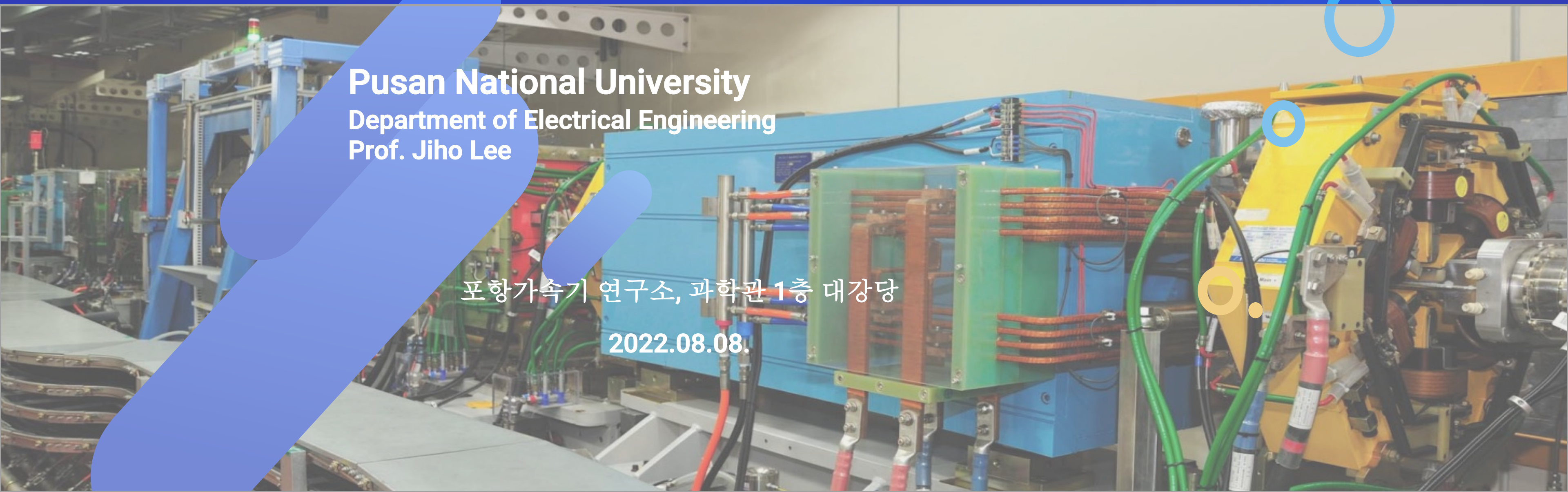
2022 ATE accelerator school

전자석의 이해
Understanding of Magnets



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Outline

- Part1: Introduction
  - Magnets
  - Accelerators and Magnets
  - Magnetic Field

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  - Multipole Field and Equations

- Part3: Superconducting Magnets
  - Superconducting magnets for high-energy accelerators

Part1: Introduction

Magnets

(Particle) accelerators and (electro-) magnets

Magnetic field

Magnet (자석)

Magnēsía (Μαγνησία) → Magnêtis (Magnesian stone) → Magnete
Greek region → Ancient Greek → Old French & middle English

- Definition
  - A magnet is a *material* or *object* that ***produces a magnetic field***.

- Magnets types by materials
  - **Permanent magnet**
    - 영구자석, <u>자석</u>
    - ~1.4 T
  - **Electromagnet (iron-dominated magnet)**
    - 전자석, 상전도 전자석, 상전도 자석, <u>자석</u>
    - ~2 T
    - <10 A/mm^2 for the most water-cooling magnets
    - Low/medium energy particle accelerators
  - **Superconducting magnet**
    - 초전도 전자석, 초전도 자석, <u>자석</u>
    - >2 T
    - >100 A/mm^2
    - High energy particle accelerators

Introduction: (Particle) Accelerators and Magnets

Particle Accelerators → Particle motion → Force

m $\mathbf{a}$

- Bending
- Focusing/defocusing
- Correcting

<Lorentz Force>

$$\mathbf{F} = m\mathbf{a} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}$$

Electric field (전기장)

Magnetic field (자기장) ← Magnet (자석)

Magnetic field? B? H?

B $[\mathrm{T} = \mathrm{Wb/m^2}]$ **H** $[\mathrm{A/m}]$

https://en.wikipedia.org/wiki/Magnetic_field

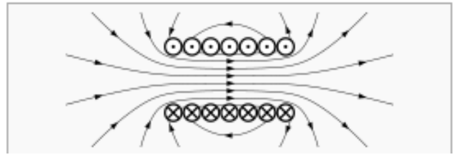
자기장

103개 언어

위키백과, 우리 모두의 백과사전.

자기장(磁氣場, 영어: magnetic field)이란 자석이나 전류에 의해 자기력이 작용하는 공간, 자기력을 매개하는 벡터장이다. 고전적으로는 움직이는 전하, 즉 전류에 의하여 발생하나, 양자역학에서는 입자 고유의 스핀도 전류와 같은 역할을 할 수 있다. (이에 따라 강자성체가 영구자성을 가질 수 있다.) 자기장의 방향은 자기장 안에 있는 나침반이 가리키는 방향과 같다. 또한 자기장의 방향을 연속적으로 이은 선의 간격이 촘촘할수록 자기장의 세기가 세다.

고전 전자기학



Alternative names for H[8]
• Magnetic field intensity[9]
• Magnetic field strength
• Magnetic field
• Magnetizing field

Alternative names for B[8]
• Magnetic flux density
• Magnetic induction[9]
• Magnetic field (ambiguous)

Magnetic field? B? H? (2)

기호와 용어 [편집]

"자기장"이라고 불리는 장은 $\mathbf{B}$와 $\mathbf{H}$ 두 개가 있다. 이 중 $\mathbf{B}$는 **자속 밀도(자기장)**(磁氣線束密度, magnetic flux density)이라 불리고, $\mathbf{H}$는 **자계 강도**(magnetic field strength)라고 부른다. 두 장은 진공에서는 서로 $\mathbf{B} = \mu_0\mathbf{H}$로 서로 비례하지만, 매질 안에서는 일반적으로 서로 다르다. 자속 밀도와 자계 강도가 서로 비례하는 매질을 **선형 매질**이라고 하는데, 이 때 비례 상수를 매질의 투자율 μ이라고 한다.

$$\mathbf{B} = \mu\mathbf{H}$$

국제단위계에서, 자속 밀도 $\mathbf{B}$의 단위는 테슬라(T)이고, 자계 강도 $\mathbf{H}$의 단위는 암페어 퍼미터(A/m)이다. CGS 단위계에서, $\mathbf{B}$의 단위는 가우스(G)이고, $\mathbf{H}$의 단위는 에르스텟(Oe)이다.

과거에는 보통 "자기장"이라고 하면 $\mathbf{H}$를 일컬었으나, 오늘날에는 $\mathbf{B}$가 로런츠 힘을 매개하는 더 근본적인 장이므로 보통 $\mathbf{B}$를 "자기장"이라고 부른다. 예를 들어, 에드워드 밀스 퍼셀은 저서 《전기와 자기》[1]에 다음과 같이 적었다.

> 심지어 $\mathbf{B}$를 기본 장으로 다루는 최근 저자들마저도 이를 "자기장"이라고 부르지 않는 경우가 있는데, 이는 역사적으로 "자기장"이라는 이름을 $\mathbf{H}$가 찬탈했기 때문이다. 이는 서투르고 고지식해 보인다. [......] 우리는 $\mathbf{B}$를 계속 "자기장"이라고 일컬을 것이다. $\mathbf{H}$의 경우에는 (다른 이름들도 제시된 바 있지만) 우리는 그냥 "$\mathbf{H}$장" 또는 "$\mathbf{H}$ 자기장"으로 부르겠다.

여기서는 현대적 용법을 따라 $\mathbf{B}$장을 "자기장"이라 부르도록 한다.

https://en.wikipedia.org/wiki/Magnetic_field

Magnitude of Magnetic Fields

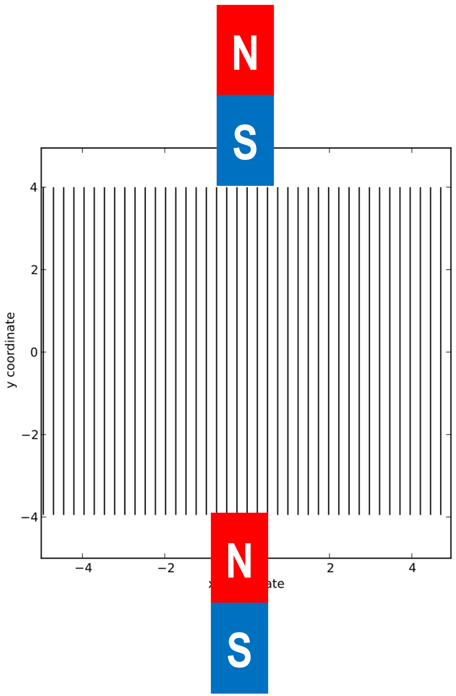
- **31-58 μT**: *Earth magnetic field* at 0º latitude
- **0.5 mT**: the suggested *exposure limit* for *cardiac pacemakers* by American Conference of Governmental Industrial Hygienists (ACGIH)
- **5 mT**: the strength of a typical *refrigerator magnet* or Apple iPhone *Magsafe*
- **1.5 T to 3 T**: strength of medical magnetic resonance imaging (*MRI*) systems in practice, experimentally up to 8 T
- 9.4 T: modern high resolution *research MRI* system
- 16 T: strength used to *levitate a frog*
- 41.4 T: strongest continuous magnetic field produced by non-superconductive *resistive magnet* (32 MW)
- ~~45 T~~: strongest continuous magnetic field yet produced in a laboratory (National High Magnetic Field Laboratory in Tallahassee, USA)
  - **45.5 T** achieved by S. Hahn with NHMFL

- Pulse magnets
  - 100.75 T: strongest (pulsed) magnetic field yet obtained *non-destructively* in a laboratory (NHMFL, Los Alamos National Laboratory, USA)
  - 730 T: strongest pulsed magnetic field yet obtained in a laboratory, *destroying* the used equipment, but not the laboratory itself (Institute for Solid State Physics, Tokyo)
  - 2.8 kT: strongest (pulsed) magnetic field ever obtained (with *explosives*) in a laboratory (VNIIEF in Sarov, Russia, 1998)

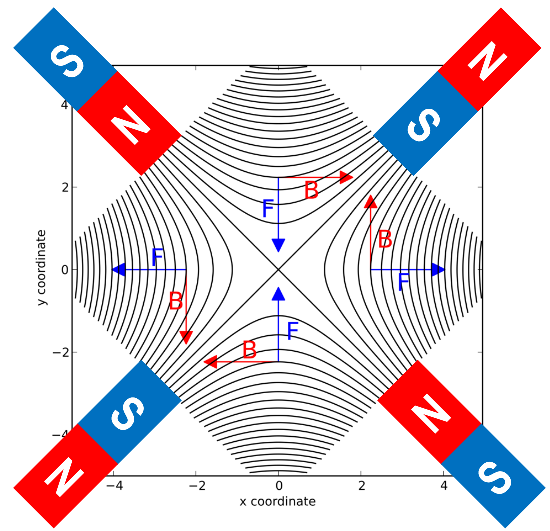
Part2: Accelerator Magnets

Multipole Magnets

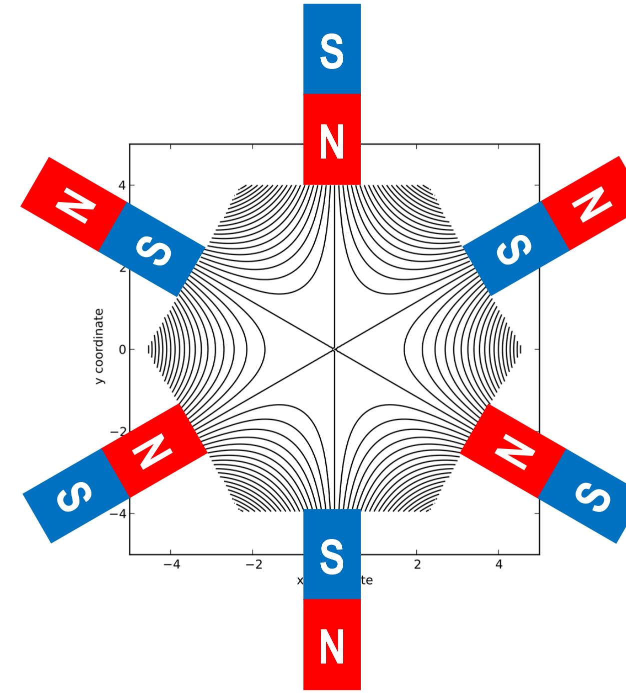
- "Multi"pole Magnets



di-: 2



quadro-: 4



hexa-: 6
(sextupole)

Magnet Types by Shapes and Functions

- The conventional (iron-dominated) magnets described herein are those whose fields are shaped by iron poles, where the maximum field level in the yoke is less than the iron saturation level (1.6-2.2 T) and whose excitation is provided by current carrying coils.

REF: "Iron Dominated Electromagnets", Jack Tanabe

- The iron dominated magnets
  - **Dipoles**
    - *Bending* the particle trajectory
  - **Quadrupoles**
    - *Focusing* and *defocusing* the charged particle beam
  - **Sextupoles**
    - Control (*correction*) of *chromatic aberrations*
  - **Correctors**
    - Vertical and horizontal steering (small angle correction)
    - Skew Quadrupoles
  - Specialized magnets
    - Gradient magnets (a specialized dipole)
    - Current Sheet Septum (Horizontal bend)
    - Lambertson Septum (Vertical bend)
    - Bump and Kicker Magnets

- Air-core magnets
  - **Solenoid**

Maxwell's Magnet (Steady State) Equations

$$\begin{cases} \nabla \times \mathbf{B} = \mu\mu_0 \mathbf{J} \\ \nabla \times \mathbf{B} = 0 \end{cases}$$

(in the absence of the sources)

$\Rightarrow$

$$\nabla^2 F = \mu\mu_0 \mathbf{J}$$ Poisson's Eq.

$$\nabla^2 F = 0$$ Laplace's Eq.

$$\nabla \cdot \mathbf{B} = 0$$

$\Rightarrow$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\mathbf{B} = -\nabla V = -\hat{x}\frac{\partial V}{\partial x} - \hat{y}\frac{\partial V}{\partial y} - \hat{z}\frac{\partial V}{\partial z}$$

A function of complex variable, F:

$$F = \mathbf{A} + iV$$

$\mathbf{A}$: the vector potential
V: the scalar potential

- An ideal *pole contour* can be computed using the *scalar equipotential*.
- The *field shape* can be computed using the *vector equipotential*.

REF: Electromagnetics (전자기학) textbook

Maxwell's Magnet (Steady State) Equations (2)

$\nabla\times\mathbf{B} = 0$ (no sources) $\Rightarrow$ Assuming that the Coulomb condition, $\nabla\cdot\mathbf{A} = 0$

$$\mathbf{B} = \nabla\times\mathbf{A}$$

$$\nabla\times\mathbf{B} = \nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\mathbf{A} = -\nabla^2\mathbf{A} = 0 \quad\Rightarrow\quad \boxed{\nabla^2\mathbf{A} = 0}$$

Vector identity

$$\nabla\cdot\mathbf{B} = 0$$

$$\mathbf{B} = -\nabla V = -\hat{x}\frac{\partial V}{\partial x} - \hat{y}\frac{\partial V}{\partial y} - \hat{z}\frac{\partial V}{\partial z} \quad\Rightarrow\quad \nabla\cdot\mathbf{B} = -\nabla^2 V = 0 \quad\Rightarrow\quad \boxed{\nabla^2 V = 0}$$

The function, $F = \mathbf{A} + iV$ must also satisfy the Laplace equation

$$\boxed{\nabla^2 F = 0}$$

The Two-Dimensional Fields

- The function: $F = \underline{\mathbf{A}} + i\underline{V}$

vector potential　　scalar potential

$$\mathbf{B} = \nabla\times\mathbf{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\mathbf{B} = -\nabla V = -\hat{x}\frac{\partial V}{\partial x} - \hat{y}\frac{\partial V}{\partial y} - \hat{z}\frac{\partial V}{\partial z}$$

$$\mathbf{B} = \nabla\times\mathbf{A} = \hat{x}\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) + \hat{y}\left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) + \hat{z}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) = \mu\mu_0\left(\hat{x}J_x + \hat{y}J_y + \hat{z}J_z\right)$$

Two-dimensional field assumption

3-dimensional form

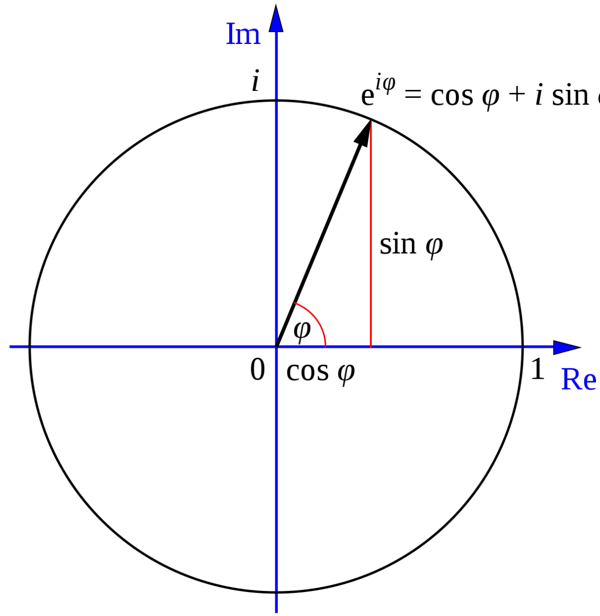
$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \mu\mu_0 J_z$$

: <u>two-dimensional magnetic fields</u> (satisfy the scalar equation).

(x, y) plane

Fundamental Relationships [Euler's identity - Wikipedia]

- Geometric interpretation
  - The any complex number $z(= x + iy)$ can be represented by the point (x, y) on the complex plane.
  - In polar coordinate, the point can be represented as (r, θ).



$$z = x + iy = re^{i\theta} = r(\cos\theta + i\sin\theta)$$

$$|z| = \sqrt{x^2 + y^2} = r$$

, where:

$$\theta = \tan^{-1}\frac{y}{x}$$

Fields from the Two-Dimensional Function of a Complex Variable

$$F = A + iV \quad \Rightarrow \quad \mathbf{B}?$$

$$B^* = B_x - iB_y = iF'(\mathbb{z}) = i\frac{dF}{d\mathbb{z}} = i\left(\frac{\partial F}{\partial x}\frac{dx}{d\mathbb{z}} + \frac{\partial F}{\partial y}\frac{dy}{d\mathbb{z}}\right) = i\left(\frac{\partial F}{\partial x} - \frac{\partial F}{\partial y}i\right) = \boxed{\frac{\partial F}{\partial y}} + i\boxed{\frac{\partial F}{\partial x}}$$

$B_x \qquad B_y$

$$\boxed{B_x = \frac{\partial F}{\partial y}} \qquad \boxed{B_y = -\frac{\partial F}{\partial x}}$$

$$\boxed{\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \mu\mu_0 J_z} \quad \Rightarrow \quad \boxed{\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = -\mu\mu_0 J_z}$$

Poisson's Equation

Two-Dimensional Fields in a Vacuum

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = -\mu\mu_0 J_z$$

Poisson's Equation

in the magnet (pole) gaps

no current sources
& permeable materials

$$\nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = 0$$

Laplace's Equation

Let's find the solution of the Laplace Equation.

$$\mathbb{z} = x + iy \qquad \frac{dx}{d\mathbb{z}} = 1 \qquad \frac{dy}{d\mathbb{z}} = -i$$

$$\frac{\partial F}{\partial x} = \frac{dF}{d\mathbb{z}}\frac{d\mathbb{z}}{dx} = \frac{dF}{d\mathbb{z}} \qquad \frac{\partial F}{\partial y} = \frac{dF}{d\mathbb{z}}\frac{d\mathbb{z}}{dy} = \frac{dF}{d\mathbb{z}}i$$

$$\frac{\partial^2 F}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial F}{\partial x} = \frac{\partial}{dx}\frac{dF}{d\mathbb{z}} = \boxed{\frac{d^2 F}{d\mathbb{z}^2}}$$

$$\frac{\partial^2 F}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial F}{\partial y} = \frac{\partial}{dy}\frac{dF}{d\mathbb{z}}i = \frac{d^2 F}{d\mathbb{z}^2}i\frac{\partial z}{\partial x} = \frac{d^2 F}{d\mathbb{z}^2}i^2 = \boxed{-\frac{d^2 F}{d\mathbb{z}^2}}$$

$$\nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = \frac{\partial^2 F}{\partial \mathbb{z}^2} - \frac{\partial^2 F}{\partial \mathbb{z}^2} = 0$$

Solution to Laplace's Equation and Ideal "Multipoles"

$$\nabla^2 F = \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = \frac{\partial^2 F}{\partial \mathbb{z}^2} - \frac{\partial^2 F}{\partial \mathbb{z}^2} = 0$$

Laplace's Equation

$F = A + V$ Solution

$F = C\mathbb{z}^n$ in another form

$$\mathbb{z} = x + iy = |\mathbb{z}|e^{i\theta}$$

<Ideal Two-Dimensional "Multipole" Magnets>

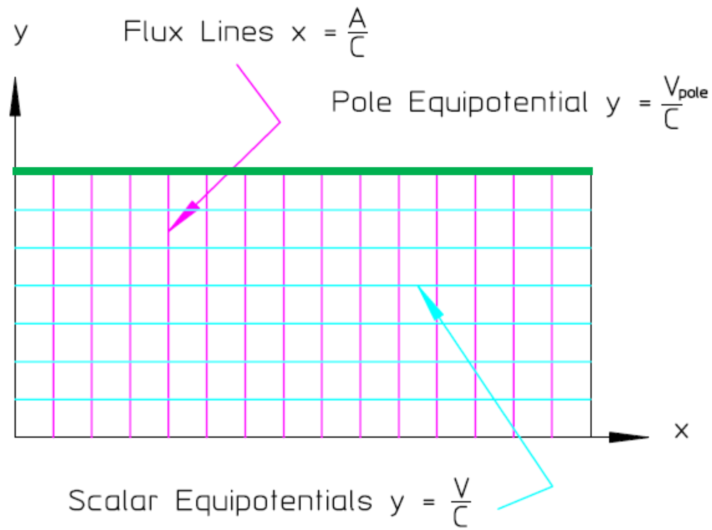
$$F = C\mathbb{z}^n \begin{cases} n = 1 \Rightarrow F = C_1\mathbb{z}^1 & \text{Dipole} \\ n = 2 \Rightarrow F = C_2\mathbb{z}^2 & \text{Quadpole} \\ n = 3 \Rightarrow F = C_3\mathbb{z}^3 & \text{Sextupole (Hexapole)} \\ n = 4 \Rightarrow F = C_4\mathbb{z}^4 & \text{Octopole} \\ \ldots \Rightarrow F = C_n\mathbb{z}^n & 2n\text{-pole} \end{cases}$$

Dipole Magnets $(n = 1)$

$$F = C_1 \mathbb{z} = C_1(x + iy) = A + iV$$

$x = \dfrac{A}{C_1}$: Vector equipotential

$y = \dfrac{V}{C_1}$: Scalar equipotential

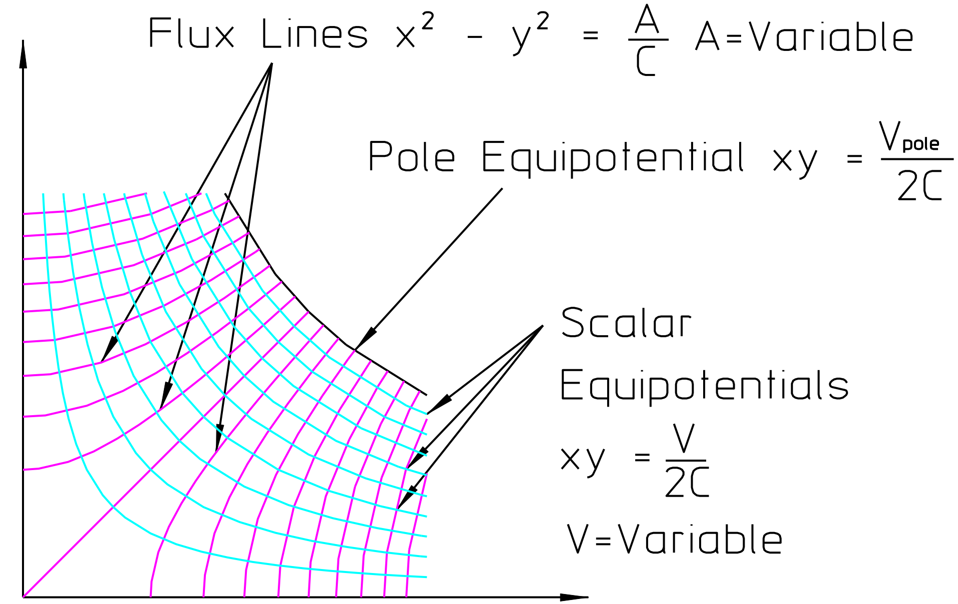


Quadrupole Magnets $(n = 2)$

$$F = C_2 \mathbb{z}^2 = C_2(x + iy)^2 = C_2(x^2 - y^2 + i2xy) = A + iV$$

$$x^2 - y^2 = \frac{A}{C_2}$$

$$xy = \frac{V}{2C_2}$$



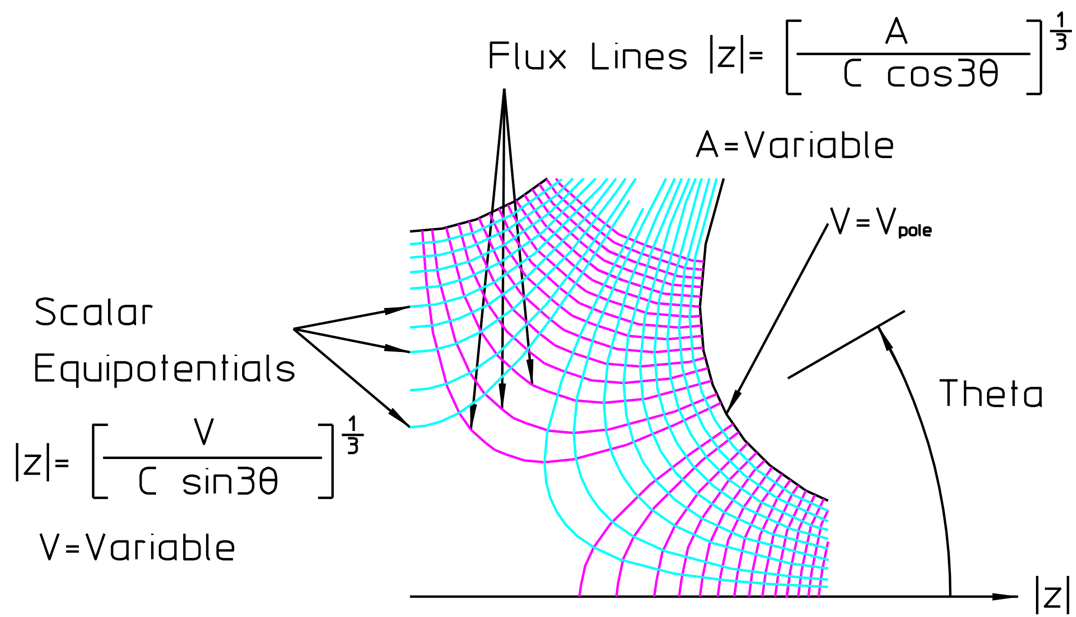
Sextupole Magnets $(n = 3)$ in polar coordinates

$$F = C_3 \mathbb{z}^3 = C_3(x+iy)^3 = C_3(x^3 - 3xy^2) + iC_3(3xy^2 - y^3) = A + iV$$

$$= C_3|\mathbb{z}|^3 e^{i3\theta} = C_3|\mathbb{z}|^3(\cos 3\theta + i\sin 3\theta) = A + iV$$

$$x^3 - 3xy^2 = \frac{A}{C_3}$$

$$3xy^2 - y^3 = \frac{V}{C_3}$$



"Real" Multipole Magnets with Multipole Errors

$F = C\mathbb{z}^n$

(Ideal) solution of the Laplace Eq.

→ Ideal multipole magnets ($n = 1,2,3, \ldots$)

$F = \Sigma C\mathbb{z}^n$

Taylor expansion of the function F

→ must also satisfy the Laplace's equation.

$$F = C_1\mathbb{z}^1 + C_2\mathbb{z}^2 + C_3\mathbb{z}^3 + C_4\mathbb{z}^4 + \cdots$$

$N = 1$ (dipole) $\quad F = C_1\mathbb{z}^1 + \sum_{n\neq 1} C_n\mathbb{z}^n$

$N = 2$ (quadrupole) $\quad F = C_2\mathbb{z}^2 + \sum_{n\neq 2} C_n\mathbb{z}^n$

General form (N-pole) $\quad F = C_N\mathbb{z}^N + \sum_{n\neq N} C_n\mathbb{z}^n$

Fundamental (desired) field component — Multipole error (undesired) component

Magnetic Field from $F(\mathbb{z})$ (1) Dipole Field

$$F = A + iV = C_n \mathbb{z}^n \quad \Rightarrow \quad \mathbf{B}?$$

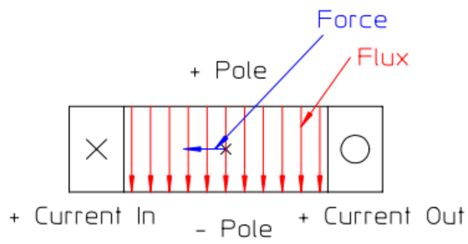
$$B^* = B_x - iB_y = iF'(\mathbb{z}) = i\frac{\partial(C_n \mathbb{z}^n)}{\partial \mathbb{z}} = inC_n \mathbb{z}^{n-1}$$

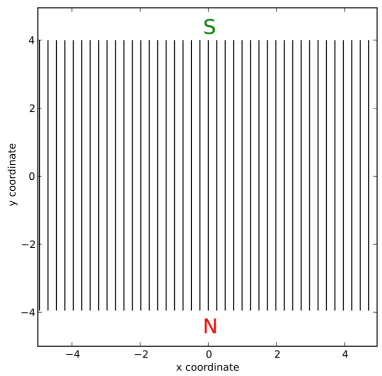
$n = 1$: dipole magnet

$$B_1^* = B_{1x} - iB_{1y} = iC_1$$

$$B_{1x} = 0$$

$$B_{1y} = -C_1$$





Magnetic Field from $F(\mathbb{z})$ (2) Quadrupole Field

$$F = A + iV = C_n \mathbb{z}^n \quad \Rightarrow \quad \mathbf{B}?$$

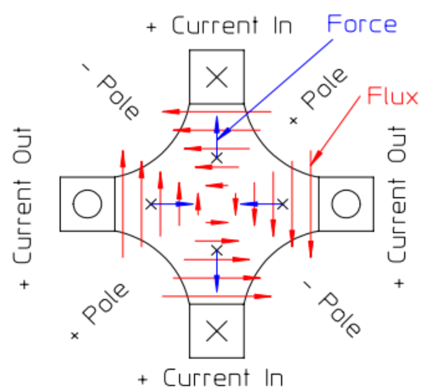
$$B^* = B_x - iB_y = iF'(\mathbb{z}) = i\frac{\partial(C_n\mathbb{z}^n)}{\partial \mathbb{z}} = inC_n\mathbb{z}^{n-1}$$

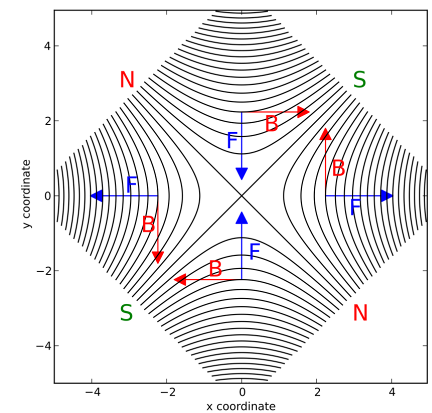
$n = 2$: quadrupole magnet

$$B_2^* = B_{2x} - iB_{2y} = i2C_2\mathbb{z} = i2C_2(x+iy) = -2C_2y + i2C_2x$$

$$B_{2x} = -2C_2y$$

$$B_{2y} = -2C_2x$$





Magnetic Field from $F(\mathbb{z})$ (3) Sextupole Field

$$F = A + iV = C_n \mathbb{z}^n \quad \Rightarrow \quad \mathbf{B}?$$

$$B^* = B_x - iB_y = iF'(\mathbb{z}) = i\frac{\partial(C_n\mathbb{z}^n)}{\partial\mathbb{z}} = inC_n\mathbb{z}^{n-1}$$

$n = 3$: sextupole magnet

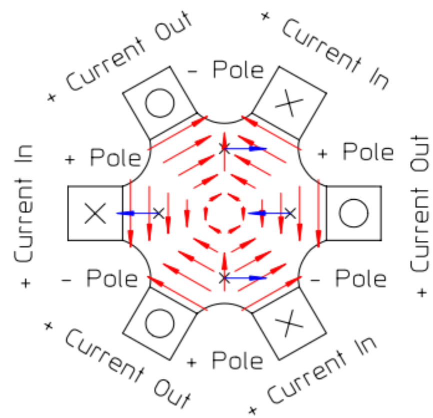
$$B_3^* = B_{3x} - iB_{3y} = i3C_3\mathbb{z}^2 = i3C_3(x+iy)^2 = i3C_3(x^2 - y^2 + i2xy)$$

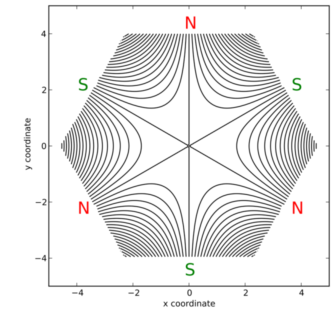
$$= i3C_3|\mathbb{z}|^2 e^{i2\theta} = i3C_3|\mathbb{z}|^2(\cos 2\theta + i\sin 2\theta)$$

$$B_{3x} = -6C_3xy = -3C_3|\mathbb{z}|^2 \sin 2\theta$$

$$B_{3y} = 3C_3(x^2 - y^2) = -3C_3|\mathbb{z}|^2 \cos 2\theta$$

$$B_{3r} = B_{3x}\cos\theta + B_{3y}\sin\theta = -3C_3|\mathbb{z}|^2 \sin 3\theta$$

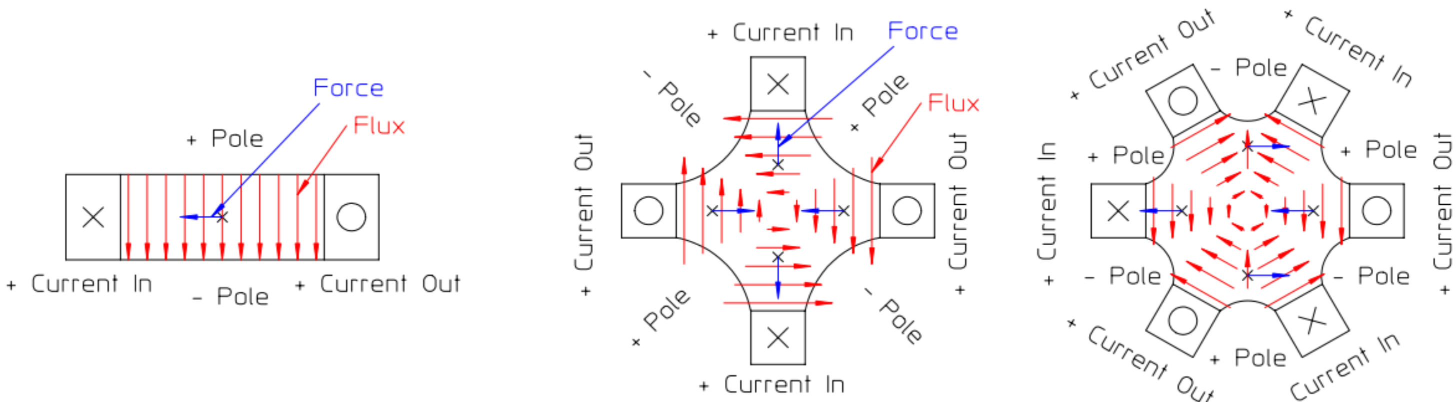




Conventions

- Current
  - Positive (+) current flows from the positive (+) lead of the power supply to the negative (-) lead.
- Field
  - ◯: *out of* the page
  - ×: *into* the page

- Ex: for the positive particle beam



Part3: Superconducting Magnets

Superconductivity

- Superconductivity (science)
  - The set of physical properties observed in certain materials, wherein electrical resistance vanishes and from which magnetic fields (flux) are expelled.

(Perfect) Superconductor: $\rho = 0, B = 0$

Normal conductor: $\rho \neq 0, B \neq 0$

Zero resistance!

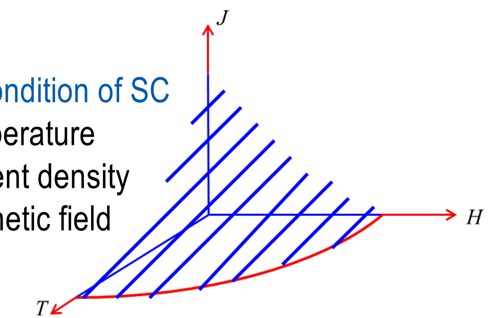
Perfect diamagnetism (Meissner effect)!

Critical condition of SC

Temperature

Current density

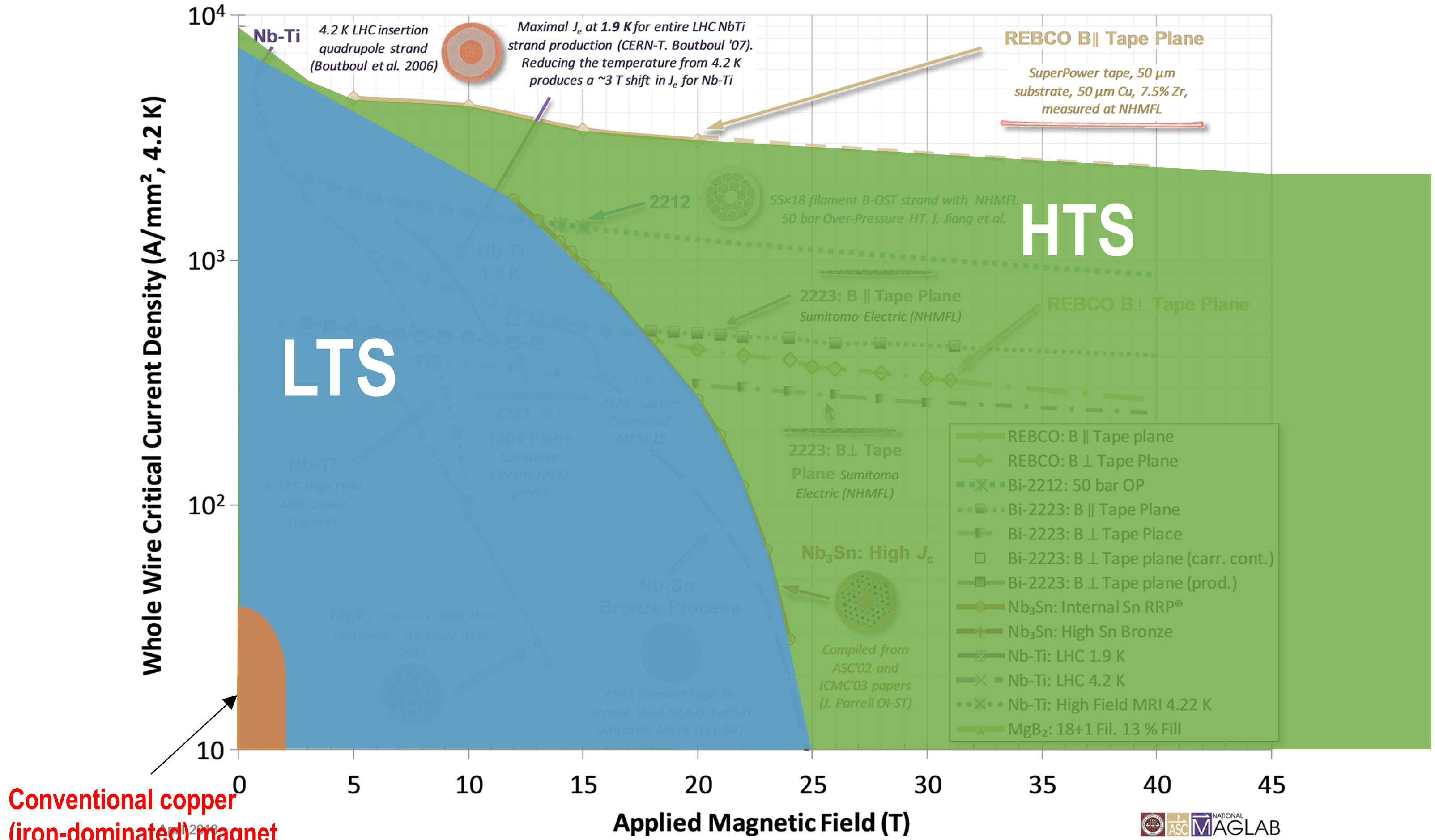
Magnetic field



- Classification of Superconductor
  - Response to a magnetic field: Type I (a single critical field) & Type II (two critical fields)
  - By theory of operation: conventional (explained by BCS theory, $T_c < 40\ K$) & unconventional ($T_c \geq 40\ K$)
  - By material: chemical elements (mercury or lead), alloys (NbTi, NbGe), cermaics (YBCO and MgB_2), superconducting pnictides (fluorine-doped LaOFeAs), or organic superconductors (fullerencs and carbone nanotubes)
  - **[engineering] By the critical temperature (LTS: <30 K)**
    - **Low-Temperature Superconductor (LTS)**: NbTi (~10 K), Nb_3Sn (18.3 K),
    - **High-Temperature Superconductor (HTS)**: REBCO (YBCO, GdBCO, ~93 K)

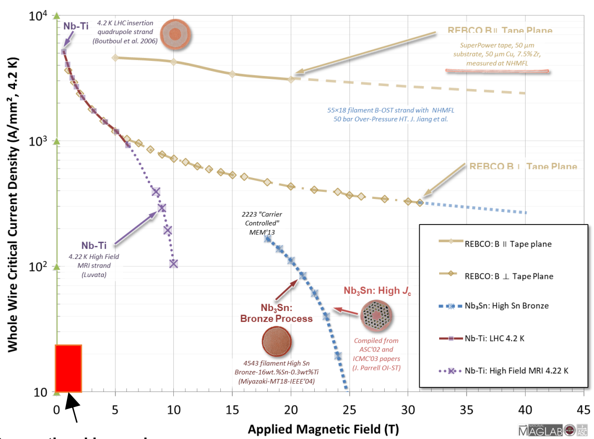
Copper vs. LTS vs. HTS

REF: Peter J. Lee, https://fs.magnet.fsu.edu/~lee/plot/plot.htm



Why HTS?

- High overall **current density**, J_e → more compact magnets
- High **stability** (energy) margin (Δe_h) → High T_{op} (versus LTS magnets)
- NI or MI REBCO coil → Dry winding (no epoxy) → Easier fabrication



Conventional iron yoke electromagnet

REF: https://fs.magnet.fsu.edu/~lee/plot/plot.htm

Table 6.4: Selected Values of T_{op}, ΔT_{op}, and Δe_h for LTS and HTS

LTS			*HTS*		
T_{op} [K]	$[\Delta T_{op}(I_{op})]_{st}$ [K]	Δe_h [J/cm^3]	T_{op} [K]	$[\Delta T_{op}(I_{op})]_{st}$ [K]	Δe_h [J/cm^3]
2.5	0.3	1.2×10^{-4}	4.2	25	1.6
4.2	0.5	0.6×10^{-3}	10	20	1.8
4.2	2	4.3×10^{-3}	30	10	3.7
10	1	9×10^{-3}	70	5	8.1

REF: Y. Iwasa, Case Studies in Superconducting Magnets (2nd ed.), Ch. 6. Stability, p357