VPP Error Minimization from a Projection Point of View

Yu Du, Jonathan C. Wong

1. *Institute of Modern Physics, Chinese Academy of Sciences*
2. *University of Chinese Academy of Sciences*

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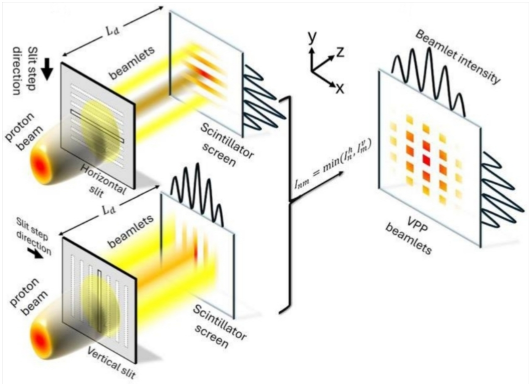
Optimal Input Distribution Selection in VPP

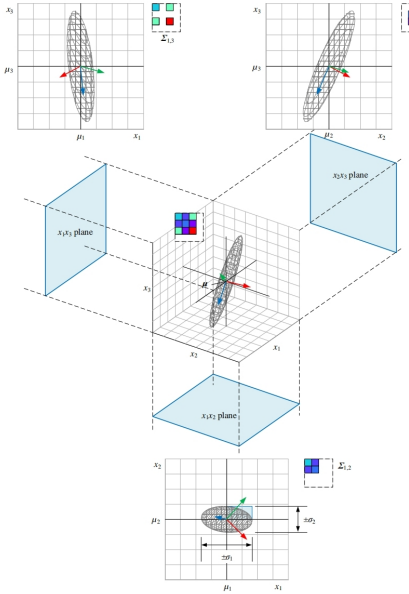
- Real input distribution profile measurement:
  - Horizontal scan: $\rho(y_i, x_f, y_f)$
  - Vertical scan: $\rho(x_i, x_f, y_f)$
- Reconstruction:
  $\rho(y_i, x_f, y_f) + \rho(x_i, x_f, y_f) \rightarrow \rho(x_i, y_i, x_f, y_f)$
- With the horizontal and vertical scanning device fixed, what input distribution maximizes reconstruction quality?
  - In terms of information content: lower information overlap between profiles (more information) $\rightarrow$ better reconstruction.



VPP Measurement Principle

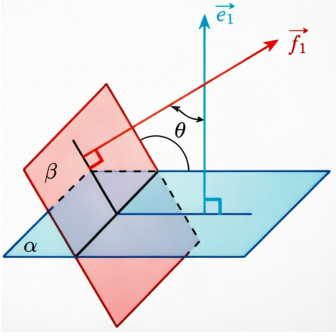
Projection Perspective

- Analyzing information content from a projection perspective:
  - Larger projection angle
  - Lower information overlap
  - Better reconstruction
- How to analyze the projection angle?
  - Using the projection geometry tool



Projection Geometry Tool

- VPP: Two 3D projections in 4D space.
- The angle between two 3D profiles in 4D space is simply the angle between their 1D normal vectors.
- Calculation formula: $\nu_n = (\vec{e}_1 \cdot \vec{f}_1)^2 = \cos^2\theta$

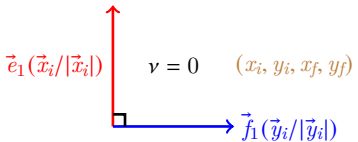


Dihedral Angle Calculation in 3D Space

Coordinate Transformation

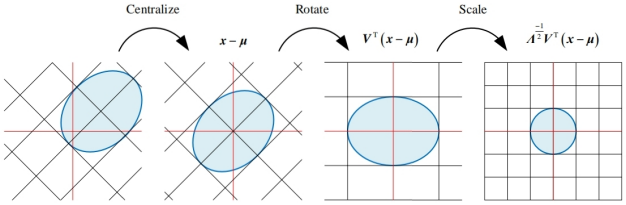
In (x_i, y_i, x_f, y_f), the scanning device is fixed:

- Normal vectors: $\rho(y_i, x_f, y_f)$ corresponds to $\frac{\vec{x}_i}{|\vec{x}_i|}$, and $\rho(x_i, x_f, y_f)$ corresponds to $\frac{\vec{y}_i}{|\vec{y}_i|}$.
- The angle between the normal vectors of the profiles is always 90° for all input distributions.
- From this perspective, the information content is the same for all different input distributions, which is clearly unreasonable.

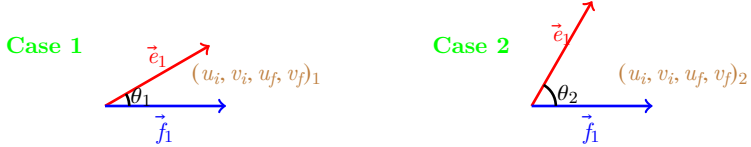


Coordinate Transformation

- From a right-angle perspective, the projection of the spherical distribution carries the maximum information.
- After transforming the 4D ellipsoid into a sphere, in (u_i, v_i, u_f, v_f):



  - The normal angle varies with the input distribution → distinguishable
  - In the figure below, the input distribution of Case 2 is better than that of Case 1.



中国科学院大学
University of Chinese Academy of Sciences

中国科学院近代物理研究所
Institute of Modern Physics, Chinese Academy of Sciences

Thanks!