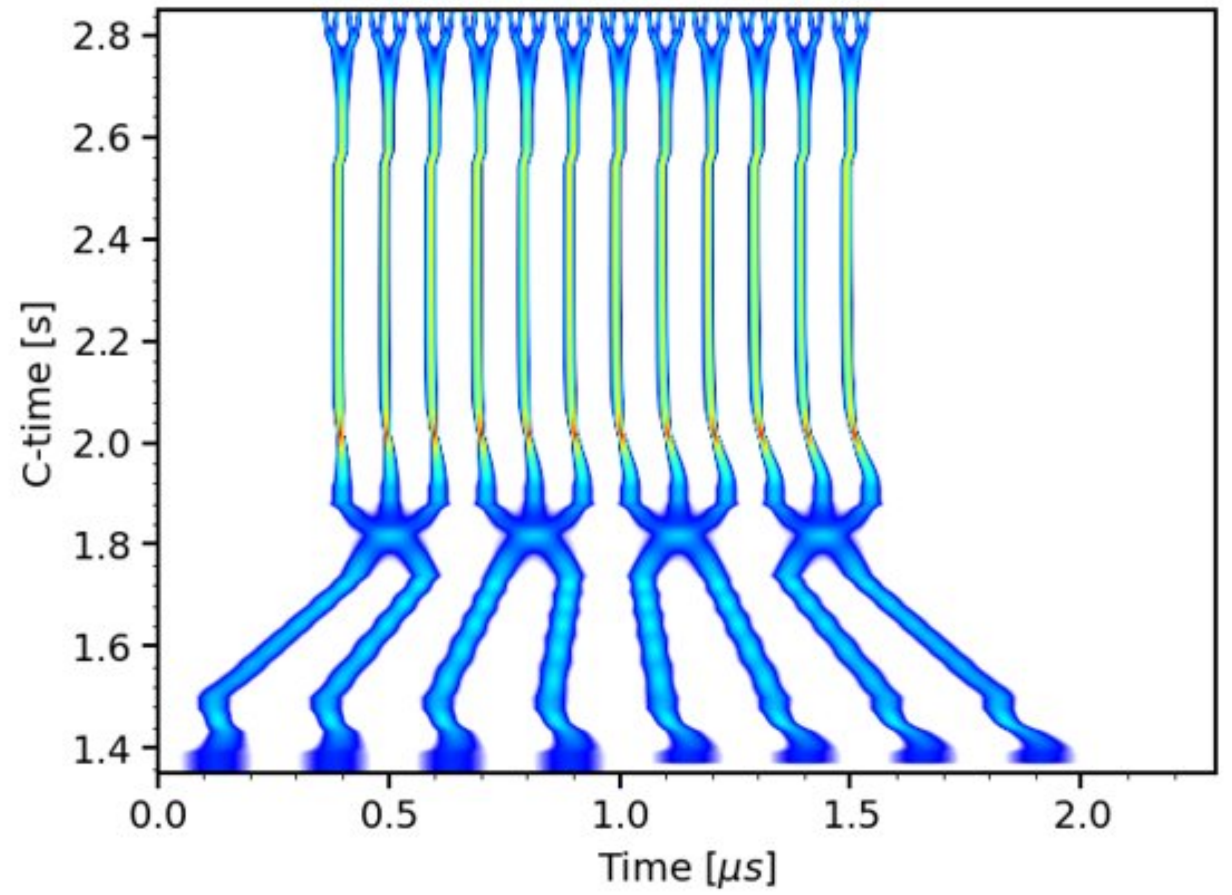
Top: RF locked to the B-field
Bottom: fixed RF frequency (bottom)

Courtesy of D. Quartullo

RF manipulations

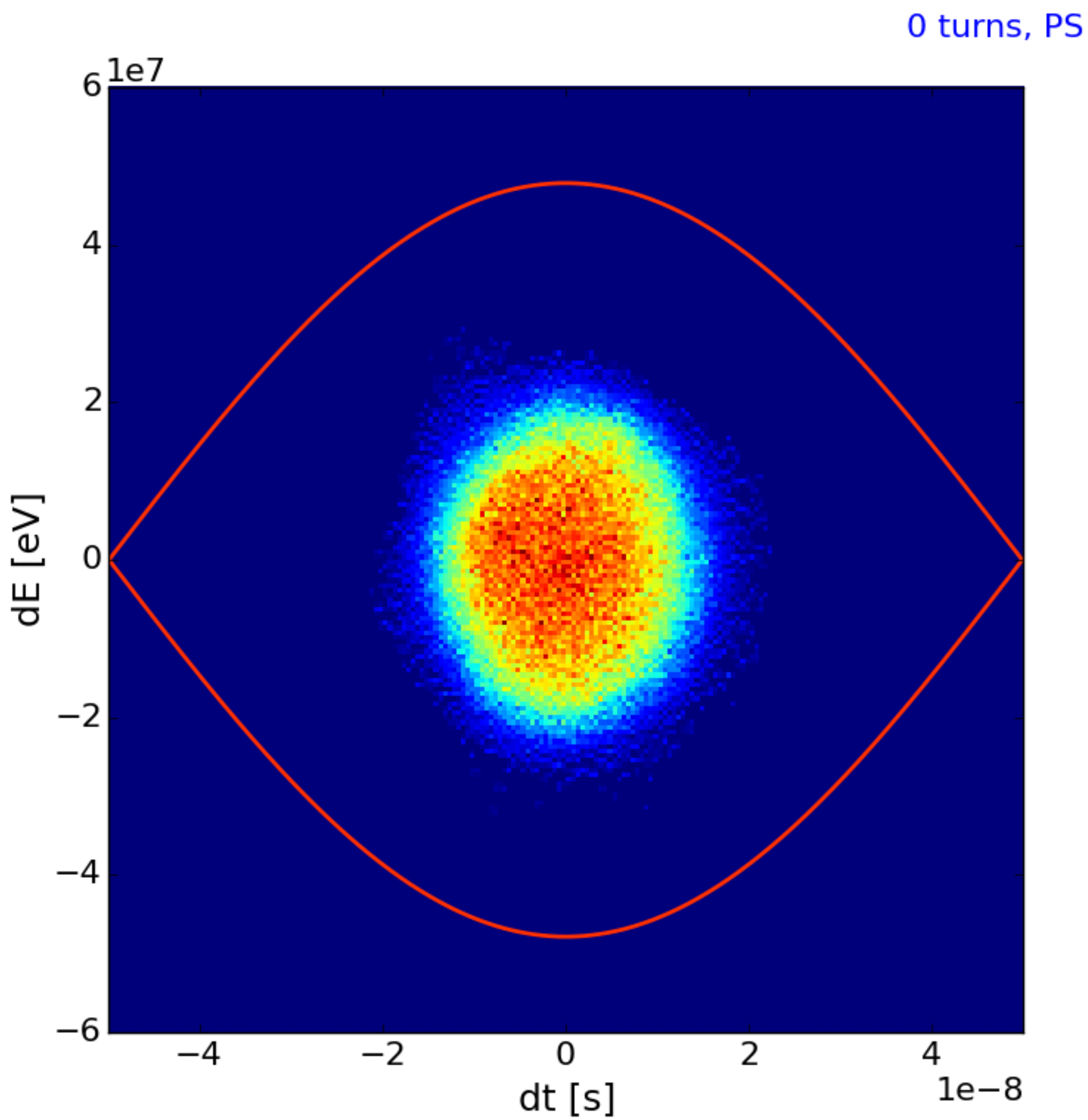
Multi-harmonic systems

- RF gymnastics defined by the RF voltage programme of several harmonics
  - Splitting or merging: adiabatic change of higher-harmonic RF voltage
  - Bunch rotation: non-adiabatic increase the RF voltage and recapture bunch after $\frac{1}{4}T_s$
  - Batch compression: change of RF harmonic



Simulated bunch profiles of the Batch Compression, Merging and Splitting (BCMS) scheme in the CERN Proton Synchrotron (PS)

Courtesy of A. Lasheen

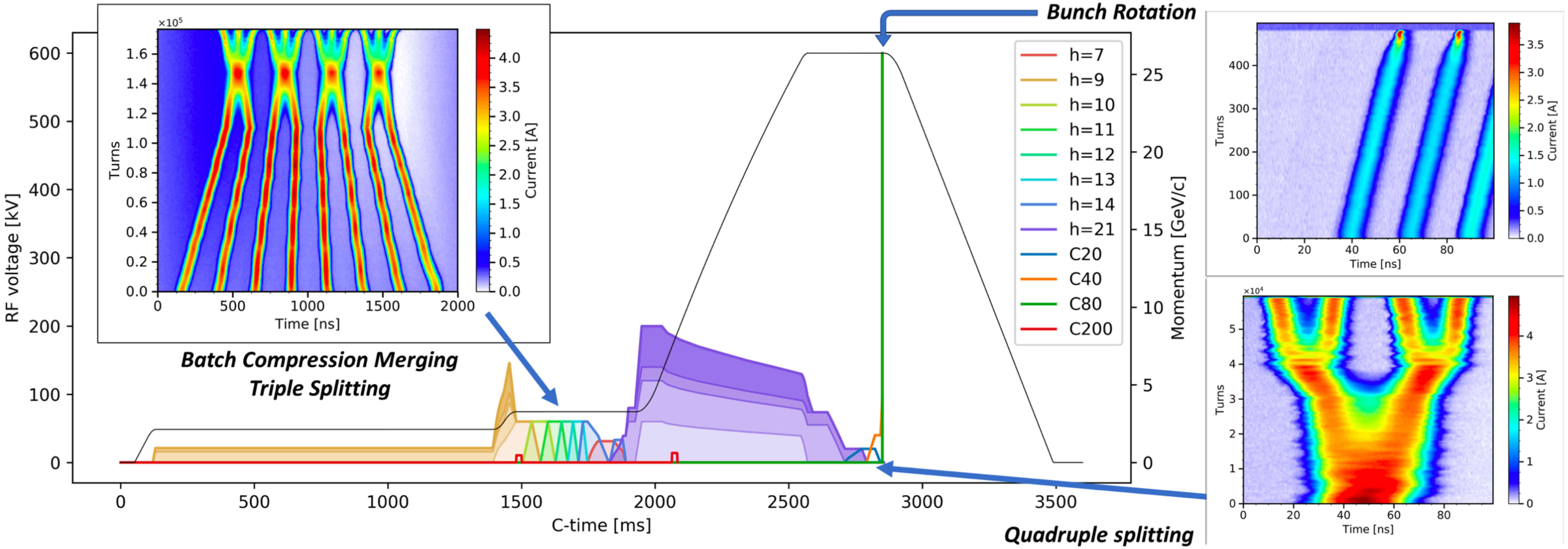


Double splitting and bunch rotation in the PS, then capture in the Super Proton Synchrotron (SPS)

RF manipulations

PS end-to-end simulations

- RF manipulations, controlled emittance blow-up, transition crossing, collective effects
  - Possible with multi-CPU (MPI) version of BLonD



PS momentum and voltage programmes for the BCMS cycle, with the bunch profile evolution as simulated in BLonD, from [3]

[3] A. Lasheen, H. Damerau, and K. Iliakis: 'End-to-end longitudinal simulations in the CERN PS', Proc. HB2021, MOP17, Batavia, USA, 2021.

Momentum slip stacking

Slip stack batches to reduce bunch spacing or double intensity [4-6]

- Capture two beams w/ two RF systems of slightly different frequency

$$V_{\mathrm{rf}} = V_{\mathrm{rf},1} \sin(\omega_{\mathrm{rf},1} t + \varphi_{\mathrm{rf},1}) + V_{\mathrm{rf},2} \sin(\omega_{\mathrm{rf},2} t - \varphi_{\mathrm{rf},2})$$

- The small frequency difference results in a phase error

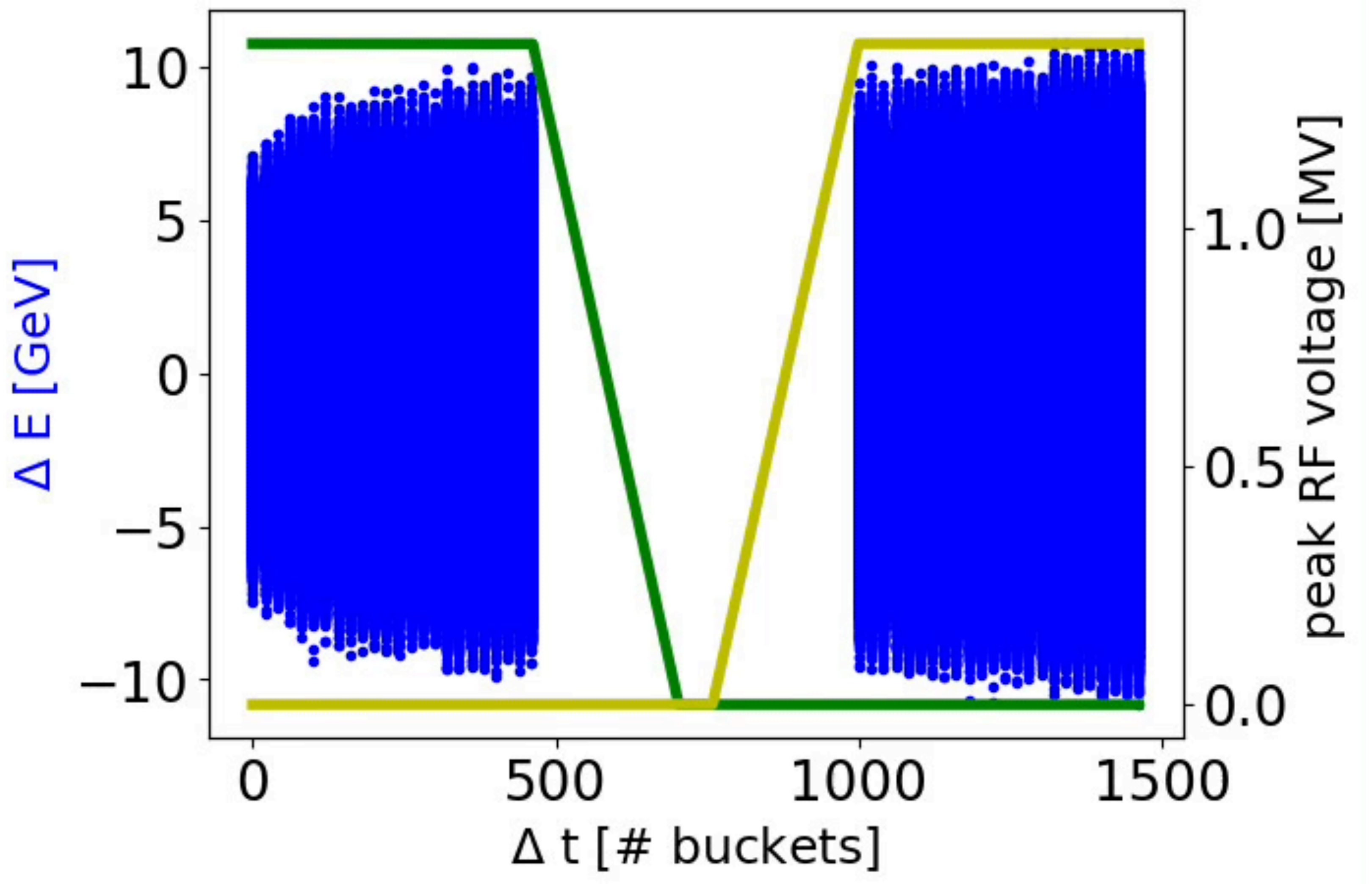
$$\Delta\varphi_{\mathrm{rf}} = \frac{2\pi h \Delta\omega_{\mathrm{rf}}}{\omega_{\mathrm{rf,d}}}$$

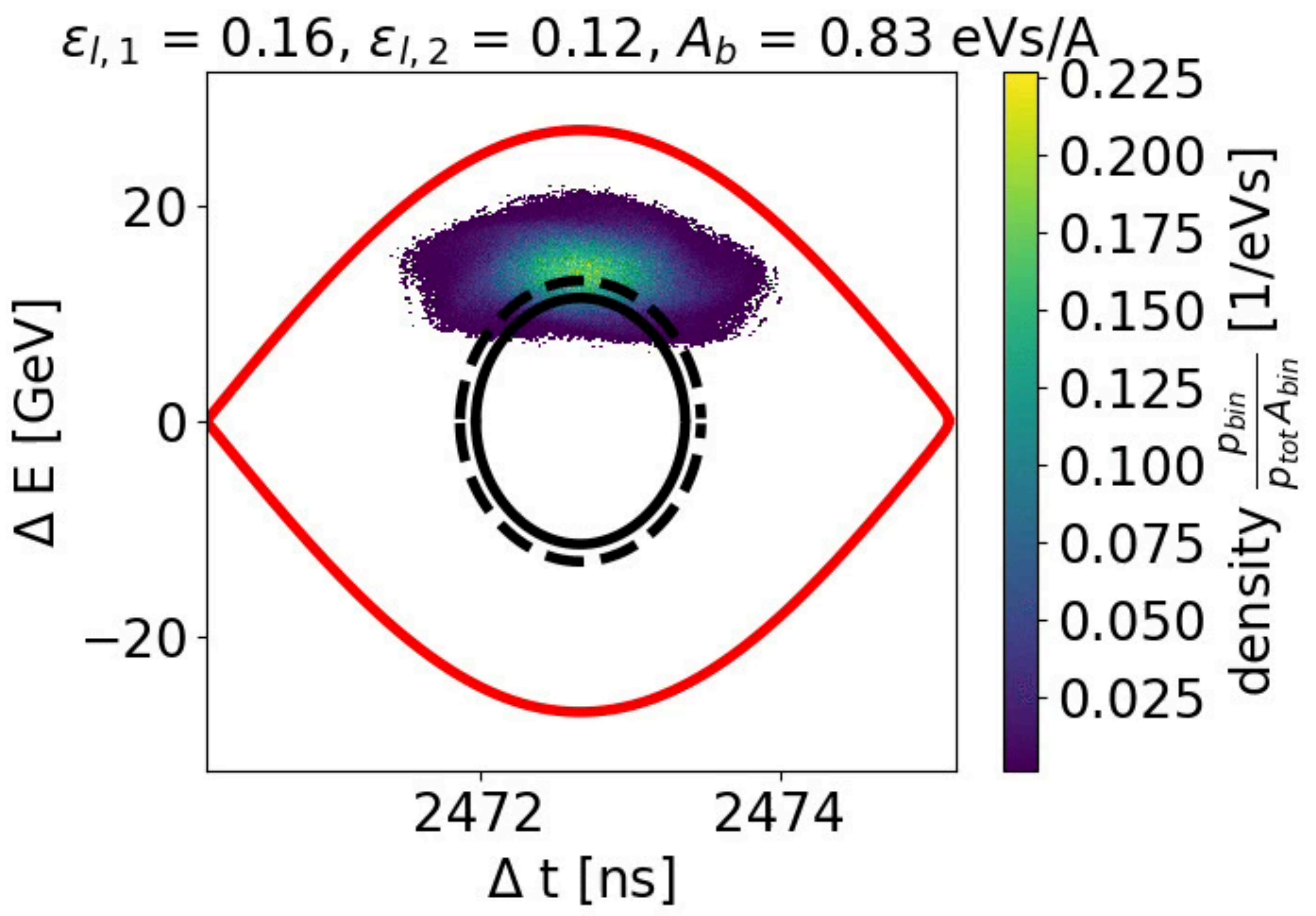
- Which at constant magnetic field translates to a slippage (drift) of

$$\frac{\Delta\omega_{\mathrm{rf}}}{\omega_{\mathrm{rf,d}}} = -\eta_0 \frac{\Delta p}{p_{\mathrm{d}}}$$

  - The two beams slip inside the same beam pipe in opposite directions

- At the desired longitudinal position, the two beams are recaptured with a higher RF voltage, at the common frequency





Top: Slip stacking 2 batches of 24 bunches
Bottom: re-capture and ramp the first bunch

Courtesy of D. Quartullo

[4] J. P. Burnet *et al.:* 'Fifty years of the CERN Proton Synchrotron: Volume 1', CERN-2011-004, 2011.
[5] J. Coupard, *et al.:* 'LHC Injectors Upgrade', Technical Design Report, CERN-ACC-2016-0041, 2016.
[6] G. Hagmann *et al.* : 'CERN SPS low-level RF architecture & implementation', Presentation at LLRF'22 workshop, Brugg-Windisch, Switzerland.

Content

Beam-induced voltage and impedance

Couples the motion of particles, enters the E_{other} term in the EOMs

- Discrete frequency-domain implementation in BLonD

  $$V_{\text{ind}}[k] = -q N_p \, \text{IFFT}\,(Z[i] \; \Lambda[i]) = -q N_p \, \text{IFFT}\left(Z[i] \; \text{FFT}(\lambda[m])\right)$$

- In time domain, we use the circular convolution theorem

  $$V_{\text{ind}}[k] = -q N_p \, \text{IFFT}\,\{\text{FFT}\,(W[n]) \; \text{FFT}\,(\lambda[n])\}$$

  - A discrete convolution would be too **time consuming**
  - Must carefully **zero-pad** to result in a linear convolution

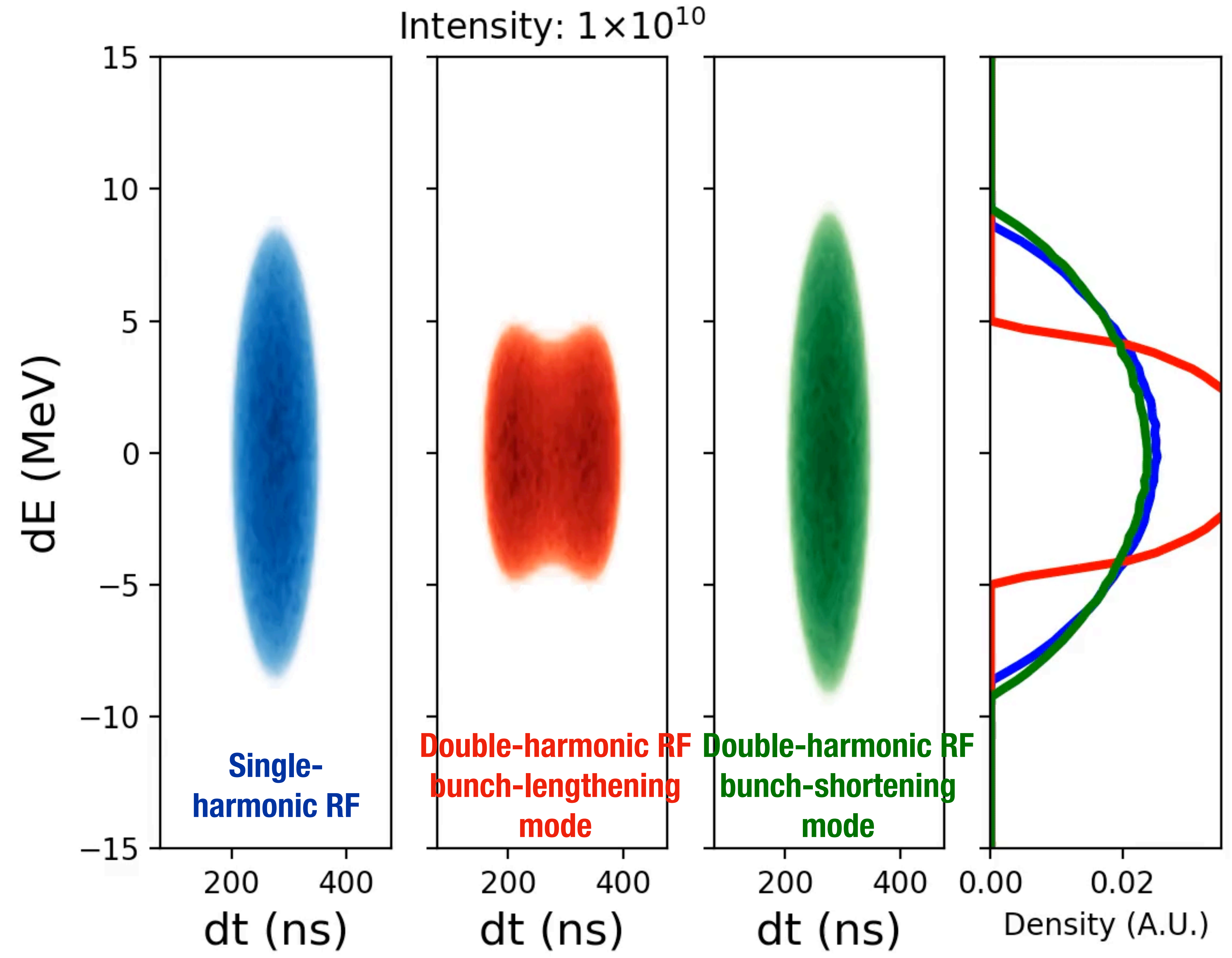
$\sum_i \lambda[i] \, T_s = 1$	beam profile, normalised to 1
$\Lambda[i] = \text{FFT}(\lambda[m])$	beam spectrum
N_p	number of real particles in the beam
$W[i]$	wake function
Z	machine impedance

Available impedance sources

- Can mix different impedance sources, in frequency and time domain
  - Impedance table, e.g. from CST
  - Resonator
  - Travelling wave cavity
  - Resistive-wall impedance
  - Constant $\mathfrak{I}(Z/n)$ e.g. for LLD or SC

Collective effects

Microwave instability in PSB

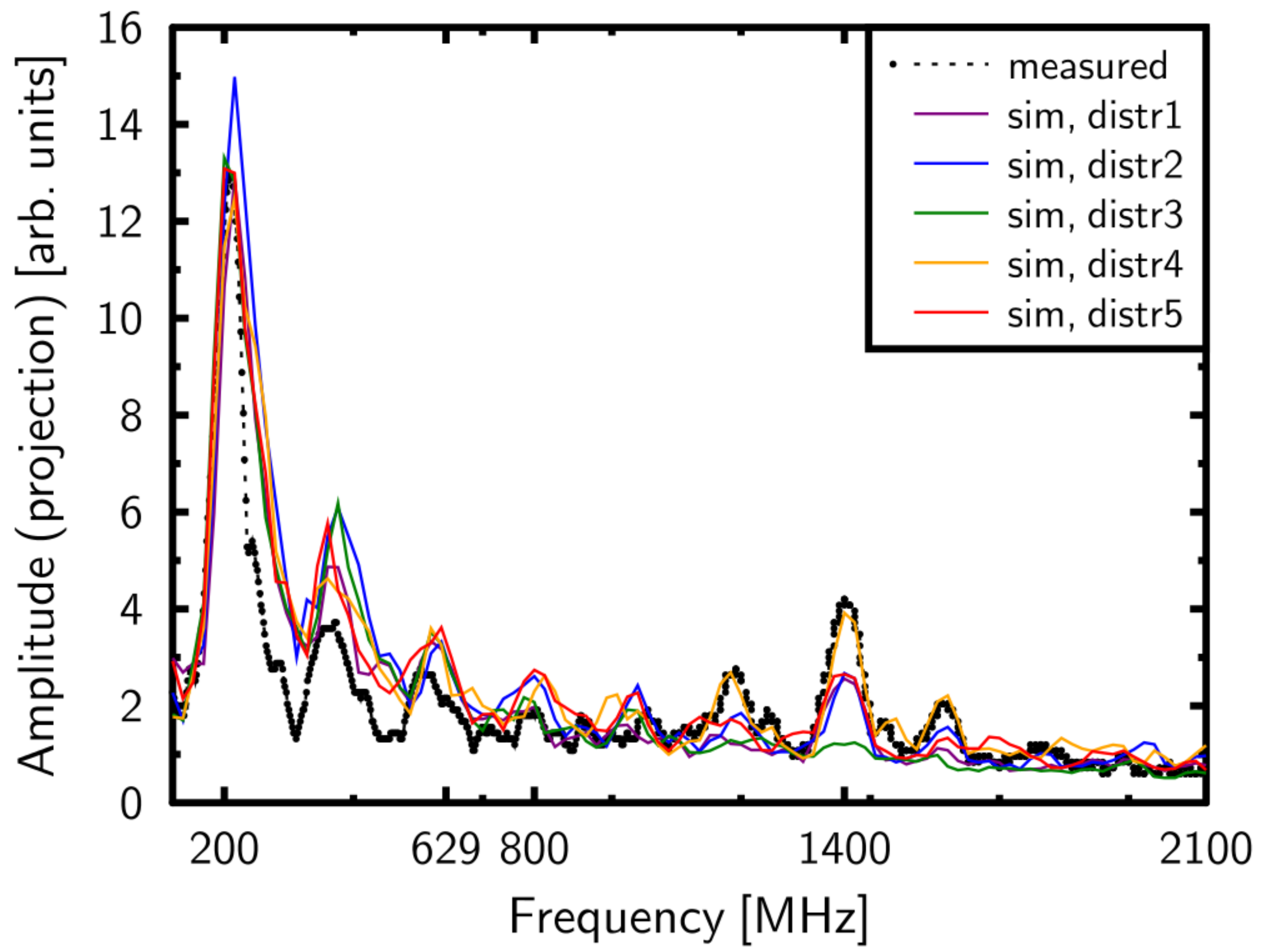


Bunch intensity threshold for microwave instability in the PSB

Courtesy of S. Albright

SPS impedance model refinement

- Identification of flange impedance for LIU-SPS

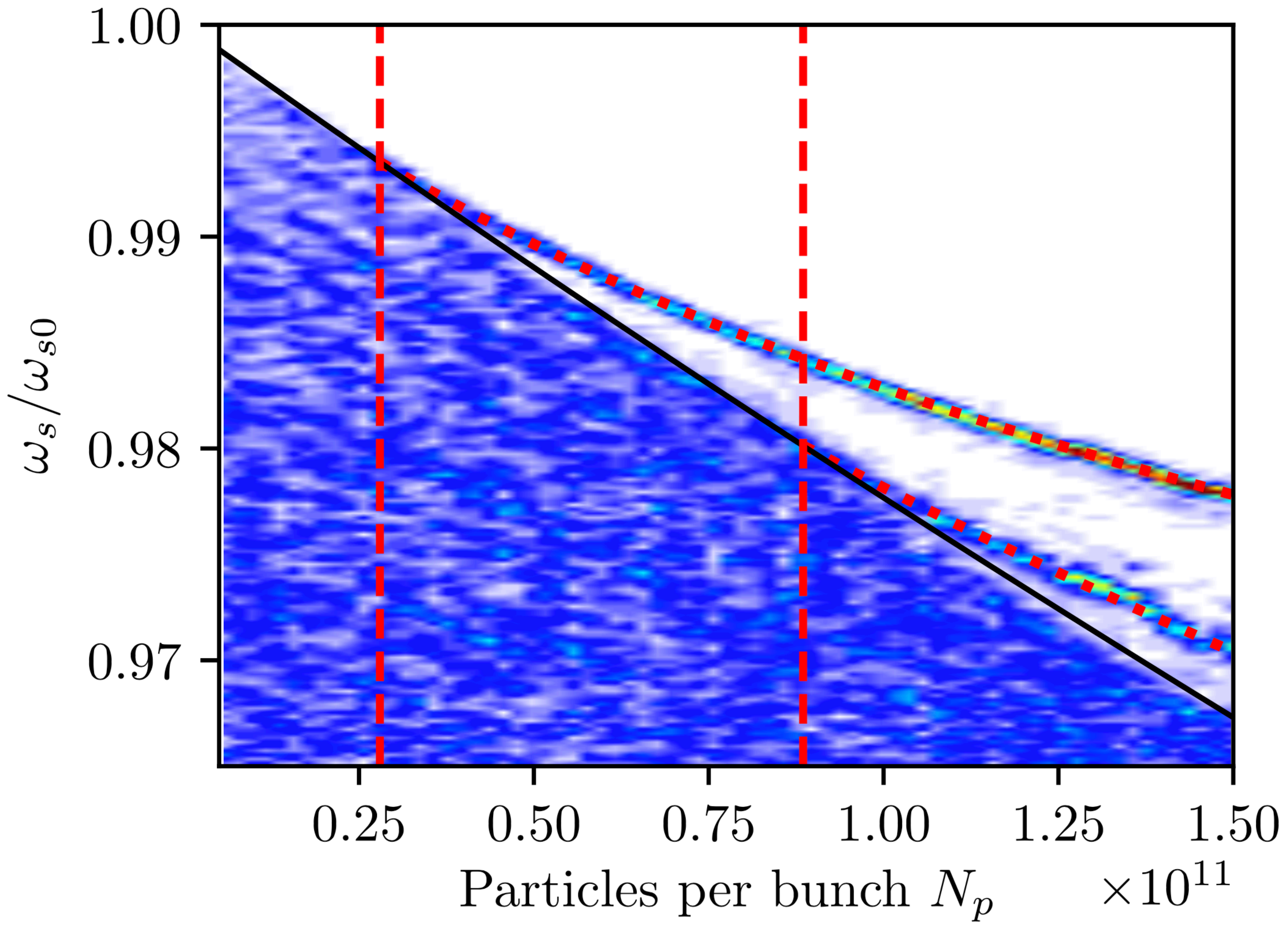


Spectrum of long debunching bunches at SPS injection; a 1.4 GHz line emerges [7]

[7] E. Shaposhnikova *et al.*: 'Identification of high-frequency resonant impedance in the CERN SPS', CERN Note CERN-ACC-2014-0099, 2014

Collective effects and multi-turn wake

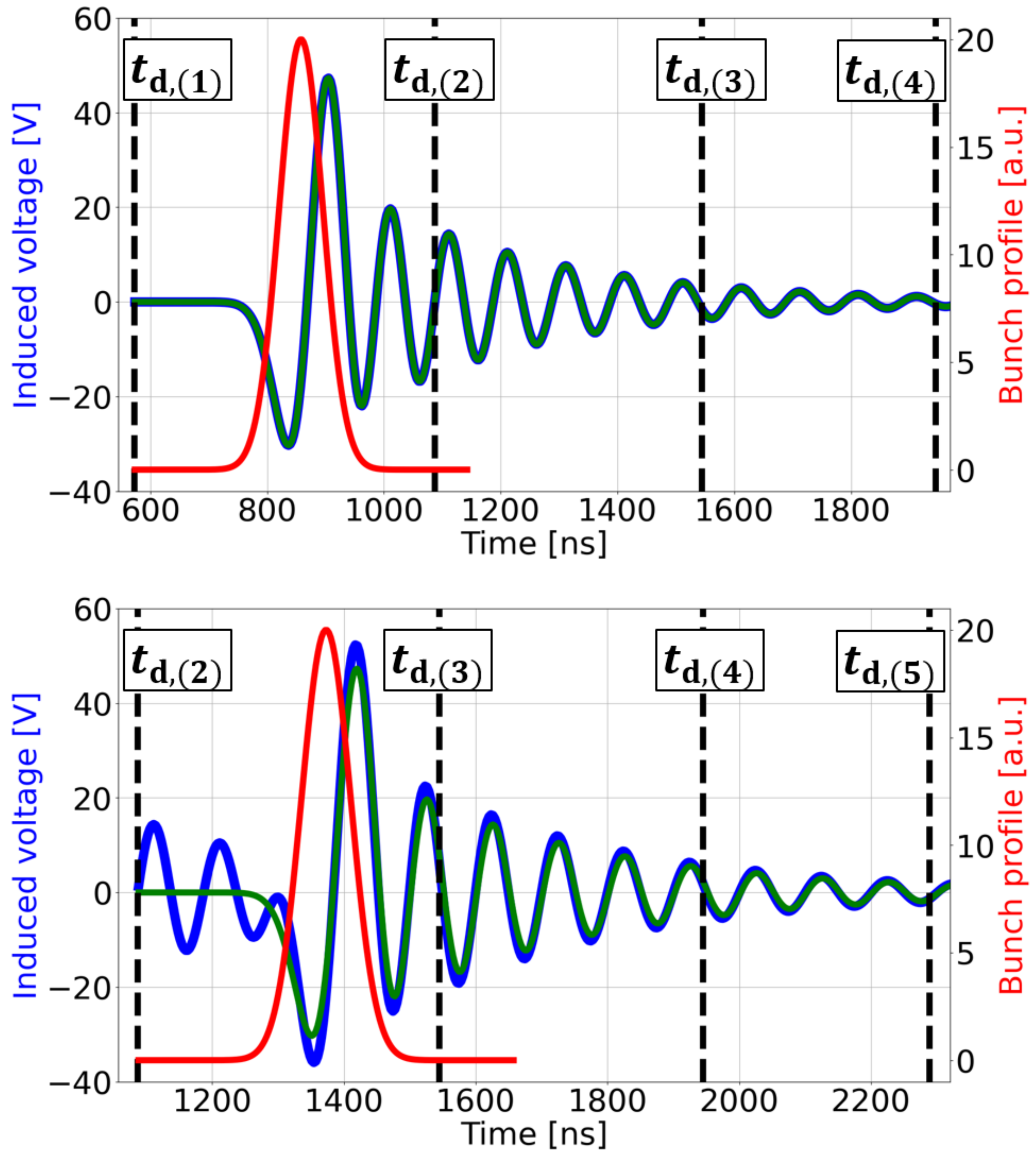
Loss of Landau damping (LHC)



Loss of Landau damping simulated with BLonD (blue spectrum) and the semi-analytic solver MELODY (red dotted line) [8]

Coupling particles over several machine turns

- Keep bunch profiles over several turns in memory



Top: first turn

Bottom: second turn

Courtesy of A. Lasheen

[8] I. Karpov, T. Argyropoulos and E. Shaposhnikova: 'Thresholds for loss of Landau damping in the longitudinal plane', PRAB **24**, 011002, 2021.

Synchrotron radiation

Synchrotron radiation & quantum excitation [9]

- SR shrinks, QE blows up the bunch emittance
- Energy loss can be applied several times per turn
  - Avoid large discrete energy kicks that can lead to fake debunching

$$E_{\text{other},(n)} = -U_0 - \frac{2}{\tau_z}\Delta E_{(n)} + 2\frac{\sigma_{\Delta E}}{\sqrt{\tau_z}} E_{d,(n)} \texttt{RANDN}$$

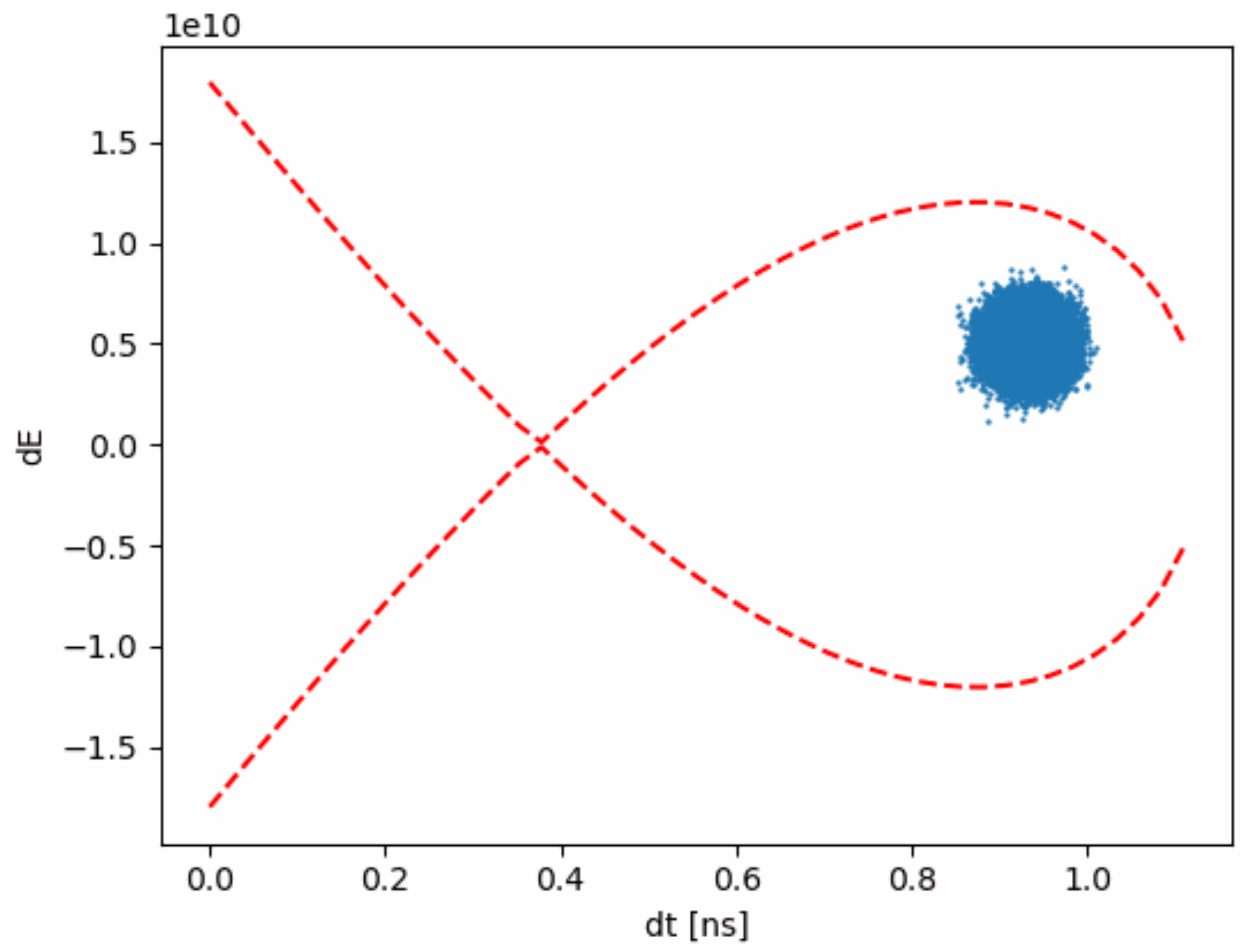
Average energy loss

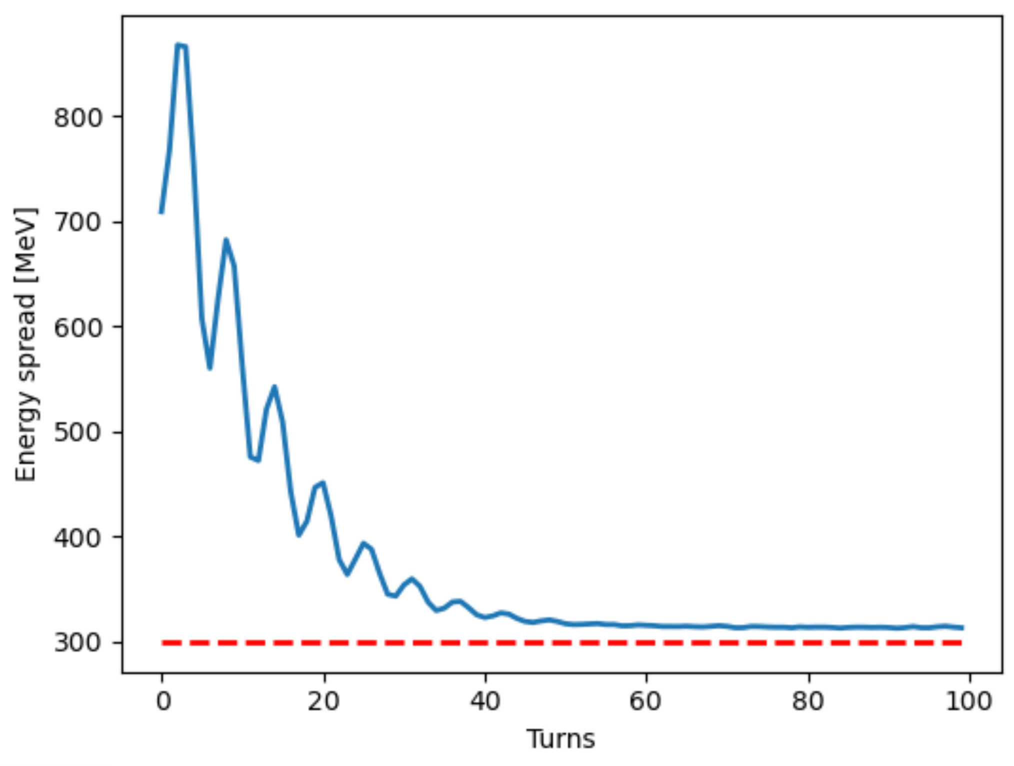
Difference in energy loss for each particle

Quantum excitation

  - Where the average energy loss is:

$$U_0 = \frac{4\pi}{3}\frac{r_{\text{cl}}}{m_p^3 c^6}\frac{1}{\rho}E_{d,(n)}^4 \frac{R}{C}$$





ρ	magnet bending radius
$\sigma_{\Delta E}$	equilibrium energy spread
τ_z	damping time [turns]
m_p	particle mass
RANDN	normal random number in (0,1)
$r_{\text{cl}} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{m_p c^2}$	classical particle radius

SR & QE in FCCee: one RF kick and 64 SR kicks per turn

The equilibrium emittance is reached within tens of turns

Courtesy of A. Vanel

[9] J. Esteban Müller: 'Modification of the simulation code BLonD for lepton rings', https://zenodo.org/record/7675649, (2017)

Content

Introduction

- BLonD and its use
- Code structure

Longitudinal tracking

- Reference frame
- Equations of motion
- RF manipulations

Collective effects

- Induced voltage
- Multi-turn wake
- Synchrotron radiation

Global control loops

- Embedding them
- CERN models
- RF noise & modulation

Local control loops

- Embedding them
- SPS and LHC models
- Use cases

Closing remarks

- Coupling loops
- Video tutorial

Global control loops

Principle: take a beam measurable and compare it to its design value

- Beam phase loop $\Delta\varphi_{\mathrm{PL}} = \varphi_b - \varphi_d$
  - Measures the beam phase w.r.t. synchronous (design) phase
  - Used to reduce injection errors and undesired RF noise (improve beam lifetime)
- Synchronisation or frequency loop $\Delta\omega_{\mathrm{SL}} = \omega_{\mathrm{rf}} - \omega_{\mathrm{rf,d}} = \omega_{\mathrm{rf}} - h\omega_{\mathrm{rev}}$
  - Measures the RF frequency
  - Used for injection/extraction and to keep the RF frequency at its design value
- Radial loop (for transition-crossing machines) $\frac{\Delta R_{\mathrm{RL}}}{R_d} = \frac{\Delta\omega_{\mathrm{rf}}}{\omega_{\mathrm{rf,d}}} \frac{\gamma^2}{\gamma_T^2 - \gamma^s}$
  - Measures the radial position of the beam
  - Used to keep the beam centred and the RF frequency at its design value

Correct the RF frequency $\Delta\omega_{\mathrm{TOT}} \equiv \omega_{\mathrm{rf}} - \omega_{\mathrm{rf,d}}$

- Can be a sum of corrections from different loops $\omega_{\mathrm{rf}} = \omega_{\mathrm{rf,d}} + \Delta\omega_{\mathrm{PL}} + \Delta\omega_{\mathrm{SL}} + \ldots$

Measure at a given harmonic, apply corrections on all RF systems (if several)

- Updated RF frequency of system k:

$$\Delta\omega_{\mathrm{rf,k}} = \frac{h_k}{h_{\mathrm{meas}}} \Delta\omega_{\mathrm{TOT}}$$

- Updated RF phase of system k:

$$\Delta\varphi_{\mathrm{rf,k}} = 2\pi h_k \frac{\omega_{\mathrm{rf,k}}}{\omega_{\mathrm{rf,d,k}}}$$

LHC global loops

Exact algorithms are machine dependent.
E.g. LHC phase & synchro loop [10]

- Correction of the beam phase loop

  $$\Delta\omega_{\mathrm{PL}} = -\, g_{\mathrm{PL}} \Delta\varphi_{\mathrm{PL}}$$

  - Reaction time: 5 turns

- Correction of the synchronisation loop

  $$\Delta\omega_{\mathrm{SL}} = -\, g_{\mathrm{SL}}(y + a\,\Delta\varphi_{\mathrm{rf}})$$

  - Reaction time: 50 turns
  - Here y is a recursive function

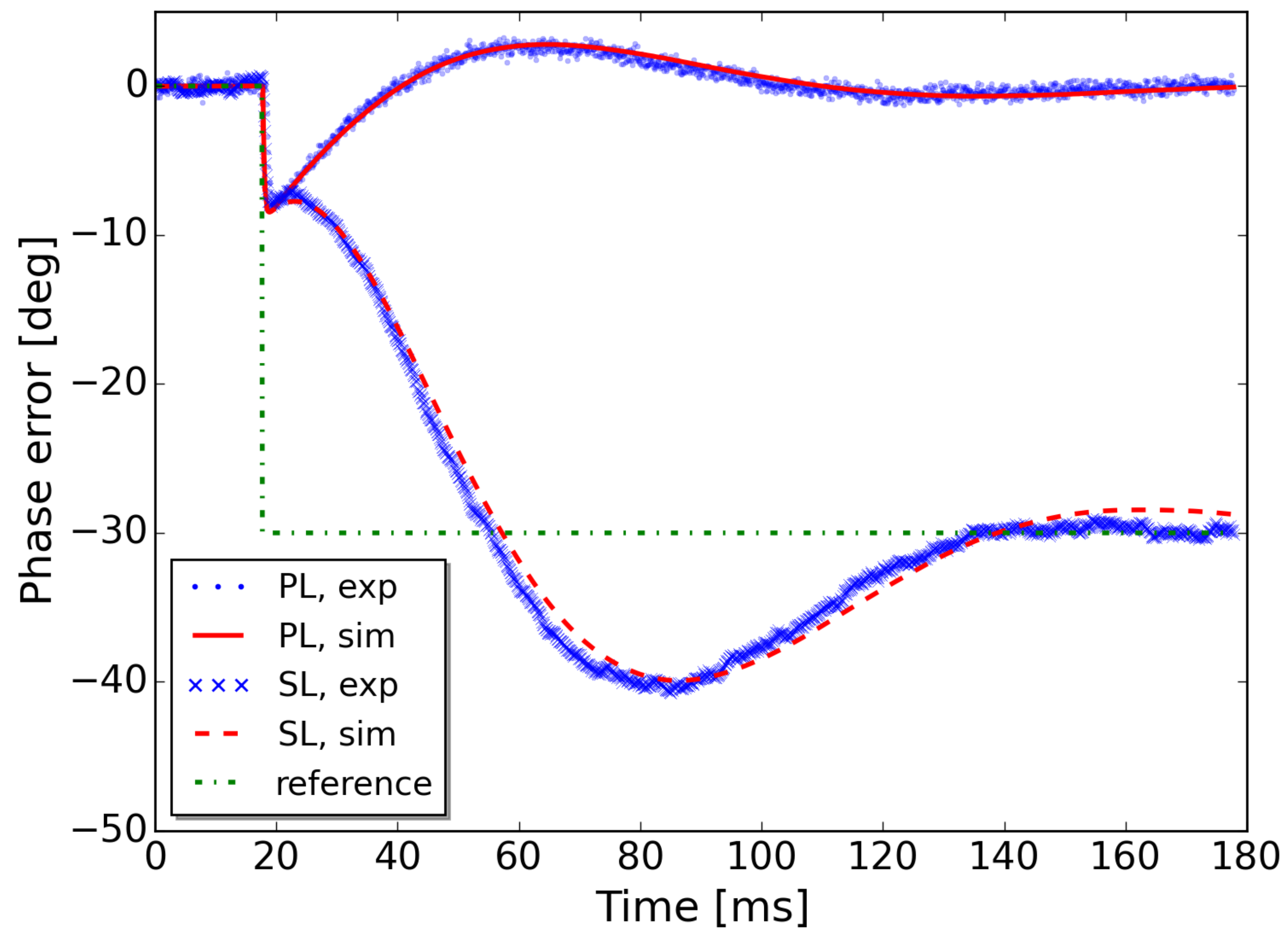
    $$y_{(n+1)} = (1-\tau)y_{(n)} + (1-a)\tau\Delta\varphi_{\mathrm{rf}} \text{ with } y_{(0)} = 0$$

  - and τ, a are functions of the synchrotron frequency

    $$a(\omega_s) \equiv 5.25 - \frac{\omega_s}{2\pi\, 20\ \mathrm{Hz}}$$

    $$\tau(Q_s) \equiv 2\pi Q_s \sqrt{\frac{a}{1 + \frac{g_{\mathrm{PL}}}{g_{\mathrm{SL}}}\sqrt{\frac{1+1/a}{1+a}}}}$$

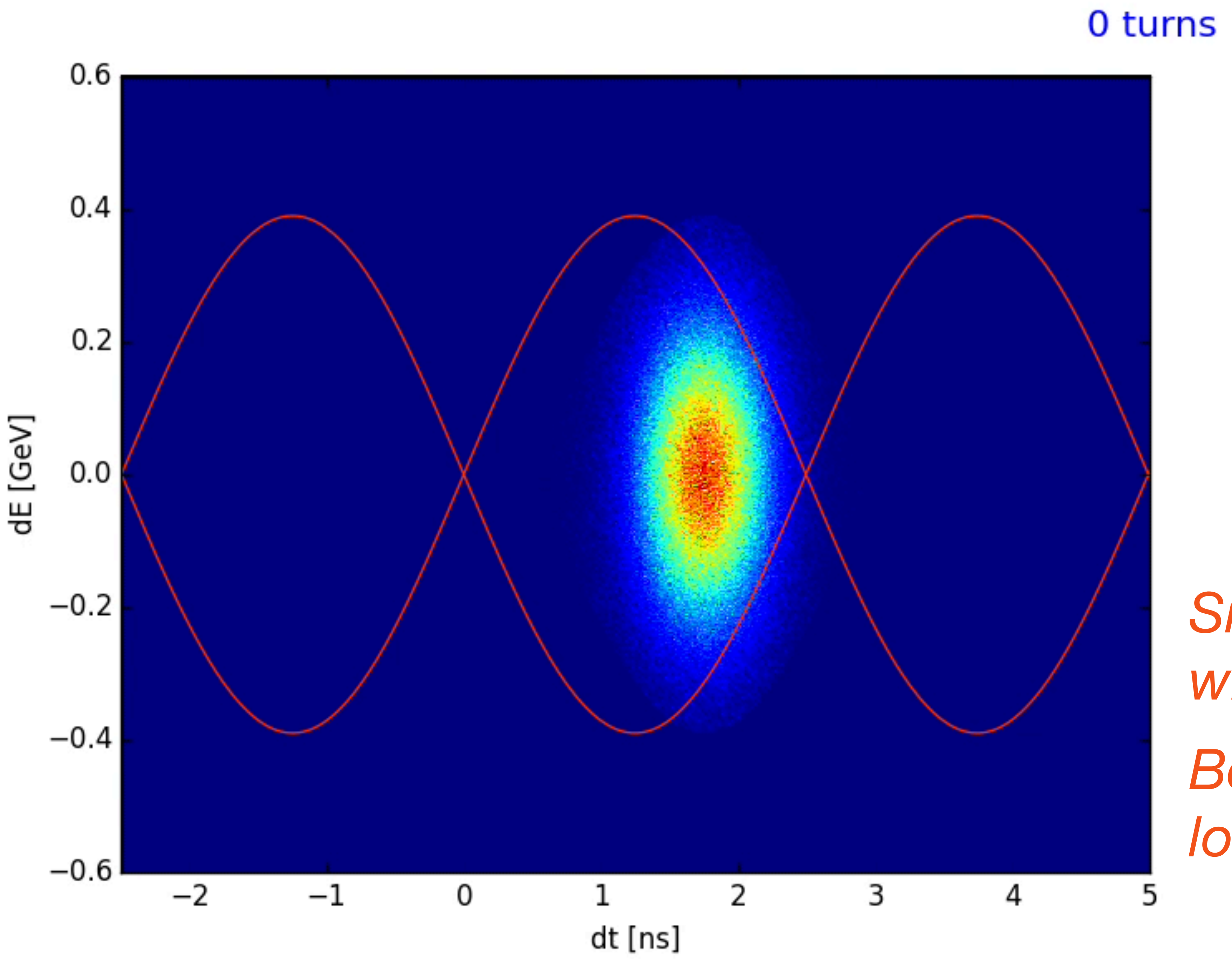
[10] P. Baudrenghien: 'The LHC Low Level Loops', unpublished, 2008.



Benchmarking

Step response of beam phase and synchro loops:

Measured (blue) vs simulated (red) phase error



Simulated LHC capture with phase error

Beam phase & synchro loops acting

PSB global loops

Another example: PSB [11]

- Beam phase loop z-domain transfer function

$$H(z) = g\frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}}$$

  - Ties the phase $\Delta\varphi_{\mathrm{PL}}$ and frequency $\Delta\omega_{\mathrm{PL}}$ corrections as

$$\Delta\omega_{\mathrm{PL}}^{n+1} = -a_1\Delta\omega_{\mathrm{PL}}^{n} + g(b_0\Delta\varphi_{\mathrm{PL}}^{n+1} + b_1\Delta\varphi_{\mathrm{PL}}^{n})$$

- Radial loop for transition crossing and orbit changes
  - Beam radial position calculated as

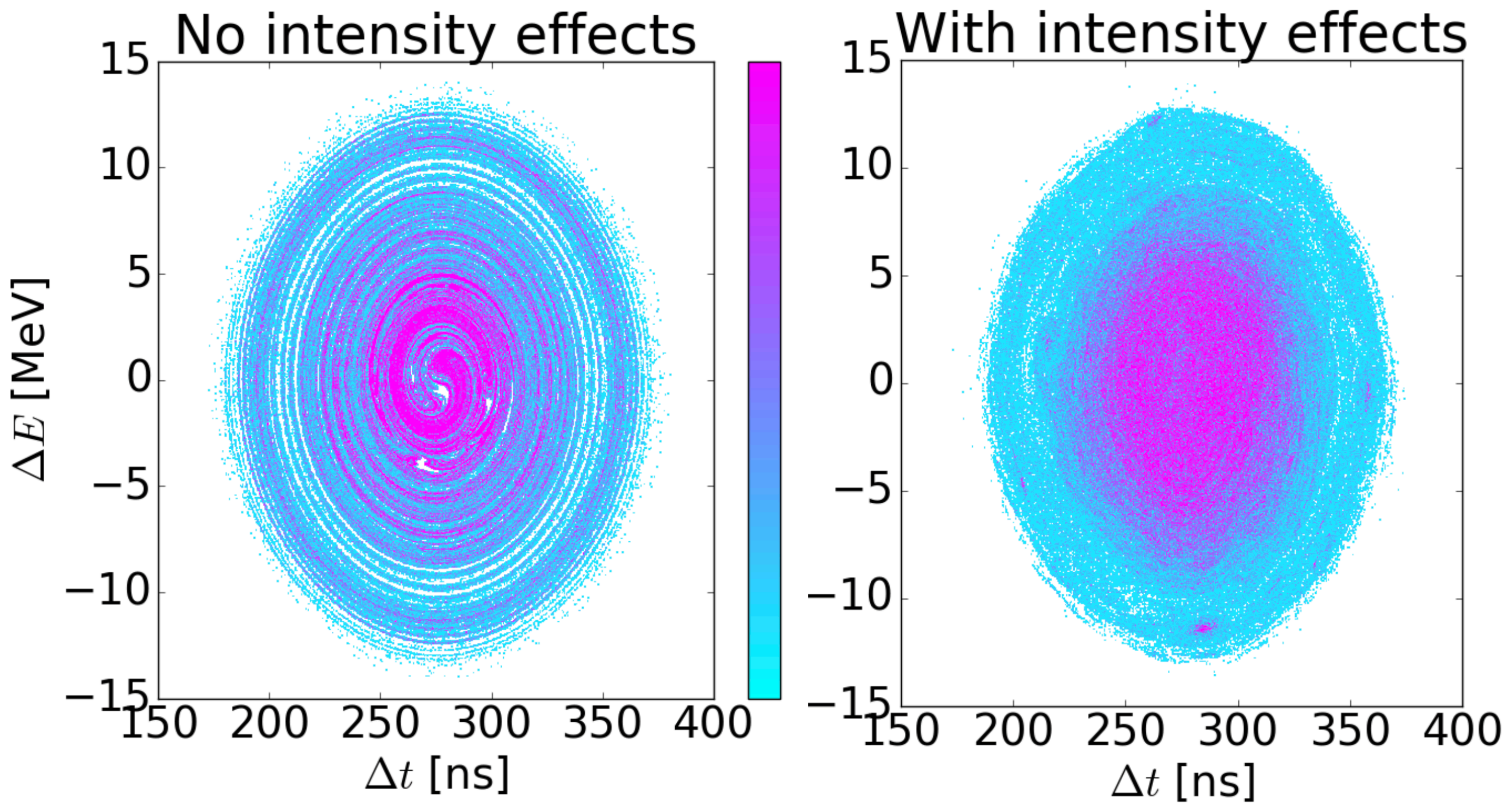
$$\frac{\Delta R}{R} = \frac{\Delta\omega_{\mathrm{rf}}}{\omega_{\mathrm{rf}}}\frac{\gamma^2}{\gamma_T^2 - \gamma^2}$$

  - Applying a proportional-integrator (PI) filter

$$\Delta\omega_{\mathrm{RL}}^{n+1} = \Delta\omega_{\mathrm{RL}}^{n} + K_P\left[\left(\frac{\Delta R}{R}\right)^n - \left(\frac{\Delta R}{R}\right)^{n-1}\right] + K_I'\left(\frac{\Delta R}{R}\right)^n$$

- The total correction becomes

$$\Delta\omega_{\mathrm{rf}}^{n+1} = \Delta\omega_{\mathrm{PL}}^{n+1} + \Delta\omega_{\mathrm{RL}}^{n+1}$$



Simulation of controlled emittance blow-up in the presence of beam phase and radial loop from [11], without (left) and with (right) collective effects

[11] D. Quartullo *et al.*: 'Controlled longitudinal emittance blow-up using band-limited phase noise in CERN PSB', Proc. IPAC'17, Copenhagen, Denmark, 2017.

RF phase modulation

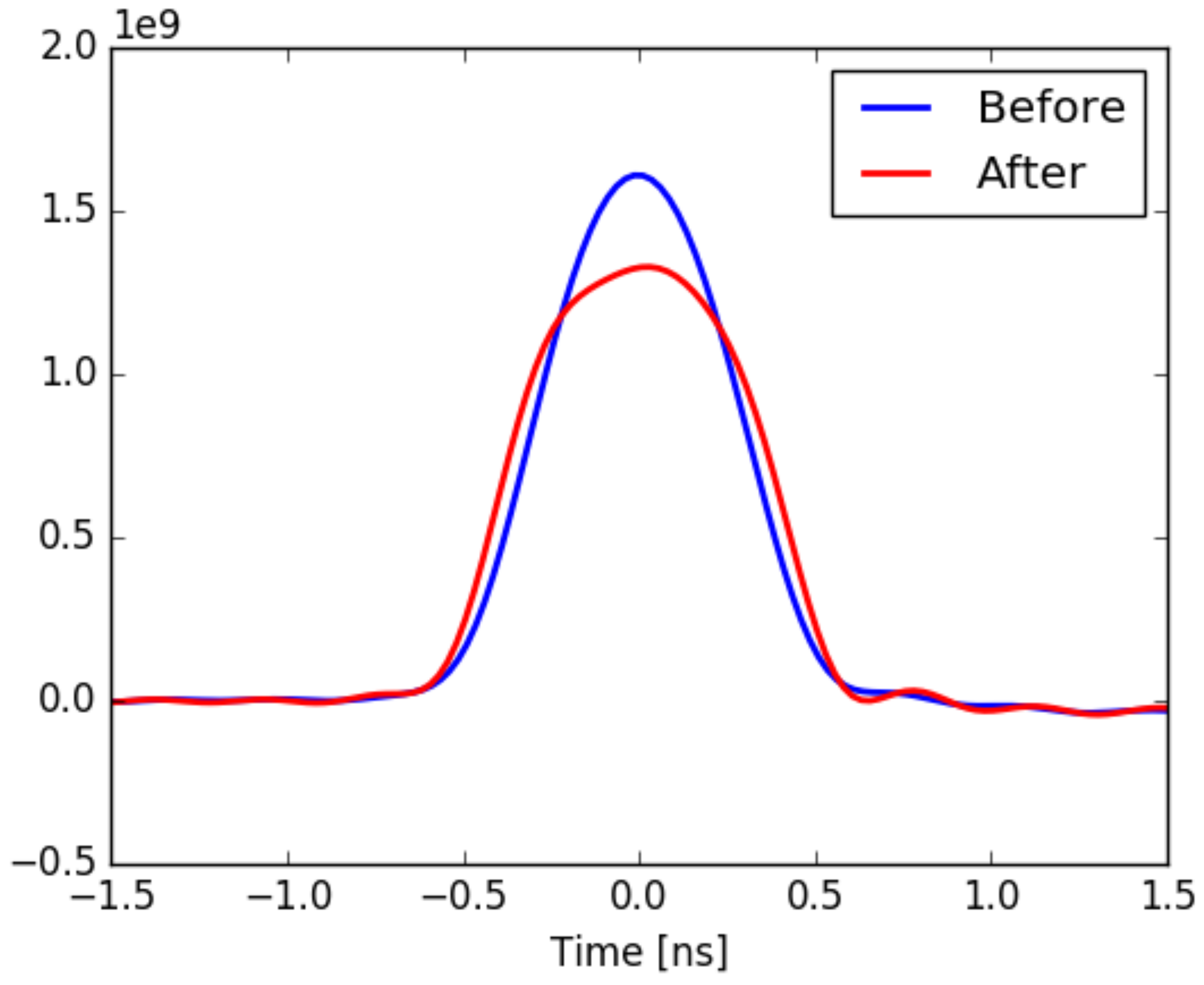
Resonant excitation using sine waves [12-14]

- Phase modulation added to the RF phase [15]:

$$\Delta\varphi_{\mathrm{rf},(n)} = A \sin\left(2\pi \sum_{k=0}^{n} f_{\mathrm{mod},(k)} T_{\mathrm{rev},(k)}\right) + \varphi_{\mathrm{off},(n)}$$

- To correctly simulate the modulation, the RF frequency has to follow as:

$$\Delta\omega_{\mathrm{rf},(n)} = \frac{d\Delta\varphi_{\mathrm{rf},(n)}}{dt} = \delta\Delta\varphi_{\mathrm{rf},(n)} \frac{\omega_{\mathrm{rf},d,(n)}}{2\pi h}$$



Measured bunch profile in the LHC before and after flattening

Example: bunch flattening during LHC collisions

- Increase in bunch length to counteract SR w/o losses
- For this, modulate close to the core $0.98 f_{s0}$ with 0.6°

[12] S. Y. Lee: 'Accelerator Physics', World Scientific, 3rd Ed., 2012.
[13] C. Y. Tan and A. Burov: 'Phase modulation of the bucket stops bunch oscillations at the Fermilab Tevatron', PRAB **15**, 044401, (2012).
[14] E. Shaposhnikova *et al.*: 'Flat Bunches in the LHC', Proc. IPAC'14, Dresden, Germany, (2014).
[15] S. Albright and D. Quartullo: Journal of Physics: Conference Series 1350, 012144 (2019).

RF phase noise

Modelling of controlled emittance blow-up via noise injection or background RF phase noise [16,17]

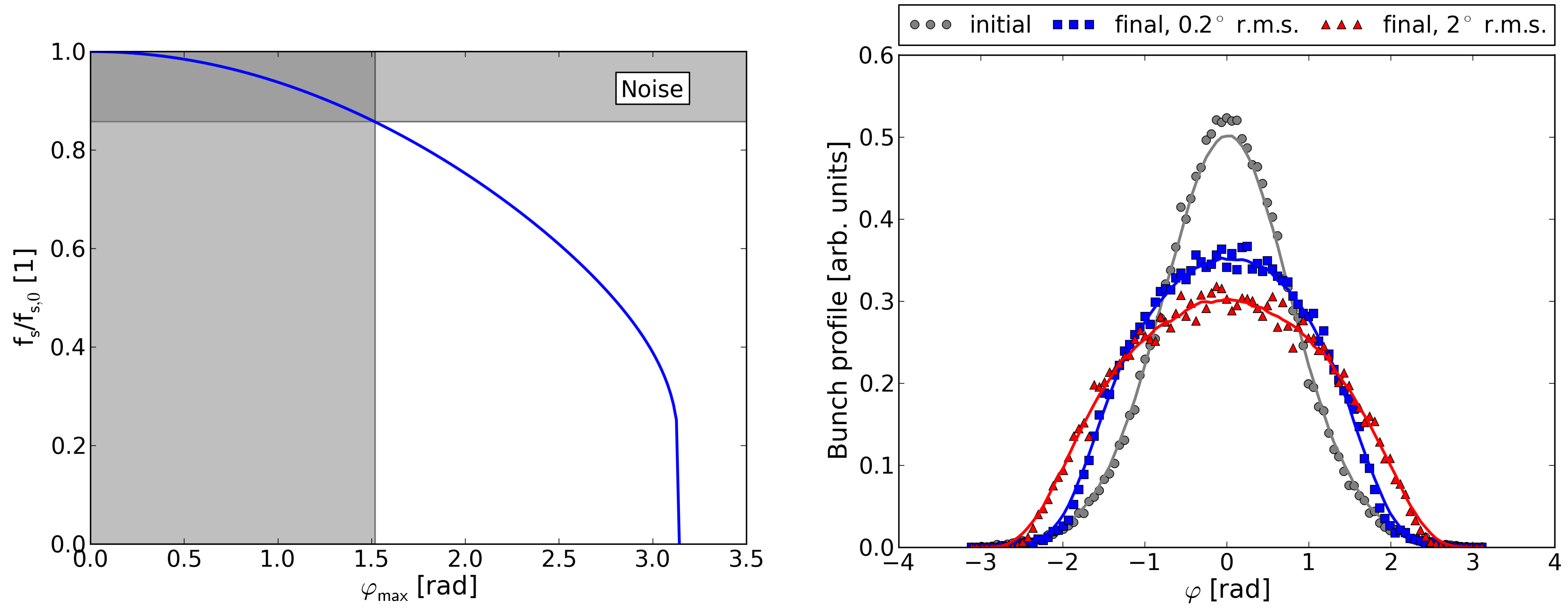
- RF phase is changed turn by turn with a noise sample:

  $\Delta\varphi_{\mathrm{rf}} = \varphi_{\mathrm{noise}}(t_{(n)})$, with $< \varphi_{\mathrm{noise}}(t_{(n)}) > = 0$

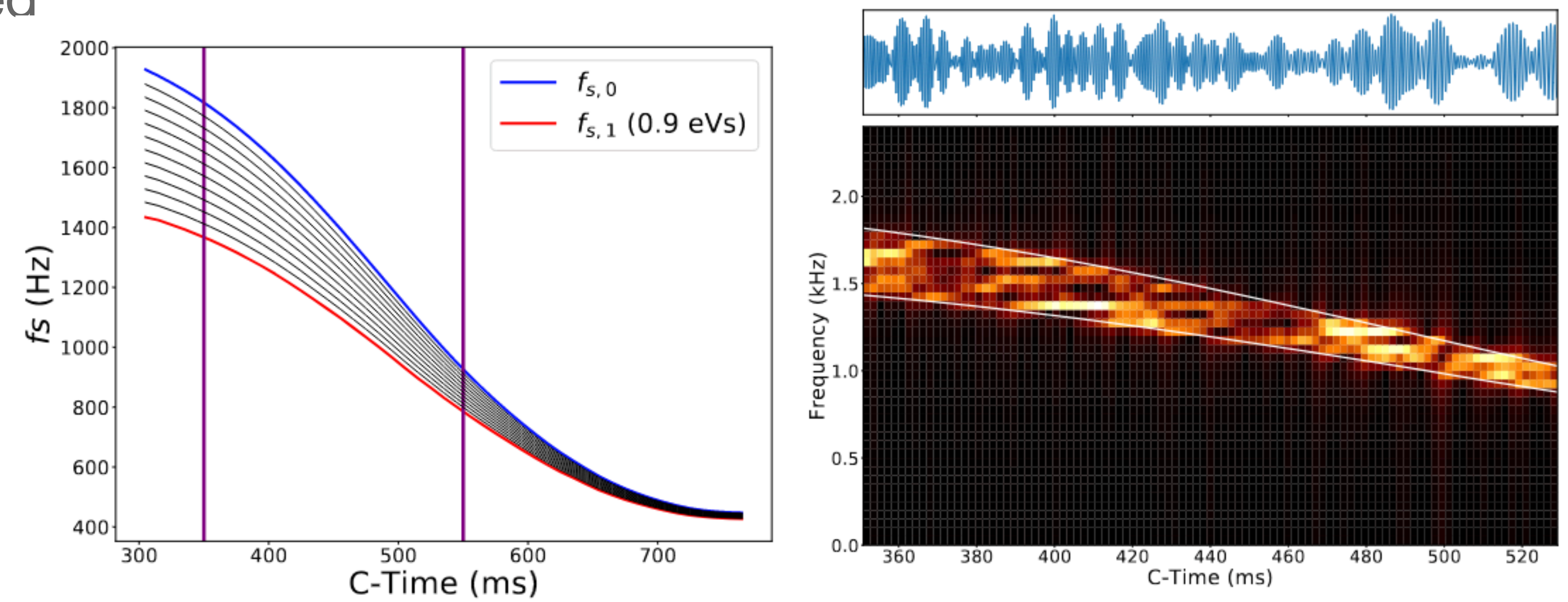
- Can shape the bunch using band-limited white noise [18]
  - The noise spectrum determines the region of the bunch affected
    - Target the core to avoid losses from the tails
  - The r.m.s. phase noise depends on the single-sided power spectral density as

$$\varphi_{\mathrm{noise}}^{\mathrm{rms}} = \sqrt{\int S_{\varphi}(f)df}$$

  - In BLonD, we use the algorithm in [19] to generate the noise sequence for band-limited noise (of any spectrum)



Diffusion due to band-limited white noise



Alternative to phase noise: a sum of single-frequency modulations (e.g. PSB) [15]

[16] S. Krinsky, J.M. Wang: 'Bunch diffusion due to RF noise', Part. Accel. 12, 107–117 (1982).

[17] G. Dôme: 'Diffusion due to RF noise', CERN Accelerator School '85, Oxford, UK, 370–401 (1985).

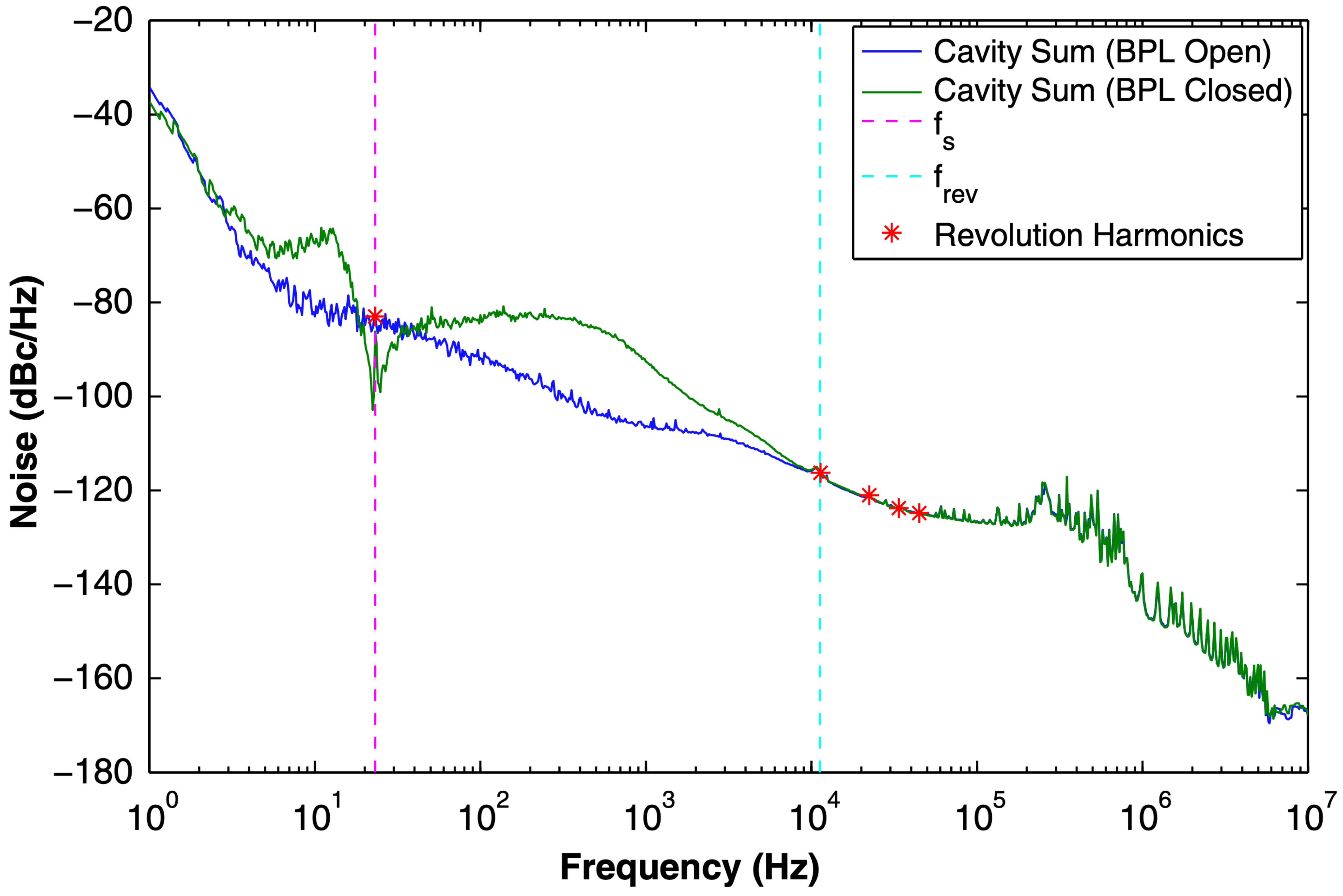
[18] T.Toyama: 'Uniform bunch formation by RF voltage modulation with a band-limited white signal', NIM A, 447, 317–327 (2000).

[19] J. Tückmantel: 'Digital generation of noise-signals with arbitrary constant or time-varying spectra', LHC-PROJECT-Report-1055, (2008).

Damping of RF phase noise

The beam lifetime relies on damping the background RF phase noise

- Without the beam phase loop in physics, the lifetime of the LHC proton beam would be of the order of 10s minutes
  - With the beam phase loop acting, proton collisions can be be maintained >24 h
- Beam dynamics simulations can help to design the right damping of background RF noise
  - RF noise feedbacks for HL-LHC crab cavities are being designed with input from 6D beam dynamics simulations
  - *See also talk by G. Hagmann, Mon PM*



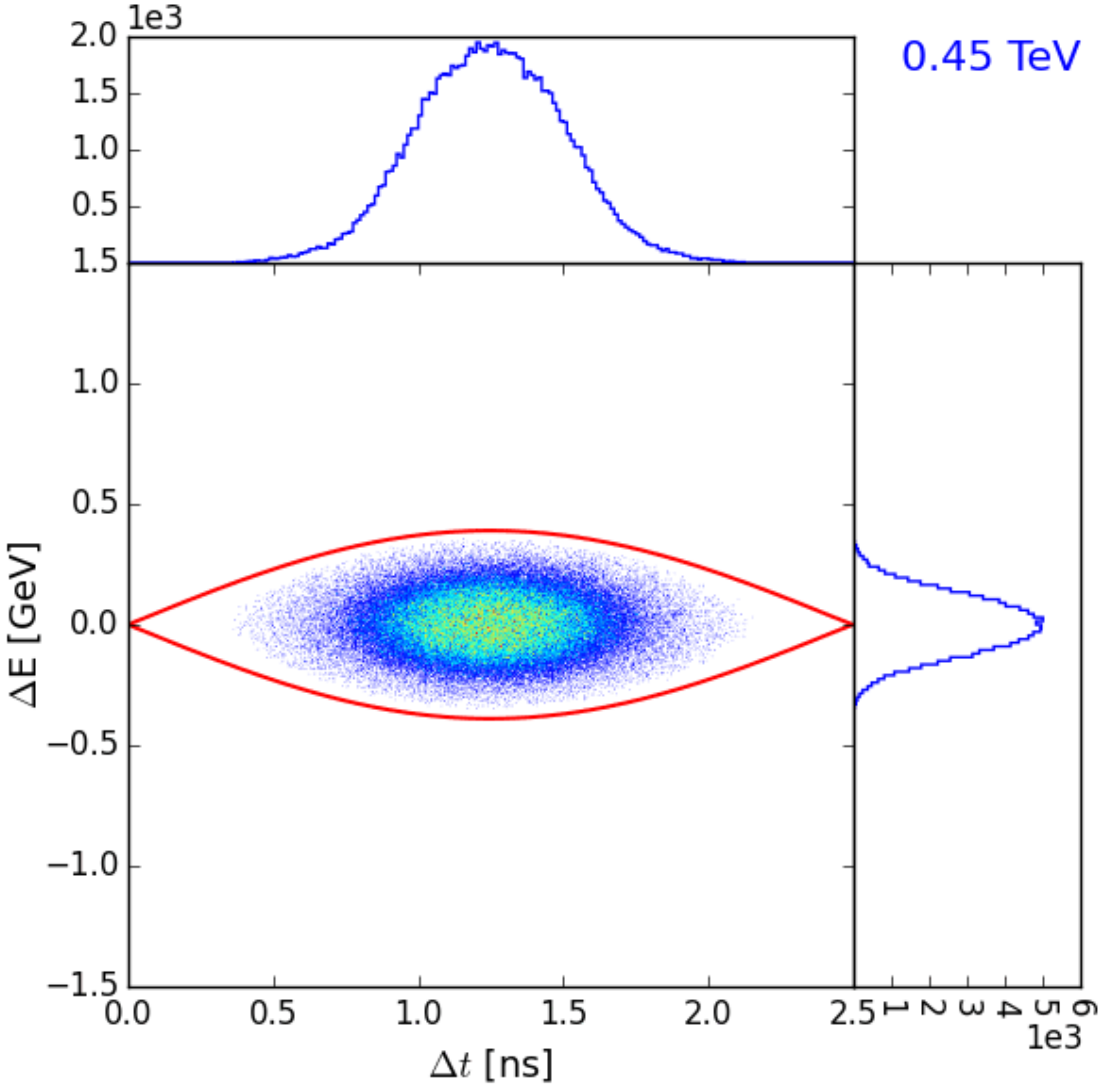
RF phase noise spectrum in the LHC, without and with the beam phase loop acting; a plot from [20]

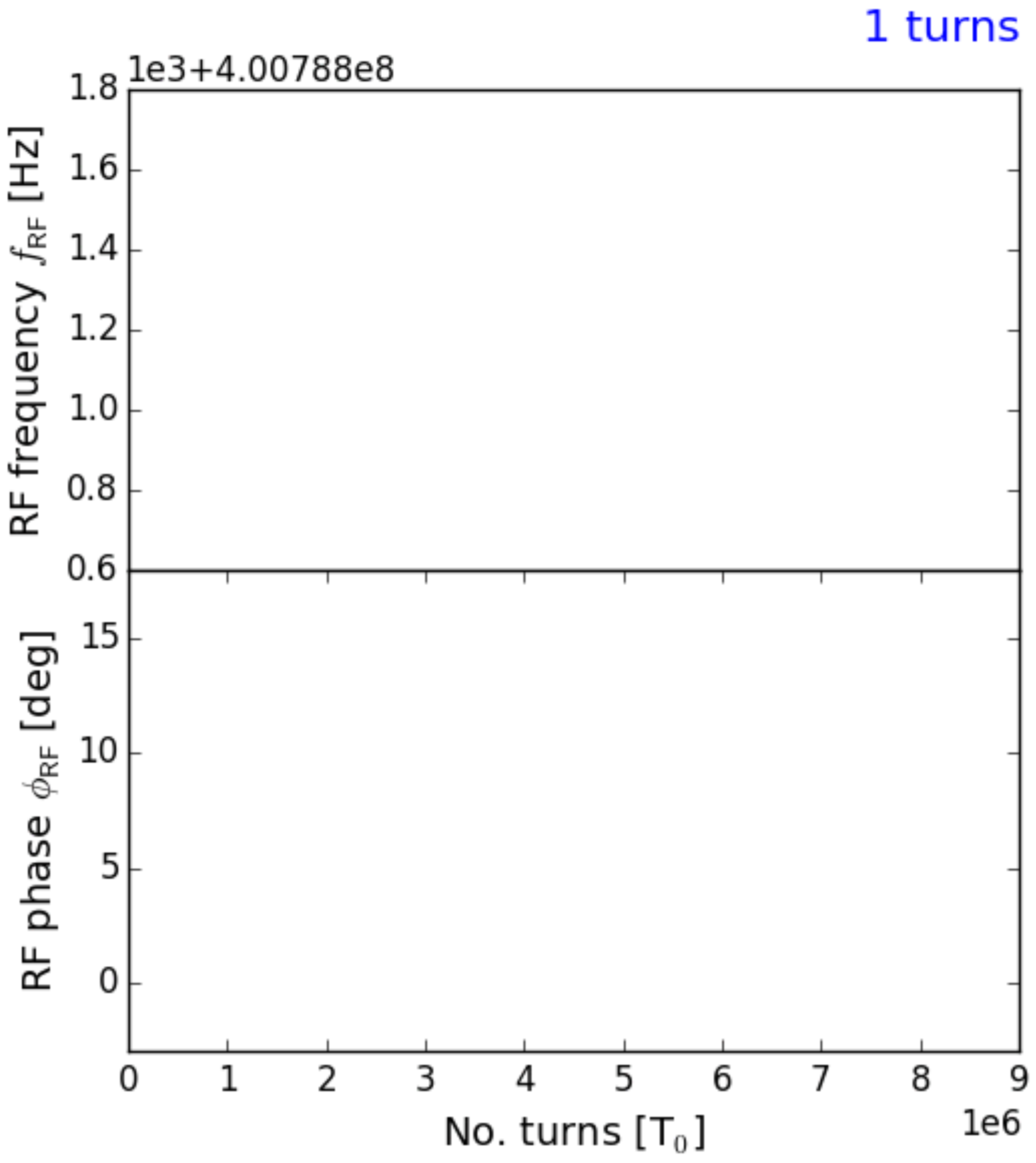
[20] T. Mastoridis et al., 'Radio frequency noise effects on the CERN Large Hadron Collider beam diffusion', Phys. Rev. Accel. Beams **14**, 092802, 2011.

LHC blow-up with global loops

A concrete example: LHC blow-up during the ramp

- RF phase noise injection with beam phase loop and synchro loop on
- Bunch length feedback regulating the strength (r.m.s. amplitude) of the noise





LHC controlled emittance blow-up with beam phase loop and synchronisation loop acting

Content

Introduction
- BLonD and its use
- Code structure

Longitudinal tracking
- Reference frame
- Equations of motion
- RF manipulations

Collective effects
- Induced voltage
- Multi-turn wake
- Synchrotron radiation

Global control loops
- Embedding them
- CERN models
- RF noise & modulation

Local control loops
- Embedding them
- SPS and LHC models
- Use cases

Closing remarks
- Coupling loops
- Video tutorial

Local control loops

Control an RF chain from transmitter to cavity

- Principal function: regulate the RF voltage amplitude and phase $\vec{V}_{\text{ant}}$ to the set point (design) value $\vec{V}_{\text{set}}$
- Tune the cavity
- Transmitter regulation (e.g. polar loop)
- Clamping/protection loops

From the beam point of view

- Reduce the cavity impedance at the RF frequency
- If necessary, damp coupled-bunch modes
  - E.g. reduce the side-bands of the cavity impedance at $nf_{\text{rev}} + kf_s$ (comb filters)

What it means in our simulations

- Voltage amplitude and phase are **now arrays** over one turn!

$$\left|V_{\text{ant}}\right| \sin\left(\omega_{\text{rf}}\Delta t + \varphi_{\text{ant}}\right) = \left|V_{\text{set}}\right| \left|V_{\text{corr}}\right| \sin\left(\omega_{\text{rf}}\Delta t + \varphi_{\text{set}} + \varphi_{\text{corr}}\right)$$

Set point voltage, one value per turn

Correction voltage, sampled with sampling time

Some built-in generic building blocks

- RF beam current calculation from beam profile
- Up- and down-modulation
- Comb filter
- FIR filter
- Moving average
- …

SPS cavity controller model

Time-domain model [21,22] in a nutshell

- Synchronous with RF frequency (~200 MHz)
  - Harmonic h samples per turn
- Normal conducting travelling wave cavities

  $$V_{\mathrm{ant}} = V_{\mathrm{gen}} + V_{\mathrm{beam}} = I_{\mathrm{gen}} Z_{\mathrm{gen}} + I_{\mathrm{beam}} Z_{\mathrm{beam}}$$

- Induced voltage calculated through impulse response

  $$\begin{pmatrix} V_I(t) \\ V_Q(t) \end{pmatrix} = \begin{pmatrix} h_c(t) & -h_s(t) \\ h_s(t) & h_c(t) \end{pmatrix} * \begin{pmatrix} I_I(t) \\ I_Q(t) \end{pmatrix}$$

- A comb filter to implement a one-turn feedback

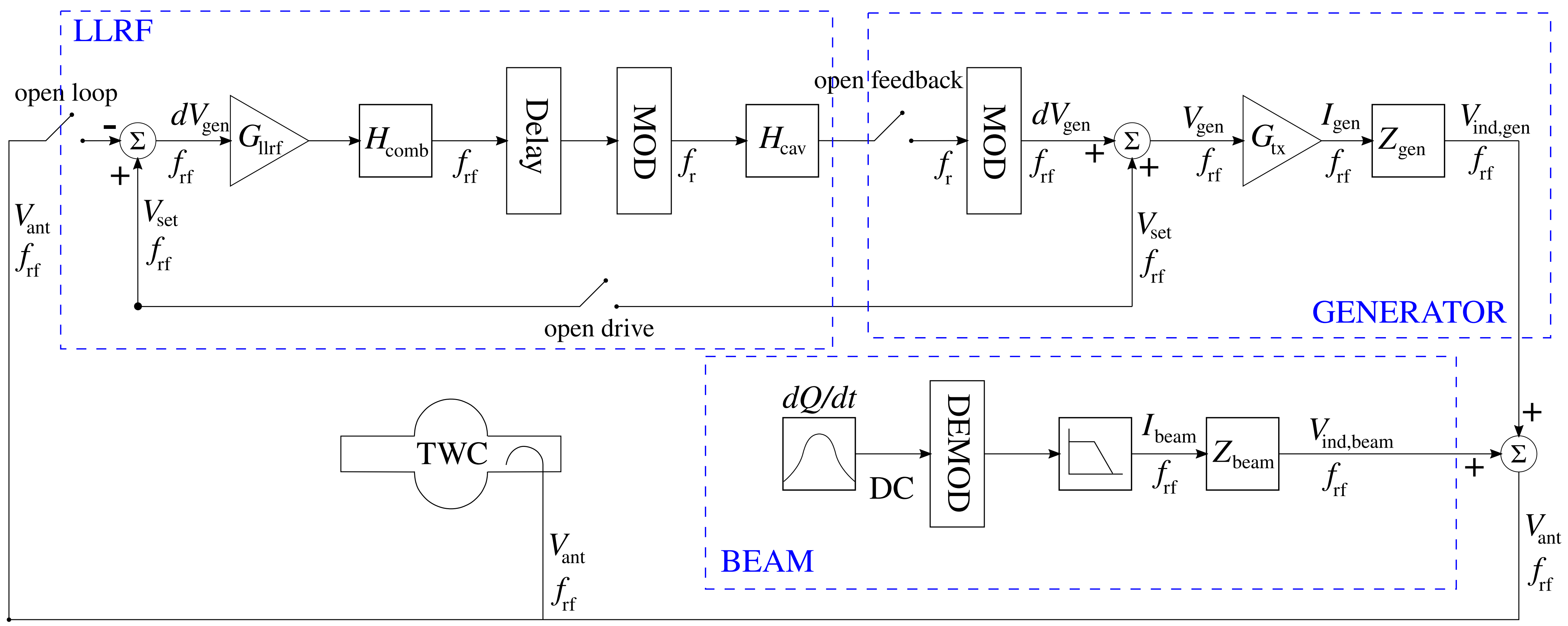
  $$dV_{\mathrm{gen,out},k,n} = a_{\mathrm{comb}}\, dV_{\mathrm{gen,out},k,n-1} + (1 - a_{\mathrm{comb}})\, dV_{\mathrm{gen,in},k,n}$$

- Cavity response is represented by a moving average

  $$dV_{\mathrm{gen,out},k} = \frac{1}{K} \sum_{i=k-K}^{k} dV_{\mathrm{gen,in},i}$$

- Feed-forward FIR filter $Z_{\mathrm{gen}}(f) H_{\mathrm{FF}}(f) = -Z_{\mathrm{beam}}(f)$

SPS One Turn Feedback



BLonD model of the SPS cavity controller

Impulse response for beam-induced voltage with triangle function:

$$h_{c,\mathrm{beam}}(t) = \frac{2R_{\mathrm{beam}}}{\tau} \mathrm{tri}\left(\frac{t}{\tau}\right) \cos((\omega_{\mathrm{rf}} - \omega_r)t) \quad \text{(s) (sin)}$$

Impulse response for generator-induced voltage with rectangle function:

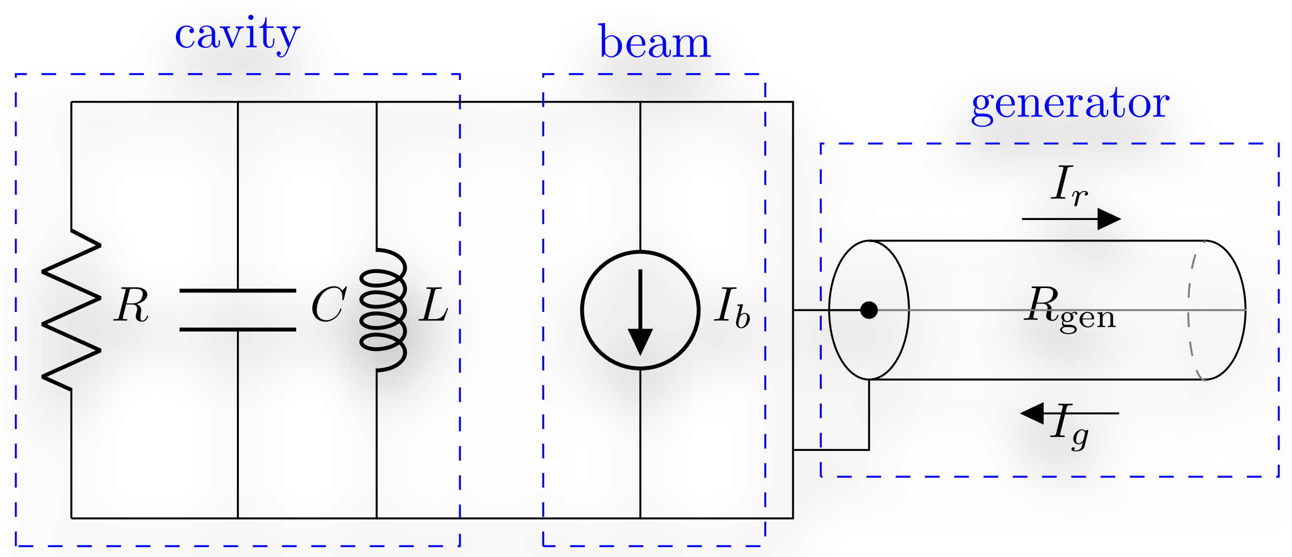
$$h_{c,\mathrm{gen}}(t) = \frac{2R_{\mathrm{gen}}}{\tau} \mathrm{rect}\left(\frac{t}{\tau}\right) \cos((\omega_{\mathrm{rf}} - \omega_r)t) \quad \text{(s) (sin)}$$

τ cavity filling time: 462 ns for 3-section, 621 ns for 4-section cavities

[21] P. Baudrenghien and T. Mastoridis: ‘I/Q model of the SPS 200 MHz travelling wave cavity and feedforward design’, CERN-ACC-NOTE-2020-0032, 2020.

[22] G. Hagmann *et al.* : ‘CERN SPS low-level RF architecture & implementation’, Presentation at LLRF’22 workshop, Brugg-Windisch, Switzerland.

LHC cavity controller model



Time-domain model [23] in a nutshell

- Synchronous with RF frequency (~400 MHz)
  - Harmonic $h/10$ samples per turn
- Tuneable superconducting cavities (resonators)
  - Tuning and clamping loops
- Direct RF feedback composed of
  - Analog high-pass branch: gain G_a, delay τ_a

$$y^{(n)} = \left[1 - \frac{T_s}{\tau_a}\right] y^{(n-1)} + G_a(x^{(n)} - x^{(n-1)})$$

  - Digital low-pass branch: gain G_d, delay τ_d

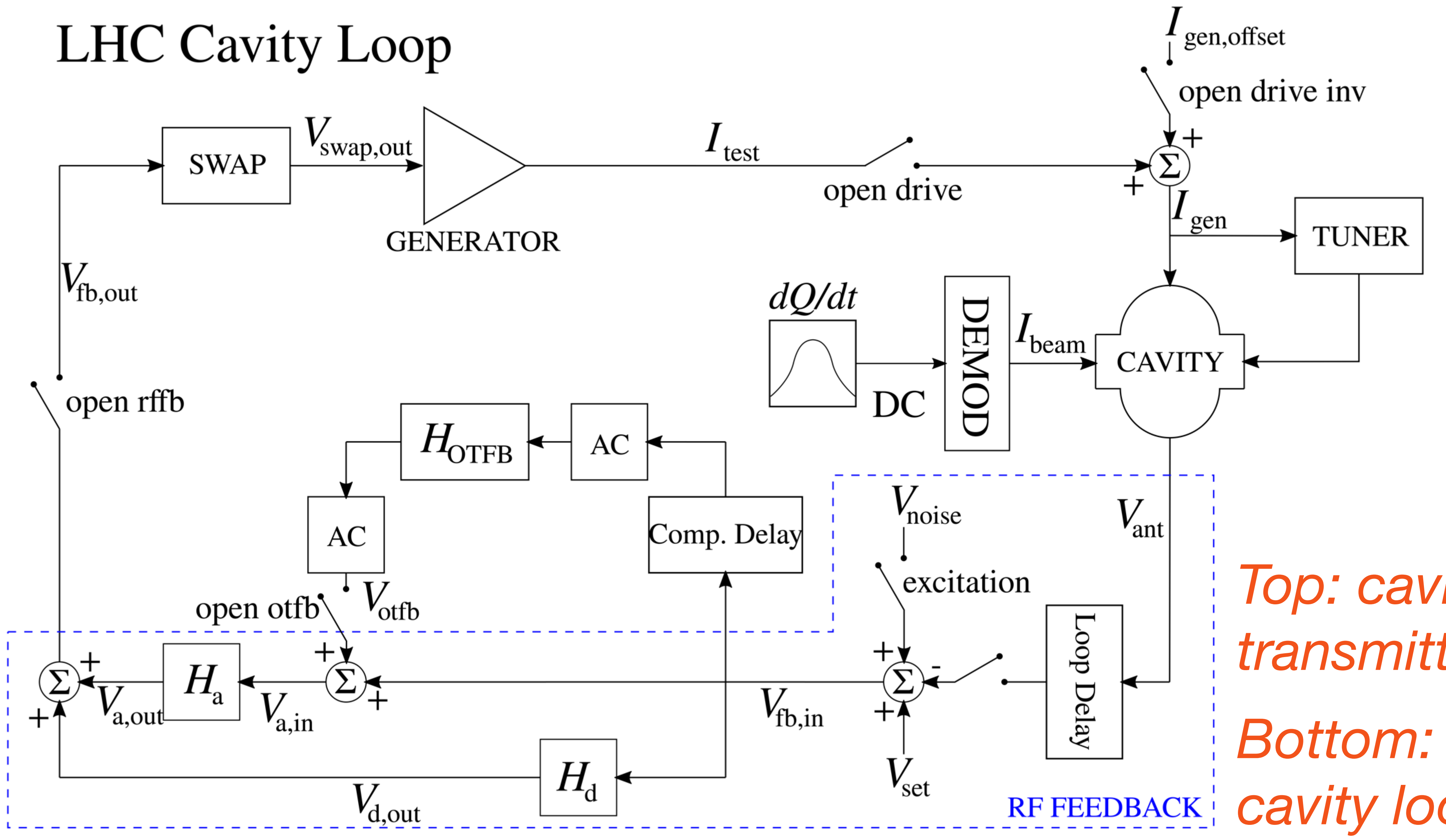
$$y^{(n)} = \left[1 - \frac{T_s}{\tau_d}\right] y^{(n-1)} + G_a G_d e^{i\Delta\varphi_{\text{ad}}} \frac{T_s}{\tau_d} x^{(n-1)}$$

- One-turn delay feedback (comb filter)
  - Gain G_o, scaling factor α, samples per turn N

$$y^{(n)} = \alpha y^{(n-N)} + G_o(1-\alpha)x^{(n-N)}$$

Cavity-transmitter-beam interaction model [24]

$$V_{\text{ant}}^{(n)} = \frac{R}{Q}\omega T_{\text{rev},(n)} I_{\text{gen}}^{(n-1)} + \left(1 - \frac{\omega T_s}{2Q_L} + i\Delta\omega T_{\text{rev},(n)}\right) V_{\text{ant}}^{(n-1)} - \frac{1}{2}\frac{R}{Q}\omega T_{\text{rev},(n)} I_{\text{b,rf}}^{(n-1)}$$



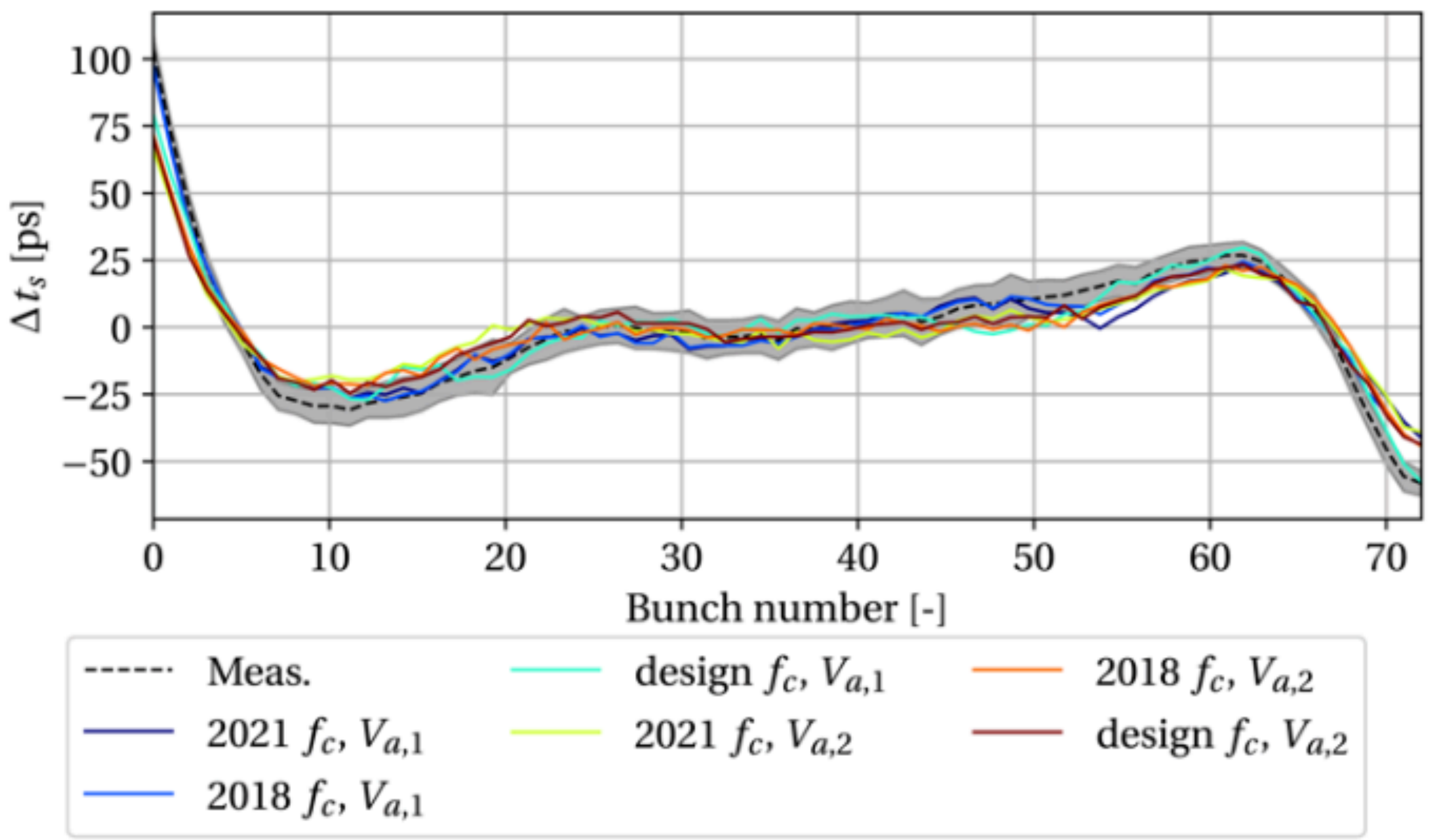
Top: cavity-beam-transmitter model

Bottom: BLonD LHC cavity loop model

[23] J. Holma: 'The model and simulations of the LHC 400 MHz cavity controller', CERN-AB-Note-2007-012, 2007.

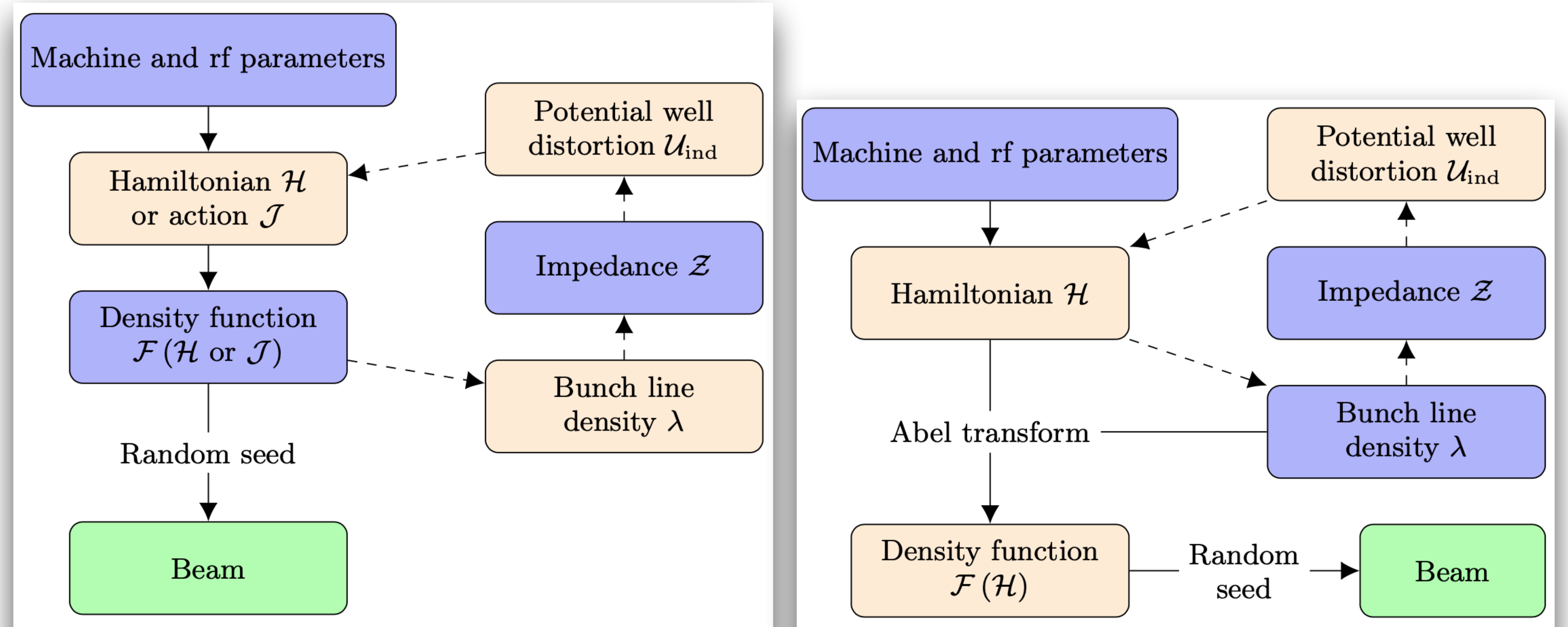
[24] J. Tückmantel: 'Cavity-beam-transmitter interaction formula collection with derivation', CERN-ATS-Note-2011-002 TECH, 2011.

Beam distribution generation

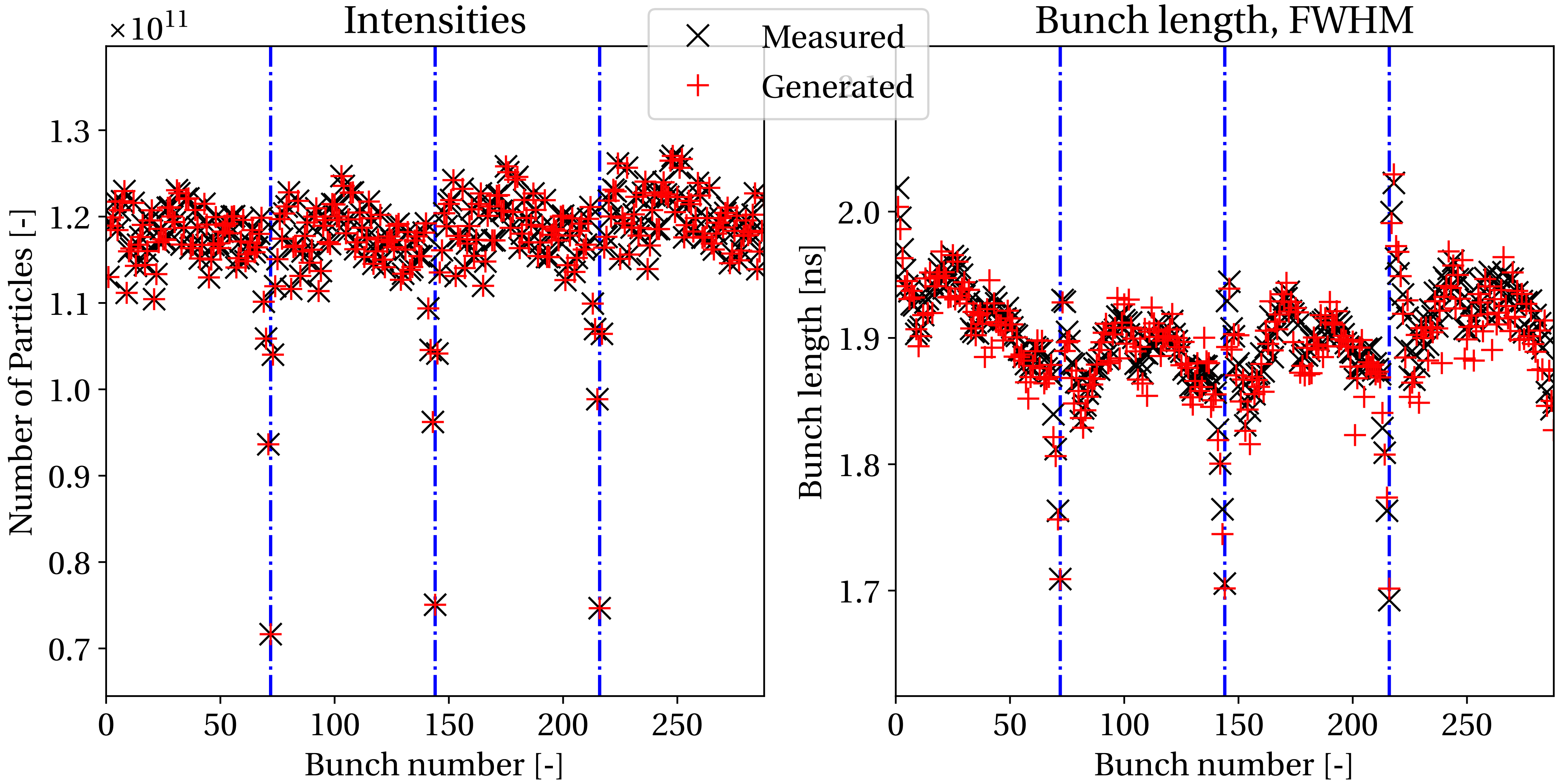


With collective effects: an iterative process

- Matching to user-defined distribution or profile
- Add LLRF to re-match in the presence of loops



(a) Using $\mathcal{F}$ as input. (b) Using λ as input.



Bunch generation algorithms in BLonD

Courtesy of A. Lasheen

Top: adding the SPS OTFB to get the right bbb offset

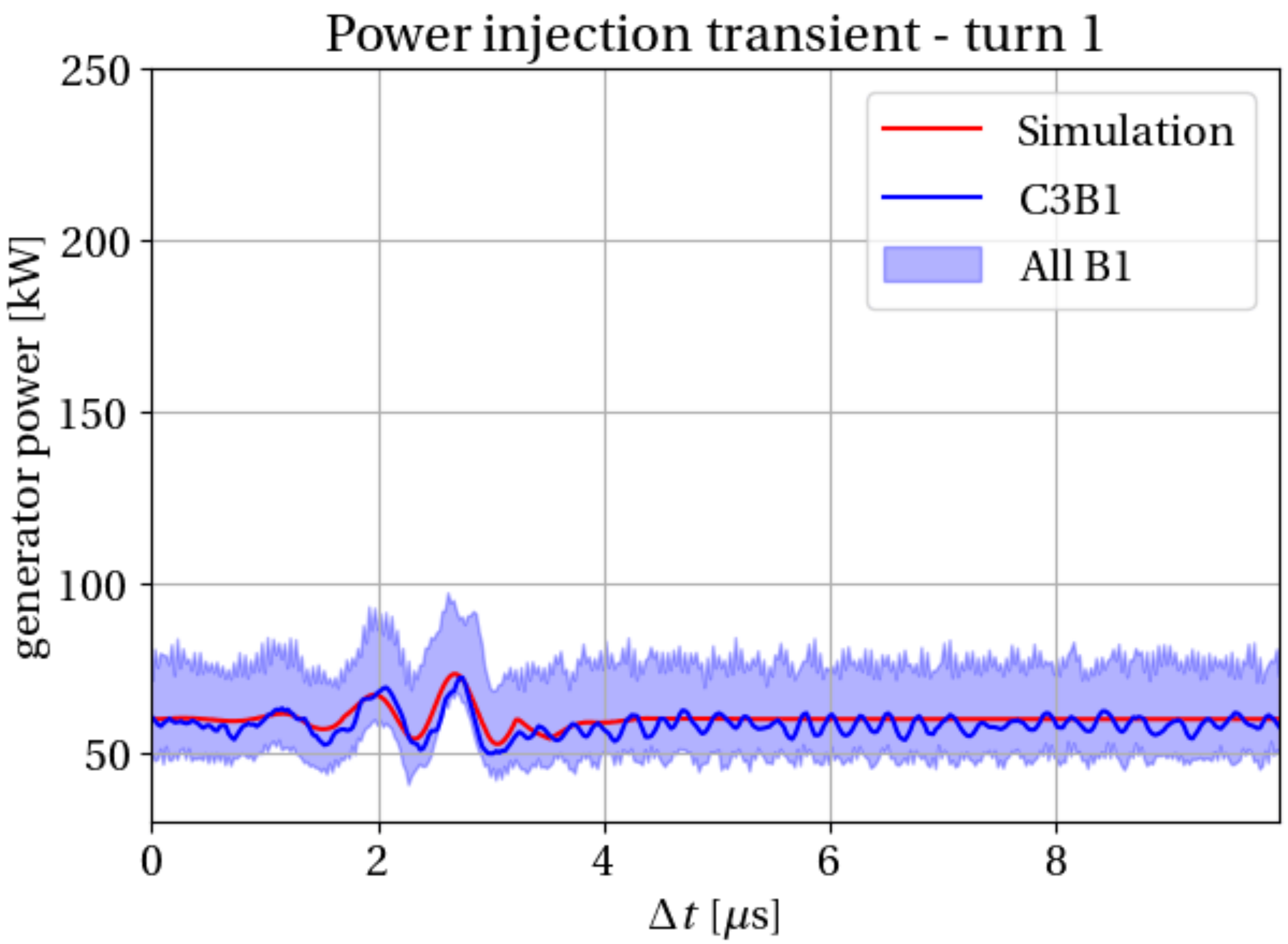
Bottom: re-generating a measured batch with intensity effects

Courtesy of B. Karlsen-Bæck

LHC injection transients

LHC RF power transients at injection

- Power transients are studied in simulation for HL-LHC beam current
  - Each cavity is fed by a 300 kW klystron
  - Power limitations encountered for the main RF system at high intensities
  - Need to optimise the operational scenario
- The study requires
  - Modelling the injected beam in the SPS with collective effects and cavity feedback
  - Modelling the LHC injection with collective effects and cavity feedback



Injection transients with a 36-bunch batch of 1.8×10^{11} p/b

Courtesy of B. Karlsen-Bæck

LHC injection transients in practise

BLonD examples

BLonD example on LHC power transients at injection

LLRF workshop 2023, Gyeongju, Korea

Import BLonD and Python modules

```python
import os, sys
sys.path.insert(0,'/Users/timko/PycharmProjects/BLonD_bkarlsen')

# Import numpy and matplotlib
import numpy as np
import matplotlib.pyplot as plt
import matplotlib
from tqdm import tqdm
%matplotlib inline

# Import blond objects
from blond.beam.beam import Beam, Proton
from blond.beam.distributions import bigaussian
from blond.beam.profile import Profile, CutOptions
from blond.input_parameters.ring import Ring
from blond.input_parameters.rf_parameters import RFStation
from blond.trackers.tracker import RingAndRFTracker
from blond.llrf.cavity_feedback import LHCCavityLoopCommissioning, LHCCavityLoop
```

[1] ✓ 1.9s Python

---------- Using the C++ computational backend ----------

https://cernbox.cern.ch/s/LNsN6zAYnc8ZznX

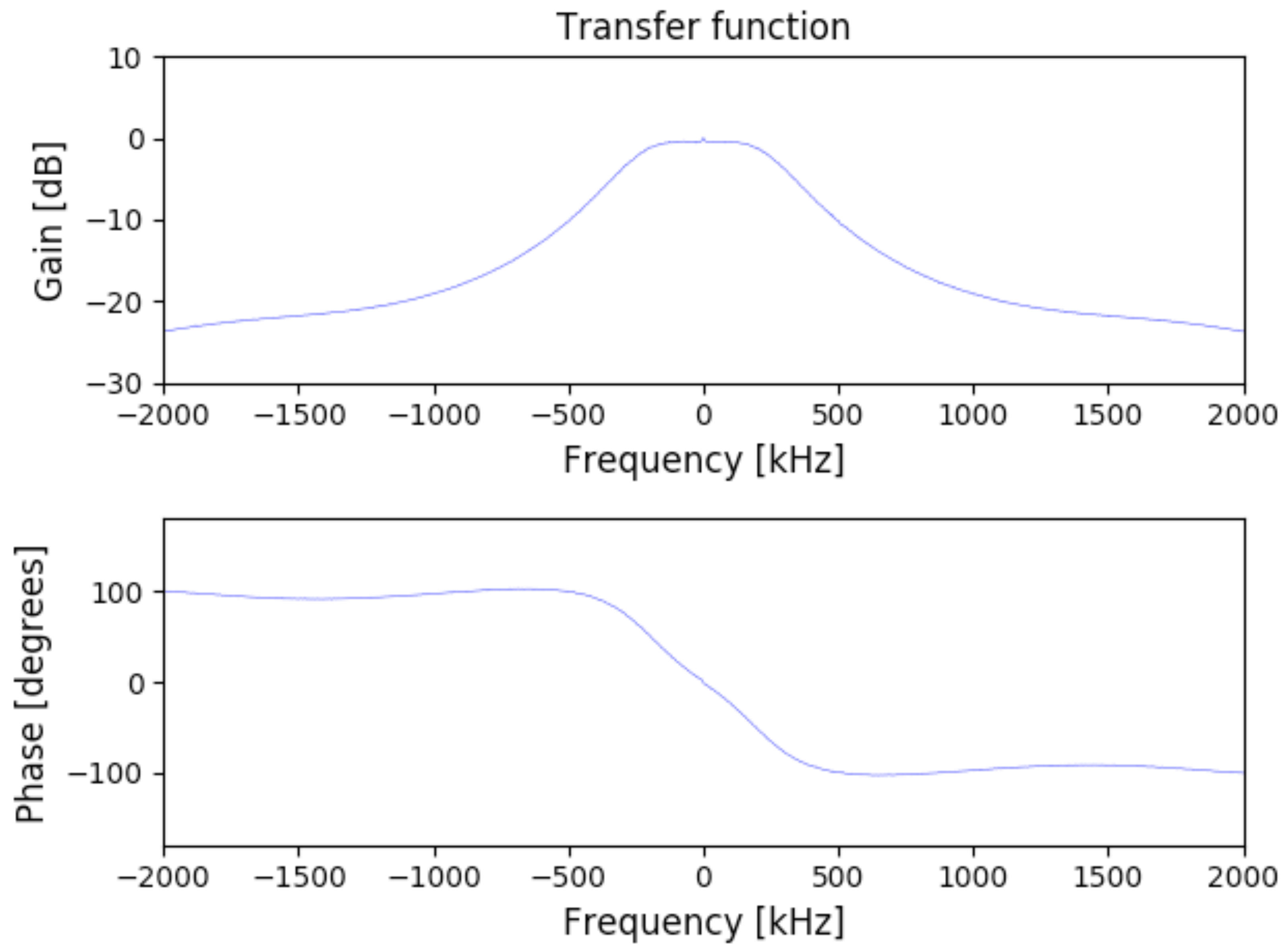
⊗ 0 ⚠ 19 0 Ln 1, Col 25 Spaces: 4 LF Cell 3 of 23

Content

Impedance reduction

Should one use a dynamic cavity controller model or effective impedance?

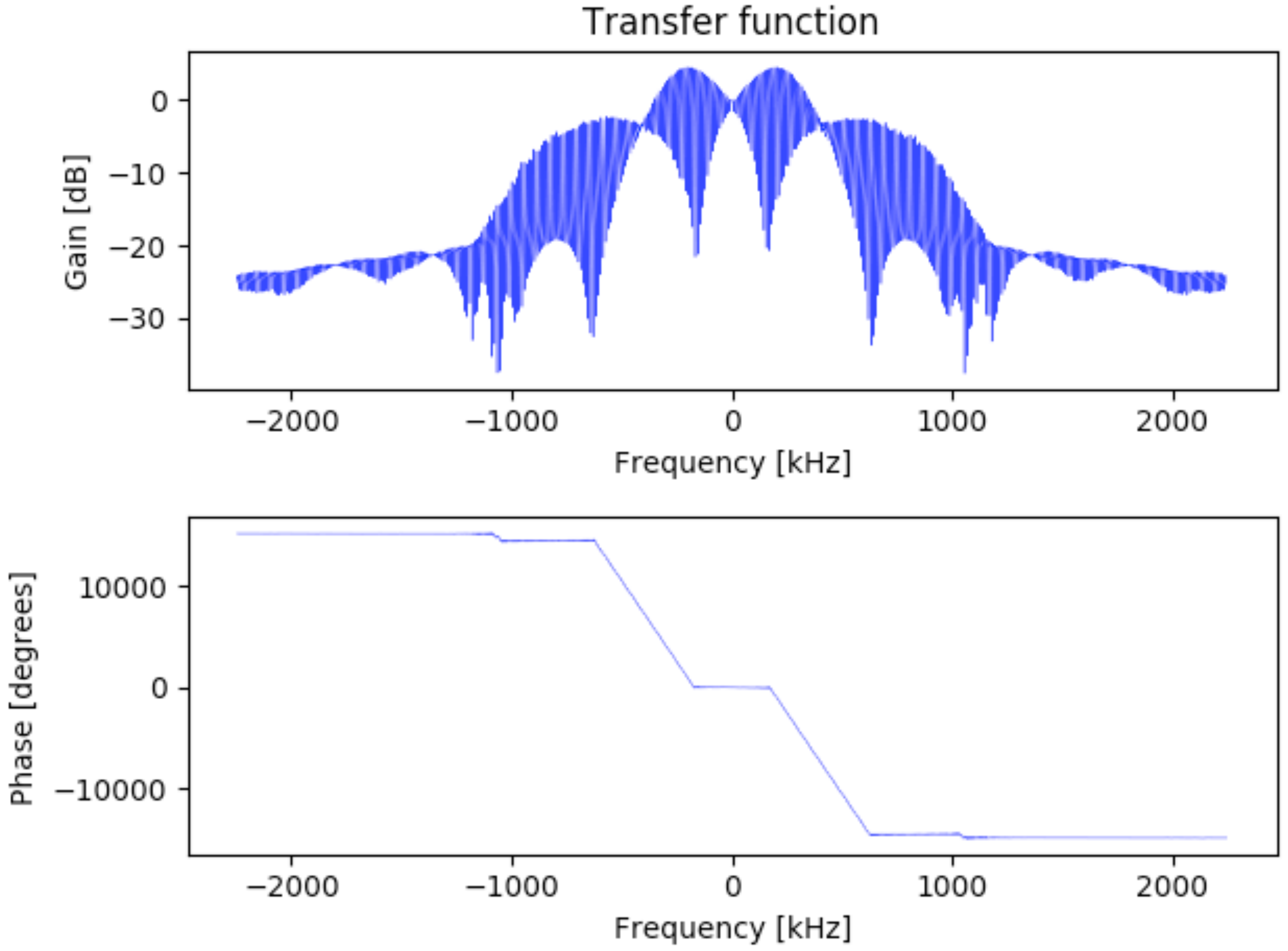
- A dynamic cavity controller model may be required
  - To simulate transient beam motion
  - For extremely narrow-band features of the impedance
- In all other cases: use the effective cavity impedance
  - Via the overall transfer function of the cavity controller
  - Computationally much faster



Example: LHC cavity impedance

- Bare cavity: resonator

$$Z(\omega) = \frac{R_s}{1 + jQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$



Simulated LHC transfer functions

Top: direct RFFB only

Bottom: direct RFFB & OTFB

Coupling global and local feedbacks

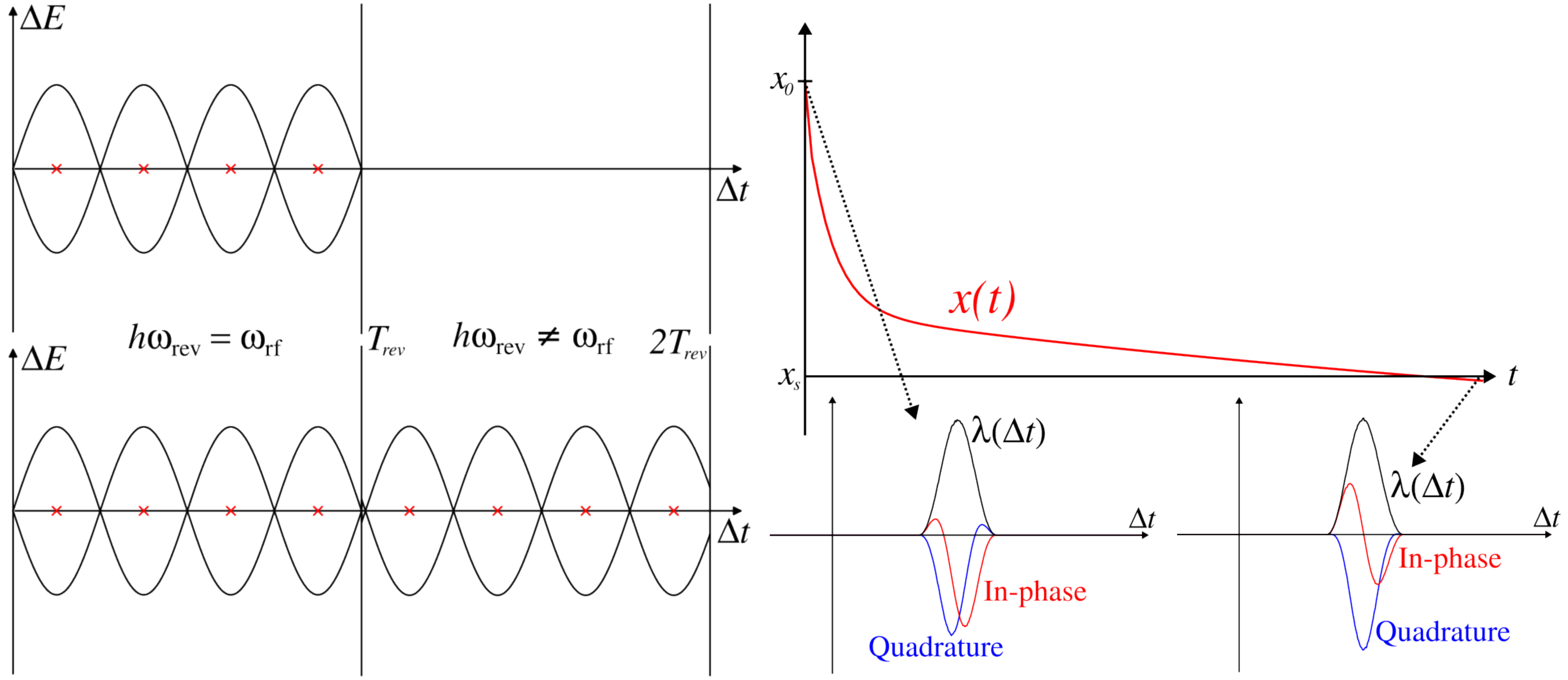
Decouping the RF from the revolution frequency

- The reference frame in BLonD is fixed to

$$T_{\mathrm{rev}} = \frac{2\pi}{\omega_{\mathrm{rev}}}$$

- E.g. when global loops are acting $\omega_{\mathrm{rf}} \neq h\omega_{\mathrm{rev}}$
- A turn-by-turn phase shift accumulates:

$$\varphi_{\mathrm{rf},k} = \sum_{i=1}^{n} \frac{\omega_{\mathrm{rf},k,(i)} - h_{k,(i)}\omega_{\mathrm{rev},(i)}}{h_{k,(i)}\omega_{\mathrm{rev},(i)}} 2\pi h_{k,(i)}$$



Implementation

- In the local loops: short of/in excess of a fraction of a sampling time
  - Move the sample centres by the equivalent time shift corresponding to $\varphi_{\mathrm{rf,k}}$
    - This phase shift also goes into the set point phase, ensuring that the RF wave is continuous w.r.t. reference frame
- Beam current: sampled with the RF wave shifted by $\varphi_{\mathrm{rf,k}}$
- In the tracker: RF voltage sampled at the bin size of the beam profile

Sampling in local loops when the RF and revolution frequencies are decoupled

Left: fraction of a sampling time missing

Right: beam current sampling

Courtesy of B. Karlsen-Bæck

How easy is it to make your own OTFB?

Make your own OTFB

LLRF workshop 2023, Gyeongju, Korea

```python
import os, sys
sys.path.insert(0,'/Users/timko/PycharmProjects/BLonD_bkarlsen')

# Import numpy and matplotlib
import numpy as np
import matplotlib.pyplot as plt
import matplotlib
from tqdm import tqdm
%matplotlib inline

# Import blond objects
from blond.beam.beam import Beam, Proton
from blond.beam.distributions import bigaussian
from blond.beam.profile import Profile, CutOptions
from blond.input_parameters.ring import Ring
from blond.input_parameters.rf_parameters import RFStation
from blond.trackers.tracker import RingAndRFTracker
from blond.llrf.cavity_feedback import CavityFeedback
from blond.llrf.signal_processing import comb_filter
from blond.llrf.impulse_response import cavity_response_sparse_matrix

from scipy.interpolate import interp1d
```

[1] ✓ 2.0s Python

```
---------- Using the C++ computational backend ----------
```

https://cernbox.cern.ch/s/YEmiRvwtN7m0uzm

Summary

Introduction
- BLonD is particle tracker for longitudinal beam dynamics in synchrotrons

Longitudinal tracking
- Used to model complicated RF manipulations

Collective effects
- Mostly applied for simulations with collective effects to study beam stability

Global control loops
- Machine-dependent implementations available for all CERN synchrotrons

Local control loops
- Require embedding in the particle tracker to expand the RF voltage vector

Closing remarks
- How to couple global and local loops

Take-home message

Particle tracking tools can also be used to design future LLRF systems based on the beam dynamics needs/limitations


관심 가져주셔서 감사합니다
Thank you for your attention!

Backup slides

Impedance sources

Impedance table e.g. from CST modelling

Resonator

$$Z(\omega) = \frac{R_s}{1 + jQ\left(\frac{\omega}{\omega_r} - \frac{\omega_r}{\omega}\right)}$$

$$W(t>0) = 2\alpha R_s e^{-\alpha t}\left(\cos\tilde{\omega}t - \frac{\alpha}{\tilde{\omega}}\sin\tilde{\omega}t\right)$$

$$W(0) = \alpha R_s, \text{ with } \alpha = \frac{\omega_r}{2Q} \text{ and } \tilde{\omega} = \sqrt{\omega_r^2 - \alpha^2}$$

ω_r resonant frequency
Q cavity Q
R_s shunt impedance

Travelling wave cavity [25]

$$Z = Z_+ + Z_- \text{ with } Z_\pm(\omega) \equiv R_s\left[\left(\frac{\sin\frac{\tilde{\omega}_\pm\tau}{2}}{\frac{\tilde{\omega}_\pm\tau}{2}}\right)^2 \mp 2i\frac{\tilde{\omega}_\pm\tau - \sin\tilde{\omega}_\pm\tau}{\left(\tilde{\omega}_\pm\tau\right)^2}\right]$$

$$W(0<t<\tau) = \frac{4R_s}{\tau}\left(1 - \frac{t}{\tau}\right)\cos\omega_r t$$

$$W(0) = \frac{2R_s}{\tau}$$

τ filling time
ω_r resonant frequency
$\tilde{\omega}_\pm \equiv \omega \pm \omega_r$
R_s shunt impedance

Resistive wall impedance

$$Z(f) = \frac{Z_0 cL}{\pi}\frac{1}{\left[1 - i\,\mathrm{sgn}f\right]2bc\sqrt{\frac{\sigma_c Z_0 c}{4\pi|f|}} + i2\pi b^2 f}$$

Constant $\Im(Z/n)$ e.g. for LLD or SC

$$\frac{Z}{n} = \frac{Z}{f/f_{\mathrm{rev}}} = \text{const.} \Rightarrow V_{\mathrm{ind}}[k] = -\frac{q\,T_{\mathrm{rev}}}{2\,\pi\,T_s}\frac{Z}{n}\frac{d\lambda[k]}{dn}$$

- Simplifies the computation of the induced voltage, replacing FFTs with the derivative of the line density
  - Caveat: line density derivative can add numerical noise!

[25] G. Dôme: ‘The SPS acceleration system travelling wave drift-tube structure for the CERN SPS’, CERN Report CERN-SPS-ARF-77-11, 1977.