KNU 강원대학교 KANGWON NATIONAL UNIVERSITY

Transverse Phase Space Reconstruction On VPP via MENT

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2026 The Beam Diagnostics and Tomography Study

(Satellite meeting at KOMAC)

Outline

I. Introduction

- The KOMAC Beam test stand(BTS) layout
- The Maximum entropy theory(MENT)

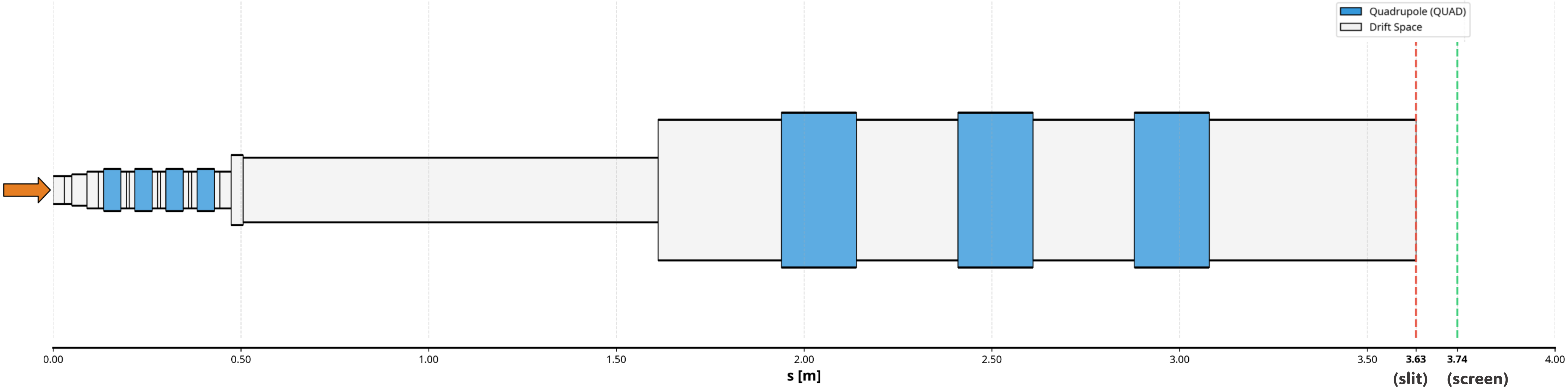
II. 4D-Reconstruction with VPP Data via MENT

- Reconstruction results
- Observation and open question

III. Conclusion

Introduction

KOMAC BTS(Beam Test Stand) implementation in x-suite



Beam parameters

Energy	1 MeV
N total	3,000,000
$\varepsilon_{x/y,rms}$	0.16 [π mm·mrad]
rms bunch length	1.141820 mm
σ_δ (Δp/p)	0.507123 %

Simulation set-up

Slit width	0.2mm
Slit step(continuous scan)	0.2mm
Distance L	0.11m
Screen resolution	$30\mu m$
Framework	X-suite

Introduction

Maximum entropy theory(MENT)

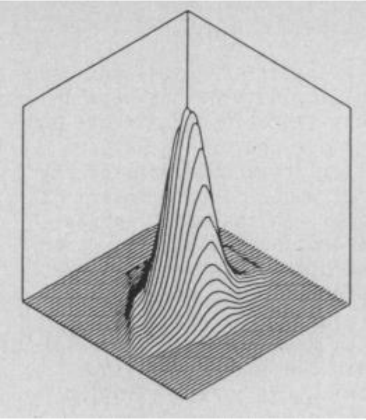
Q. How should we assign a probability density function(pdf) when 'testable information' is available?

A. Jaynes(1957) has suggested that we should make the assignment by using the principle of maximum entropy.

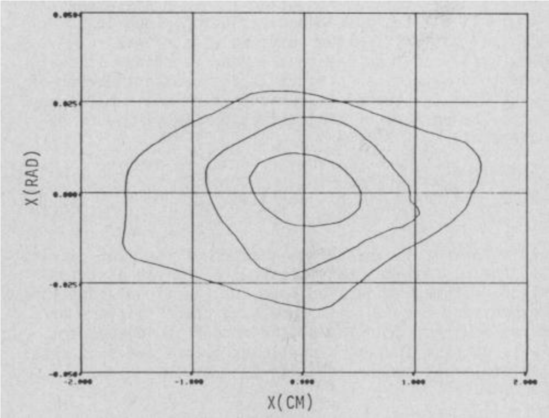
That is, we should **choose that pdf which has the most entropy S while satisfying all the available constraints**.

⇒ Need to maximize

$$S = -\sum_{i=1}^{M} p_i \ln[p_i] \qquad S[\rho] = -\iiiint \rho_{4D} \ln \rho_{4D} \, dx dx' dy dy'$$



(isometric display of MENT solution)



(contour plot of MENT solution)

4D-Reconstruction with VPP Data via MENT

New reconstruction scheme with MENT

The measured 3D data are projections of the 4D distribution in the $\tilde{\rho}(x_i, y_i, x_f, y_f)$ space. And the transformed $\tilde{\rho}$-space is mapped to the original phase space via the L^2.

$$\rho(x_i, x_i', y_i, y_i') = \rho\left(x_i, \frac{x_f - x_i}{L}, y_i, \frac{y_f - y_i}{L},\right) = L^2 \tilde{\rho}(x_i, x_f, y_i, y_f)$$

Each VPP measurement provides a 3D marginal of the 4D transformed phase-space density.

Horizontal scan:

$$\rho_h^{(meas.)}(x_i, x_f, y_f) = \int \tilde{\rho}(x_i, x_f, y_i, y_f) dy_i$$

Vertical scan:

$$\rho_v^{(meas.)}(y_i, x_f, y_f) = \int \tilde{\rho}(x_i, x_f, y_i, y_f) dx_i$$

$$\tilde{\rho}_{3D}(x_i, x_f, y_f) = \frac{H[i, f_x, f_y]}{\Delta x_i \Delta x_f(f_x) \Delta y_f(f_y)}$$

The MENT Method in Code

Gauss-Seidel iterative method

$\rho_1, \rho_2 \cdots = constraints$

$M = \int \rho \, dx_{1/2} : \; marginal$

$\rho^{(0)} = gaussian \; distribution$

Gauss – Seidel. $\;\; if \; we \; have \; two \; constraints [\tilde{\rho}(x_i, x_f, y_f), \tilde{\rho}(y_i, x_f, y_f)]$

$$R = \frac{\tilde{\rho}(x_i, x_f, y_f)}{\int \rho_{4D}^{(0)} dy_i}$$

$$\begin{bmatrix} \rho_{4D}^{(1)} = \rho_{4D}^{(0)} \dfrac{\tilde{\rho}(x_i, x_f, y_f)}{\int \rho_{4D}^{(0)} dy_i} \\ \rho_{4D}^{(2)} = \rho_{4D}^{(1)} \dfrac{\tilde{\rho}(y_i, x_f, y_f)}{\int \rho_{4D}^{(1)} dx_i} \end{bmatrix} \Rightarrow \begin{matrix} \rho_{4D}^{(3)} = \rho_{4D}^{(2)} \dfrac{\tilde{\rho}(x_i, x_f, y_f)}{\int \rho_{4D}^{(2)} dy_i} \\ \rho_{4D}^{(4)} = \rho_{4D}^{(3)} \dfrac{\tilde{\rho}(y_i, x_f, y_f)}{\int \rho_{4D}^{(3)} dx_i} \end{matrix} \cdots$$

Convergence condition: $\|\rho^{m+1} - \rho^m\| < \epsilon.$

How accurate is the reconstruction?

Emittance vs. Total variable distance(TVD)

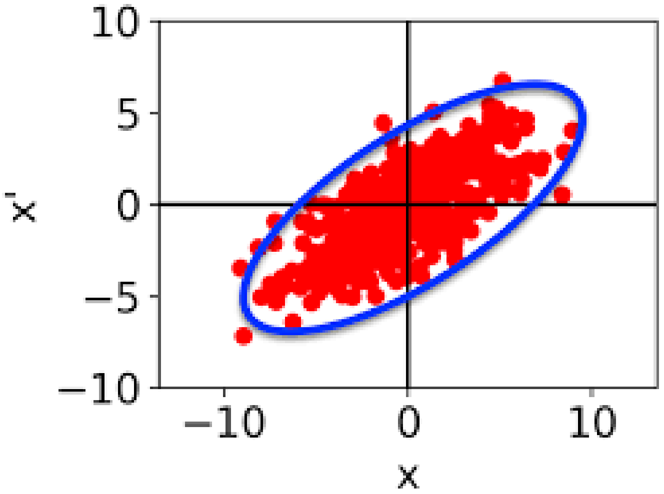
- ***4D emittance error***

$$\varepsilon_{4D} = \sqrt{det\Sigma} \qquad \Sigma = \begin{pmatrix} <xx> & <xx'> & <xy> & <xy'> \\ <x'x> & <x'x'> & <x'y> & <x'y'> \\ <yx> & <yx'> & <yy> & <yy'> \\ <y'x> & <y'x'> & <y'y> & <y'y'> \end{pmatrix}$$

- ***Total variable distance(TVD)*** : to quantify the difference

  - The TVD ranges from 0 for identical to 1for completely disjoint.

$$TVD(P,Q) = \frac{1}{2}\sum |P_i - Q_i|$$



➔ **TVD** : well suited for comparing non-gaussian structure(masking), space charge effect, and coupling.

Effect of Masking on Reconstruction

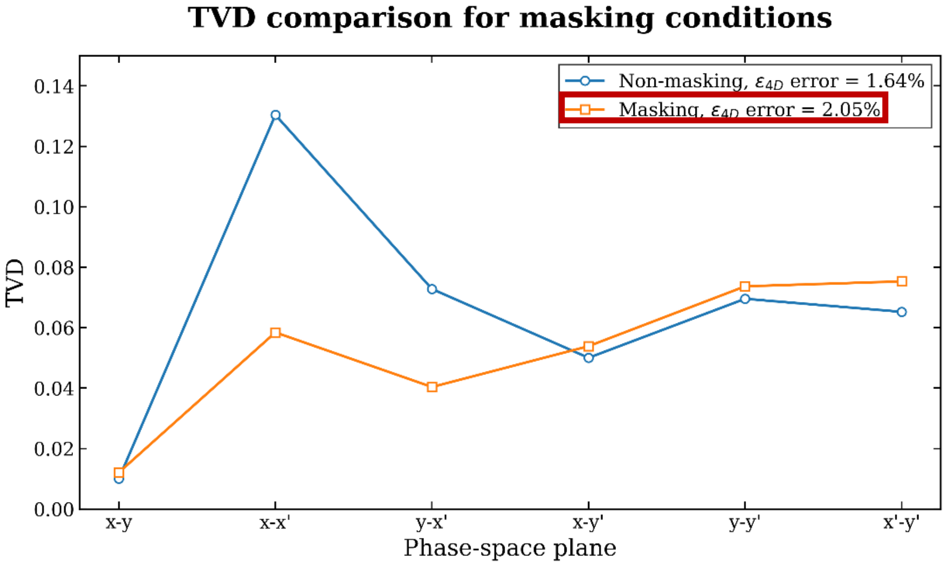
Non-coupling, Non-space charge

Scan condition

- Continuous scan
- Drift length : 0.11m
- 300,000 macroparticles
- Resolution : 30μm

Key point

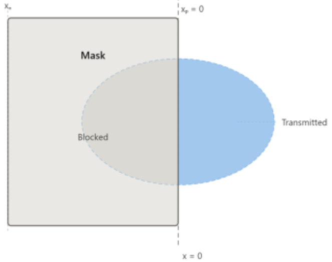
Does the MENT accurately reconstruct Non-Gaussian distributions?

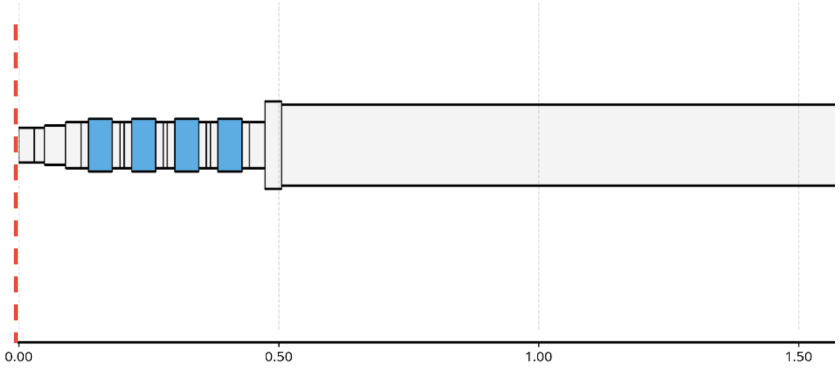


Masking point

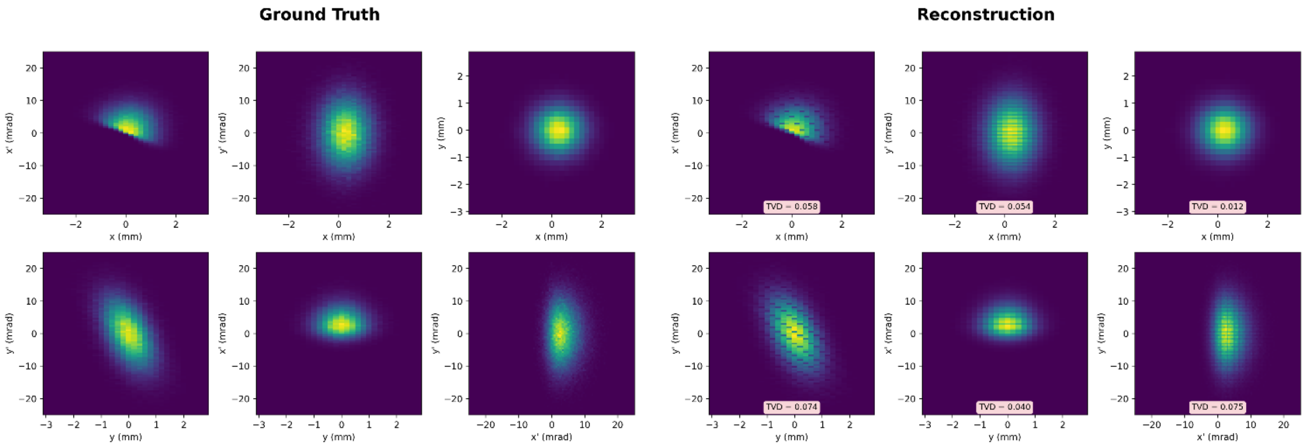
- Range :

$$x \geq 0(alive)$$





- Phase space distribution(masking)



Effect of Space-Charge on Reconstruction

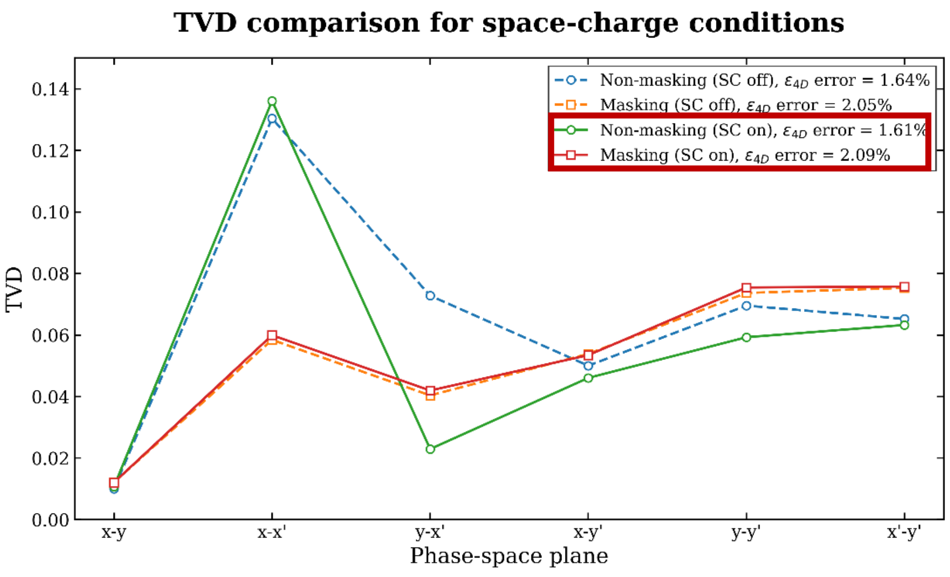
Non-coupling

Scan condition(same)

- Continuous scan
- Drift length : 0.11m
- 300,000 macroparticles
- Resolution : 30μm

Key point

How well does the MENT perform under space charge conditions?



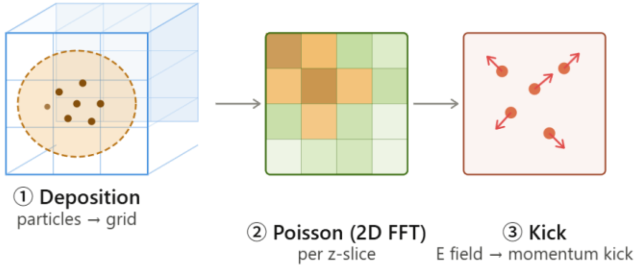
Space-charge mode(in X-suite)

- **Particle-In-Cell(PIC) mode**

① Macroparticles deposit charge onto a 3D grid.

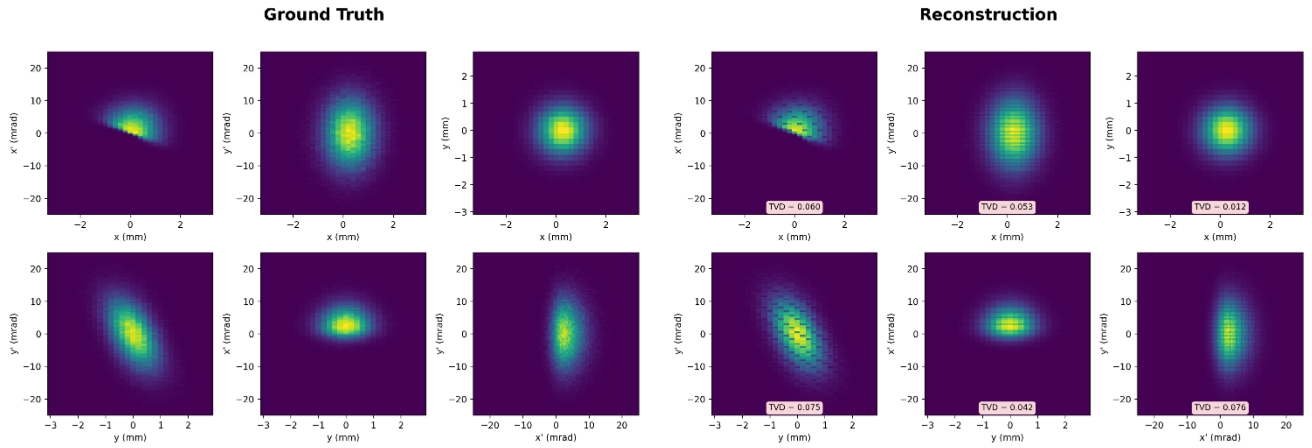
② The Poisson equation is solved per z-slice via 2D FFT.

③ The resulting electric field is applied as a momentum kick to each particle.



-> 'It applicable to Non-Gaussian distributions !'

- **Phase space distribution(masking& SC on)**



Effect of Coupling on Reconstruction

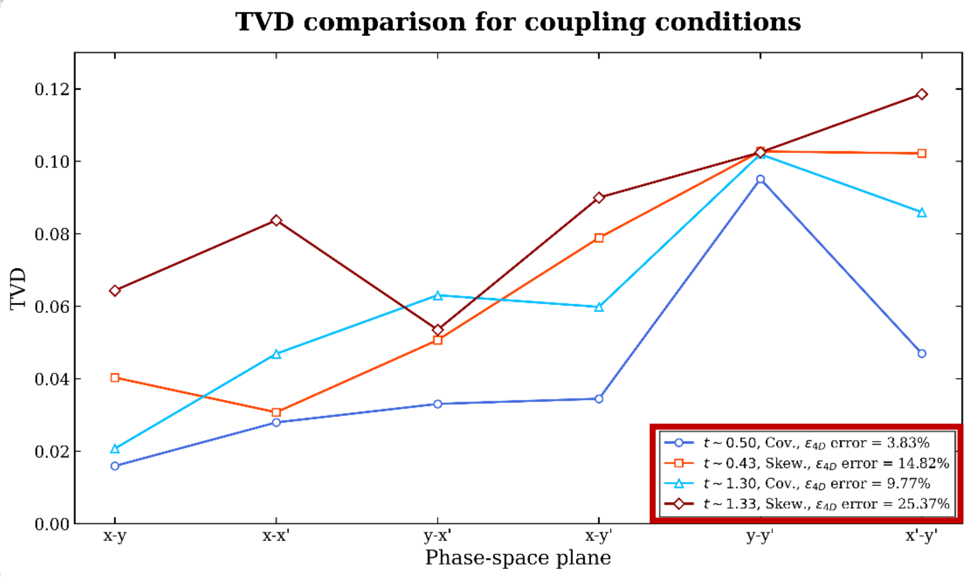
Non-masking, Non-space charge

Scan condition(same)

- Continuous scan
- Drift length : 0.11m
- 300,000 macroparticles
- Resolution : 30μm

Key point

How does coupling affect the reconstruction?



Coupling methods

- Introduce coupling factor :

$$t = \frac{\epsilon_x \epsilon_y}{\epsilon_1 \epsilon_2} - 1 \geq 0$$

t ≈ 0 : uncoupled
($\epsilon_x \epsilon_y$: projected emittance, $\epsilon_1 \epsilon_2$: eigen emittance)

- Comparison of the covariance-based and the skew QD

  - covariance-based method

$$\langle x, y' \rangle = r\, \sigma_x\, \sigma_{y'}$$

```
Sigma_4D[0,3] = Sigma_4D[3,0] = r * sig_x * sig_yp
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$$\langle x', y \rangle = r\, \sigma_{x'}\, \sigma_y$$

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Sigma_4D[1,2] = Sigma_4D[2,1] = r * sig_xp * sig_y
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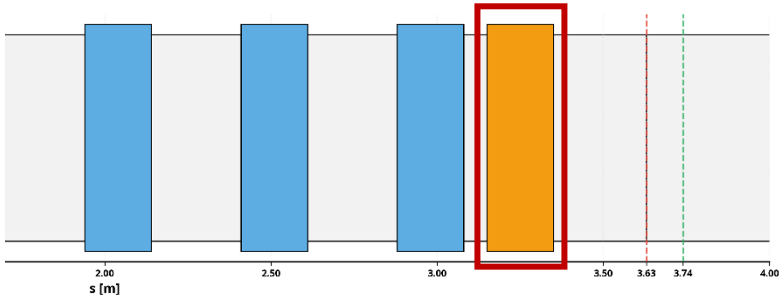
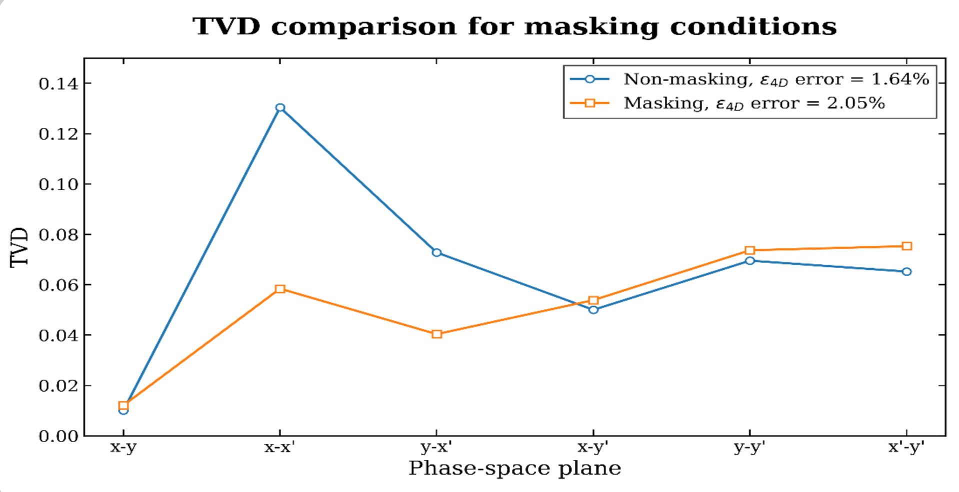
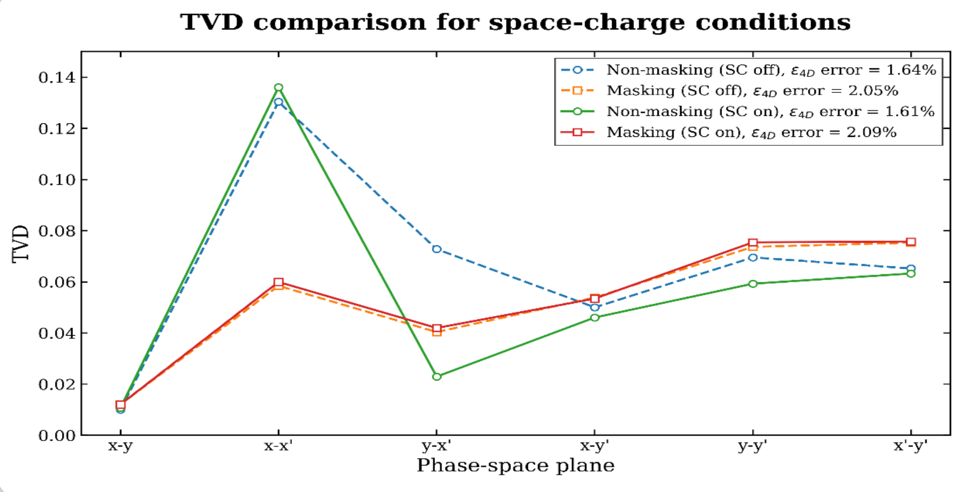
  - skew quadrupole method



Observation and Open Question

Why does the reconstruction error tend to be larger in only one phase-space plane?



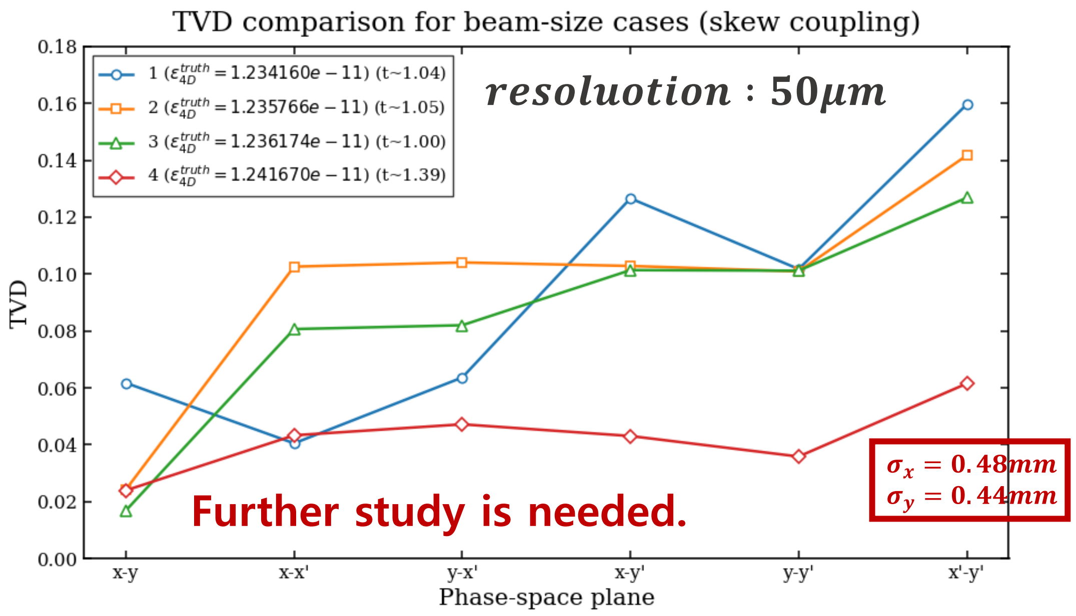


- **Observed trend :**

  In several cases, the reconstruction error is not balanced the two phase-space planes, x-x′ or y-y′.

- **Open question :**

  Why does one plane show a systematically larger deviation?



Conclusion

1) The MENT is a method for assigning an appropriate distribution that satisfies the given constraints.

2) The proposed method provides a simpler reconstruction process than VPP and allows direct observation of the beam distribution, including non-Gaussian structures.

3) Overall, the reconstruction performs well. However, a spike in the TVD value was observed in one plane. (but, the emittance errors are all at a satisfactory level.)

4) To address issue 3, diagonal-scan experiments are currently being carried out.

Thank you !