Current status of Tomography and WCM

25. 11. 07.

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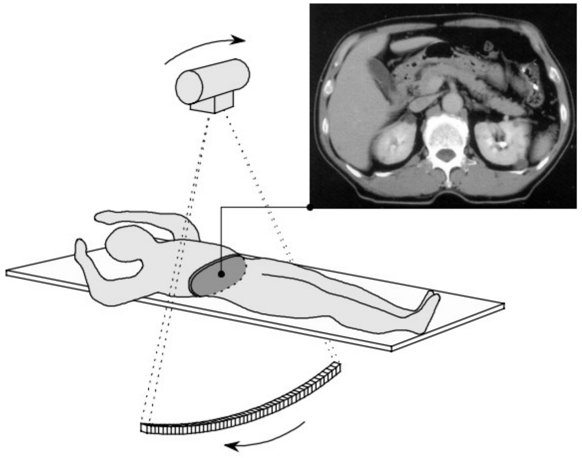
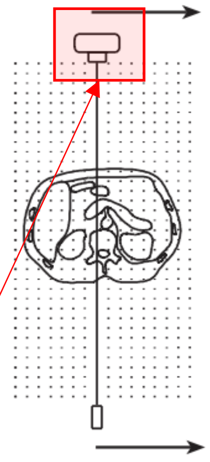
DANE, POSTECH

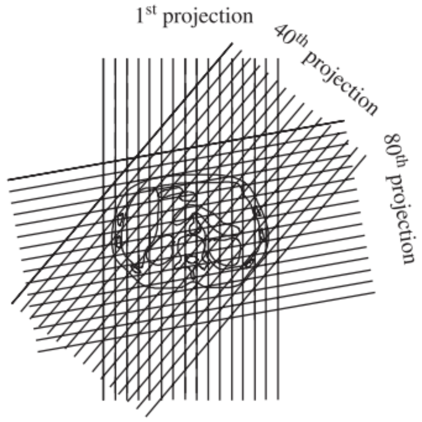
Review

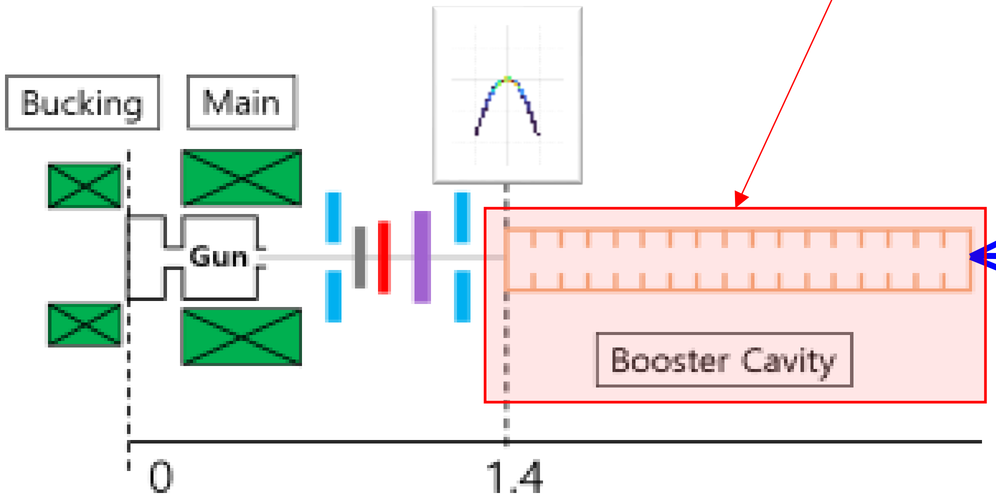
- **Basic principle of tomography**
    1. 1D distribution
    2. Projection angle

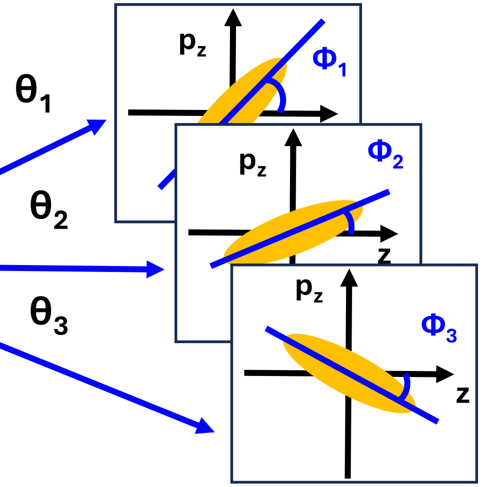


Scanner



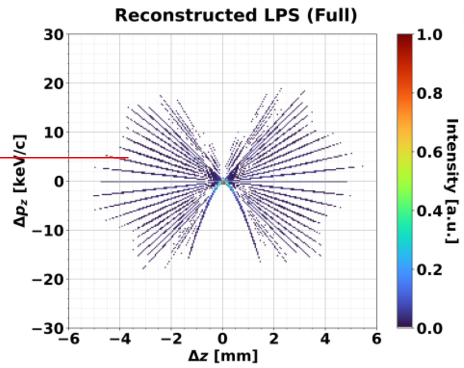


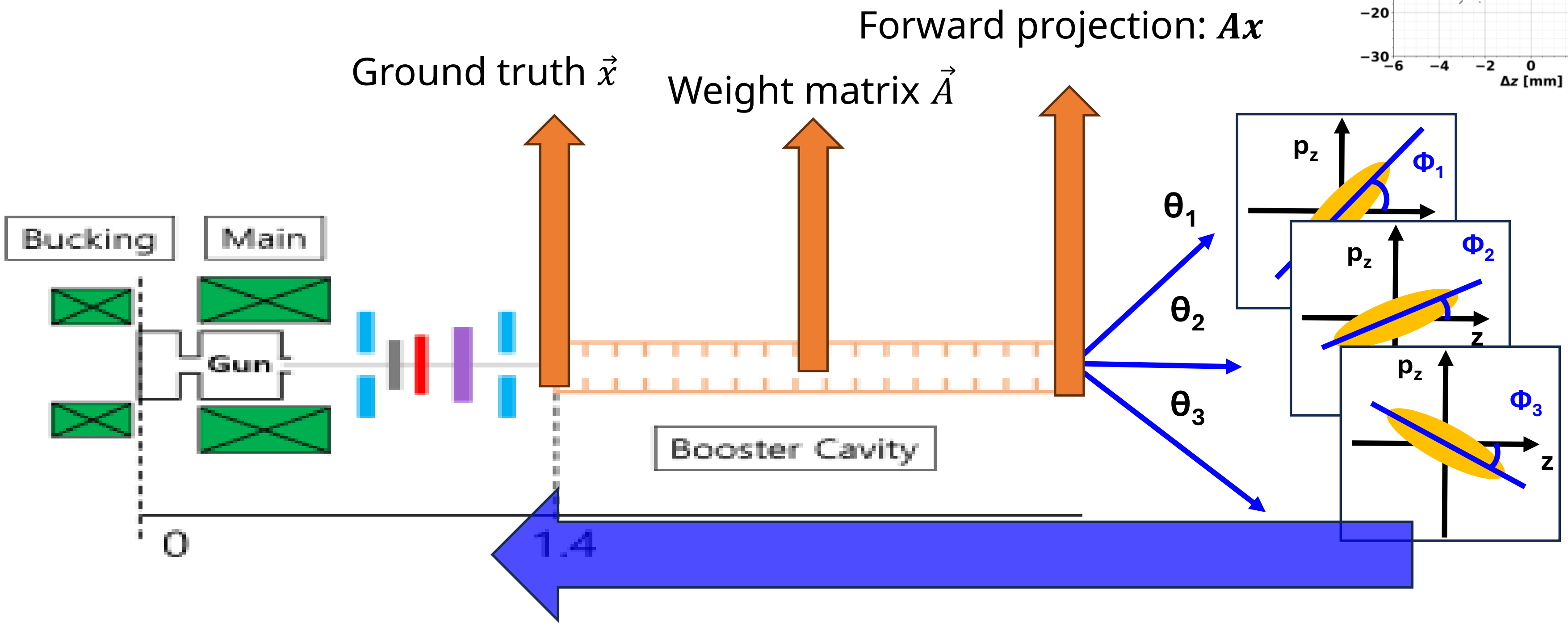




Review

- **Using different method to suppress stripe artifacts**





Iterative reconstruction algorithm

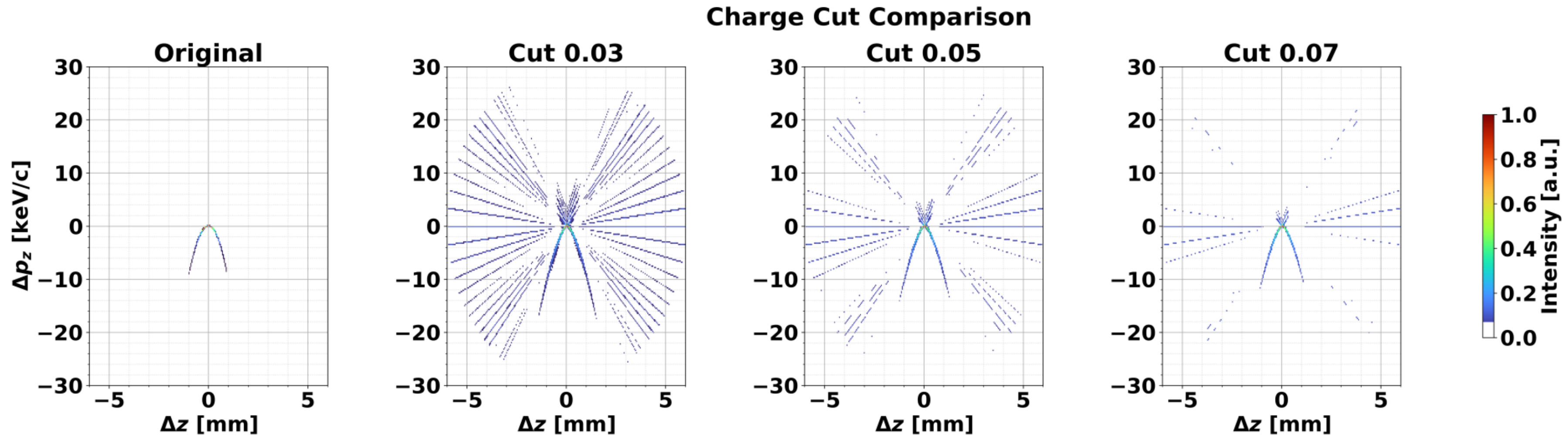
Backward projection

1. $\vec{x}^{k+1} = \vec{x}^{k} + \frac{1}{\vec{A}^T 1} \cdot \frac{\vec{A}^T(\vec{m} - \vec{A}^T \vec{x}^k)}{\vec{A}^T 1}$ (SART)
2. $\vec{x}^{k+1} = \vec{x}^{k} \cdot \frac{\vec{A}^T \vec{m}}{\vec{A}^T \vec{A} \vec{x}^k}$ (ISRA)

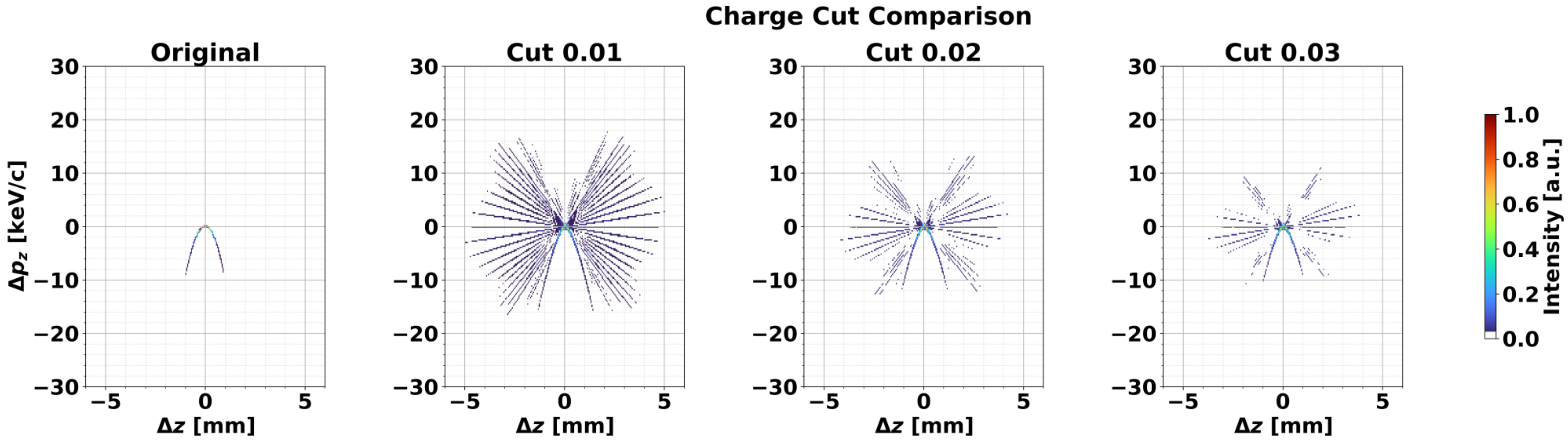
Determine $\vec{\boldsymbol{A}}$ representing the extent of the beam's physical location.

Recall the previous results: Python library [Done]

- **Filtered Back Projection (FBP)**: Direct, fast, non-iterative reconstruction.

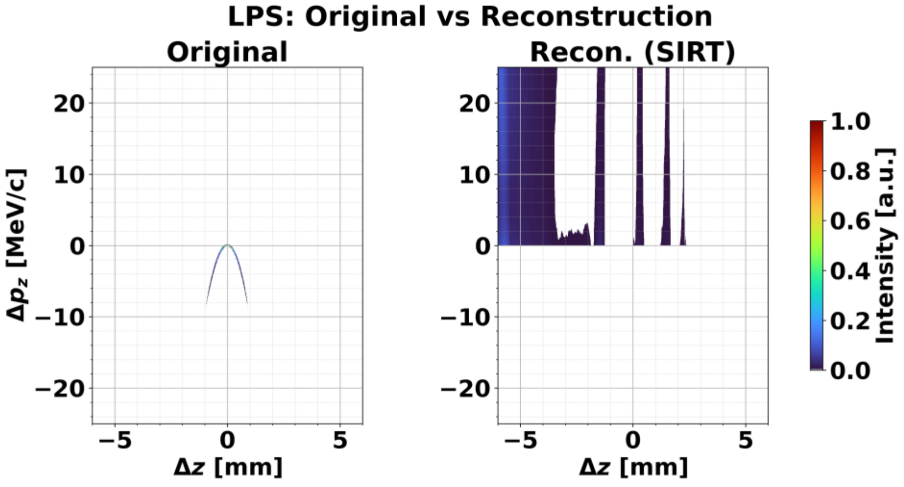


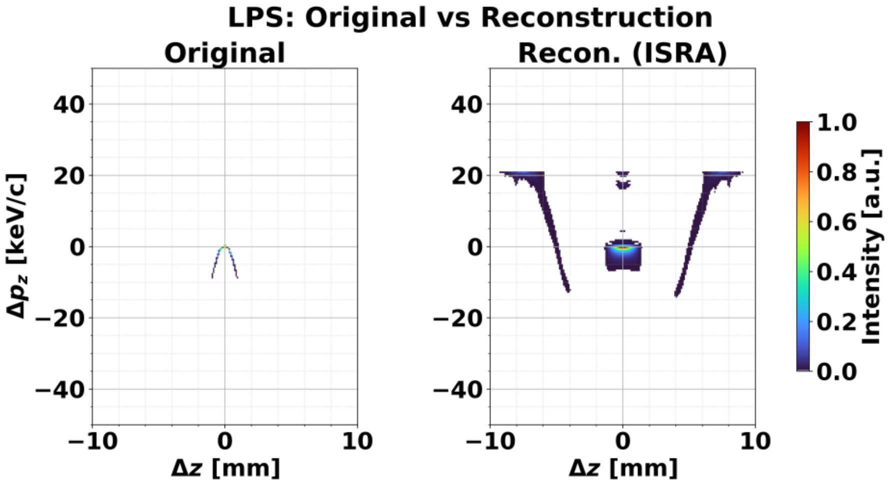
- **Simultaneous algebraic reconstruction technique (SART):** Iterative, weighted matrix, quality improvement.



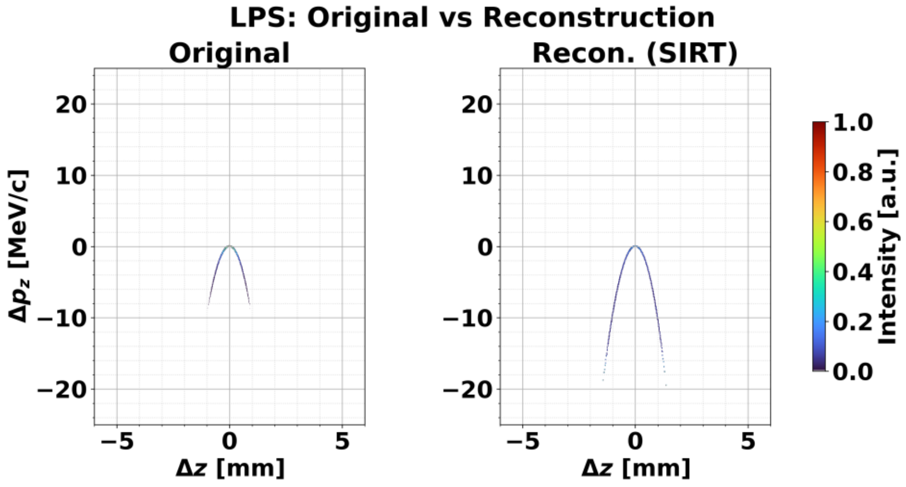
Recall the previous results: construction $\vec{A}$ [On-going]

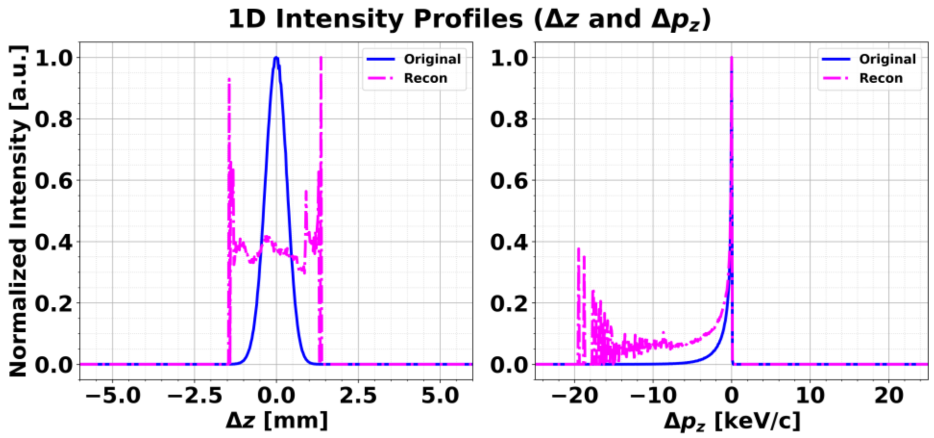
x^1: Uniform





x^1: Ground truth





Algorithm of weight matrix $\vec{A}$

- **Key parameters**
  1. z(t): Not bunch length.
  2. p_{min}^{boo} :Not min of measured value.

Algorithm 1: Weight matrix for LPS Reconstruction

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input : φ, p_z^{gun}, V, z

output: $WeightMatrix$ (consists of 1s and 0s)

Function Loop(A)

  $N_\varphi \leftarrow$ length(booster phases)

  $N_s \leftarrow$ length(momentum bins)

  $N_t \leftarrow$ length(longitudinal bins)

  for $i \leftarrow 1$ *to* φ_{max} **do**

    for $s \leftarrow 1$ *to* N_s **do**

      for $t \leftarrow 1$ *to* N_t **do**

        $p_{cal}^{boo} \leftarrow p_z^{gun}(s) + V cos(\varphi(i) + z(t))$

        $j \leftarrow \frac{p_{cal}^{boo} - p_{min}^{boo}}{\Delta p_{boo}}$

        $W(j + N_s(i-1), t + N_t(s-1)) \leftarrow 1$

  return $WeightMatrix$

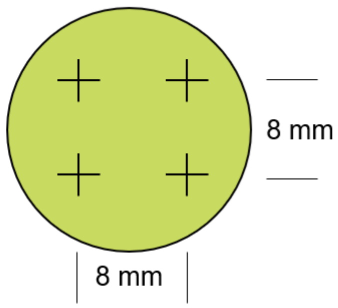
Algorithm verification

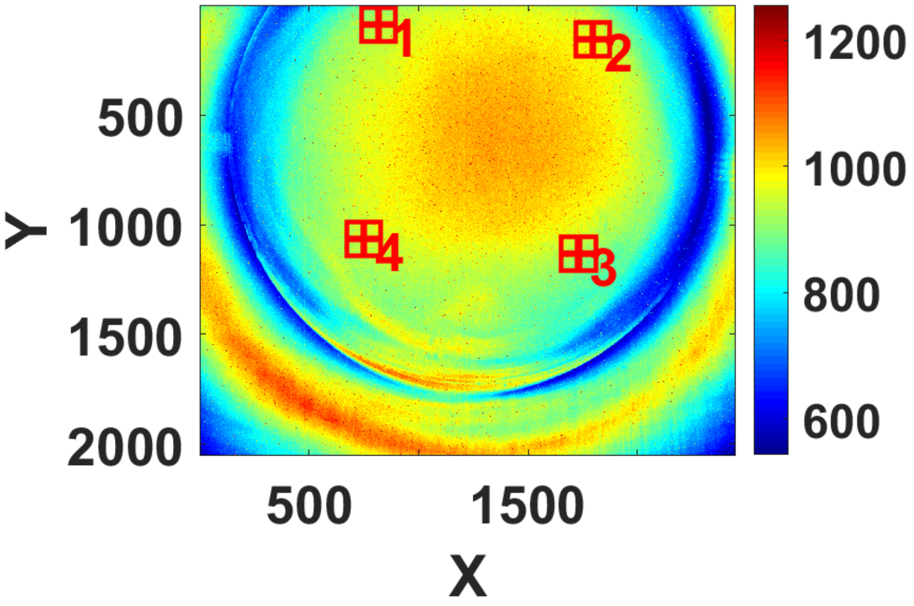
$$p_{cal}^{boo} \leftarrow p_z^{gun}(s) + V\cos(\varphi(i) + \boldsymbol{z(t)})$$

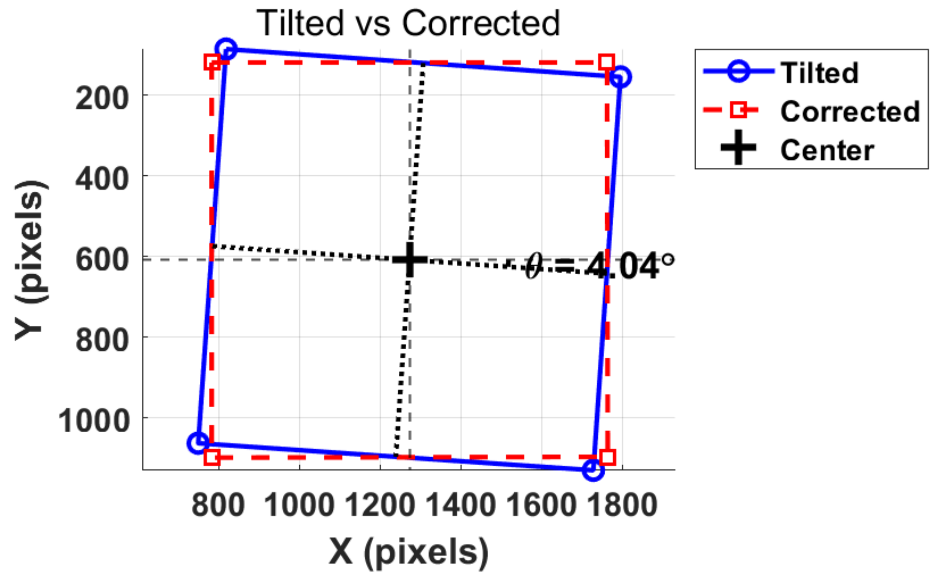
1. z(t): Not bunch length. → Calculated as **(Screen pixels) x (Screen resolution mm/pixels)**
   - It is not significant element in my situation (simulation).

SCM04

- 4도 기울어짐
- 중심: (0.35, 3.38)



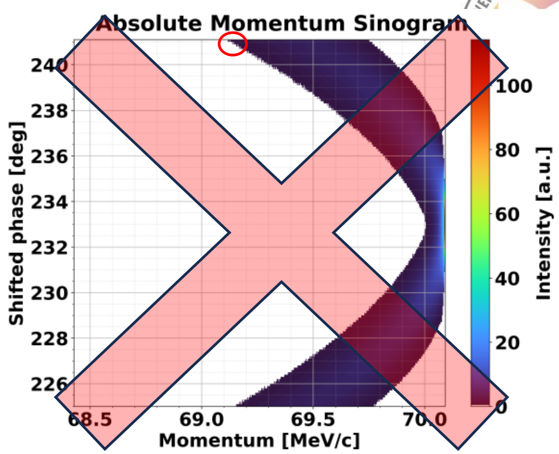




- **X Resolution : 8.16 um/pixel**
- Y Resolution: 8.18 um/pixel

Algorithm verification

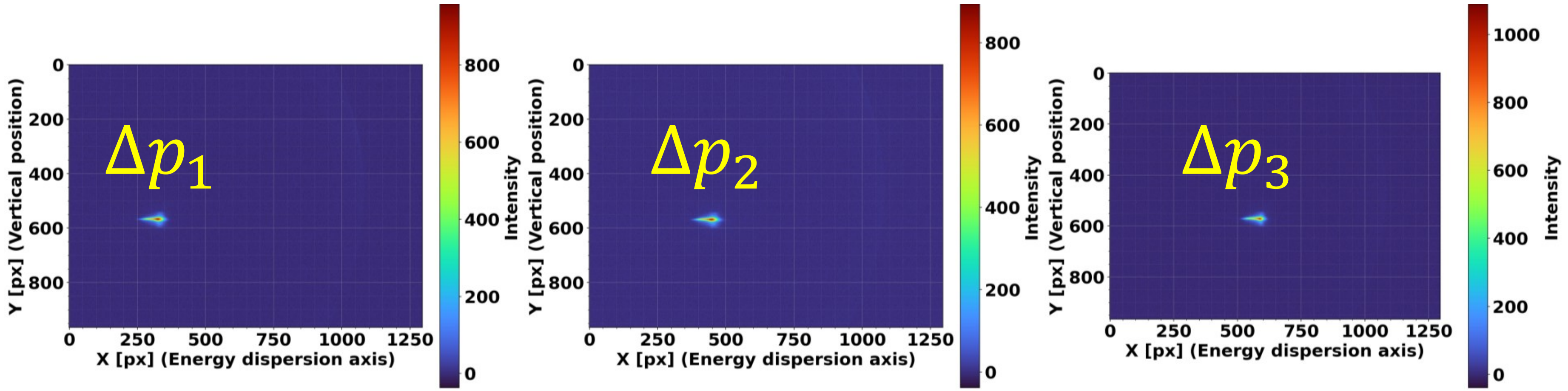
$$j \leftarrow \frac{p_{cal}^{boo} - \mathbf{p_{min}^{boo}}}{\Delta p_{boo}}$$



2. $\mathbf{p_{min}^{boo}}$: Minimum measurable momentum on screen.

- $X(s) = x_\beta(s) + D(s) \cdot \left(\frac{\Delta p}{p}\right)$

From XFEL beam data (same as elabs)



$\mathbf{p_{min}^{boo}}$: 68.396 MeV/c

In elabs case,

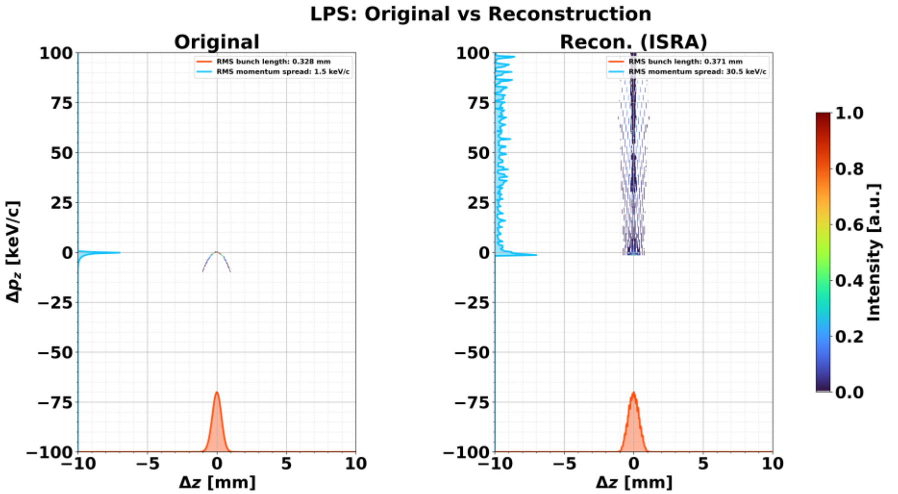
phase_deg	all_withi
220	FALSE
222	FALSE
224	TRUE
226	TRUE
228	TRUE
230	TRUE
232	TRUE
234	TRUE
236	TRUE
238	TRUE
240	TRUE
242	FALSE
244	FALSE
246	FALSE

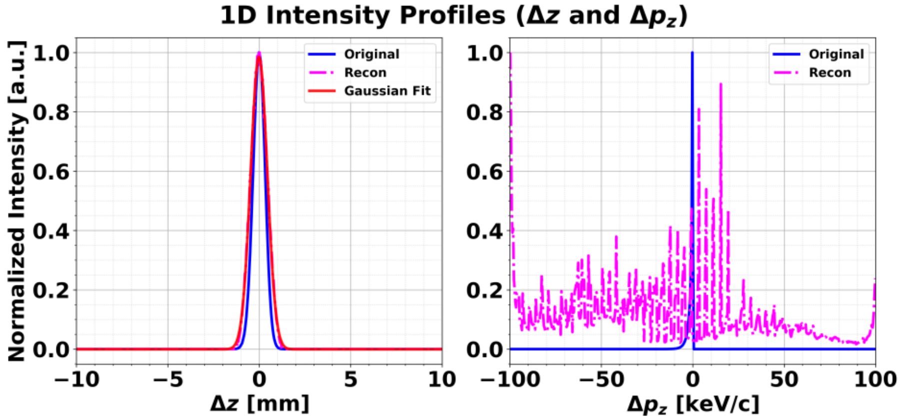
Results

$$\vec{x}^{k+1} = \vec{x}^{k} \cdot \frac{\vec{A}^T \vec{m}}{\vec{A}^T \vec{A} \vec{x}^k} \text{ (ISRA)}$$

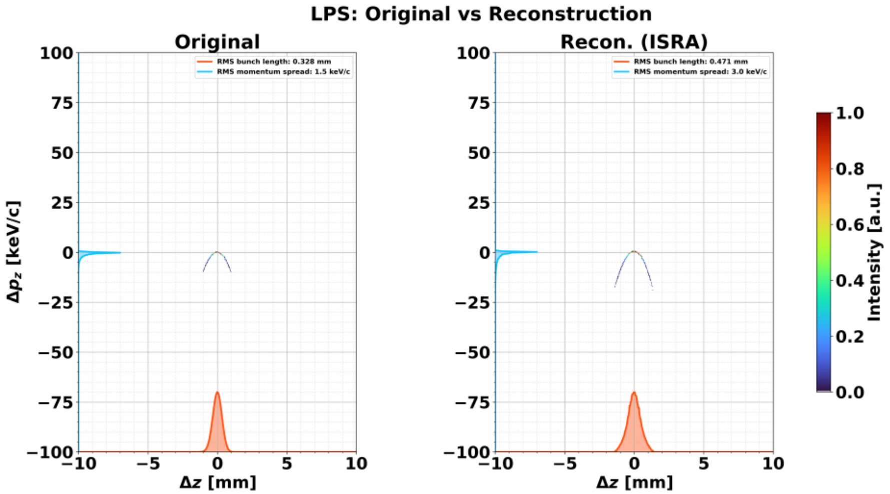
Good agreement to $z(t)$, but, obtained worse Δp results.

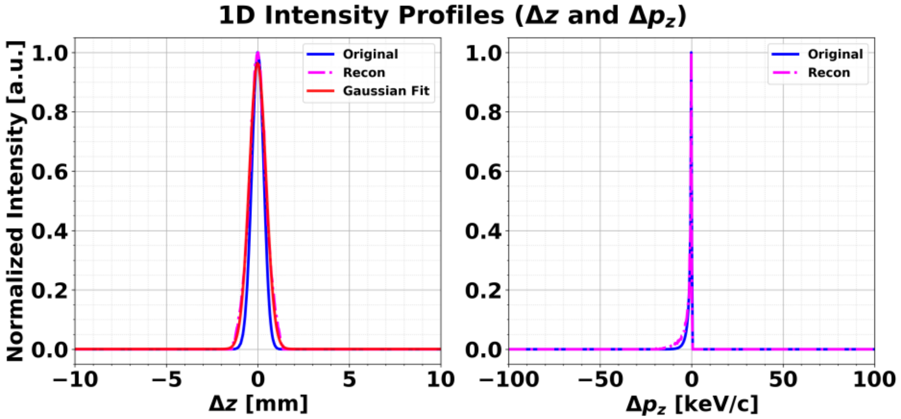
x^1: Uniform



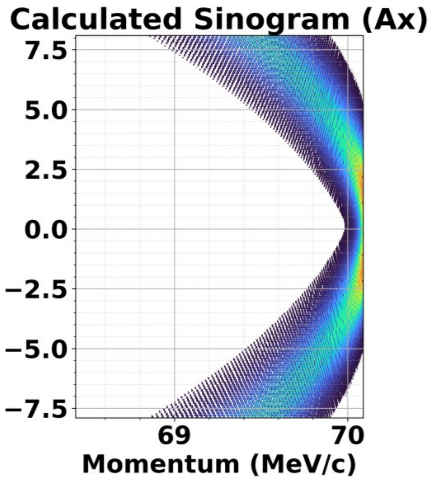


x^1: Ground truth

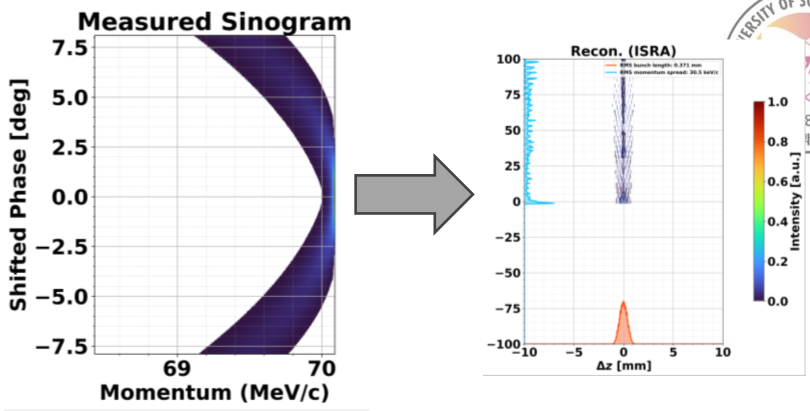




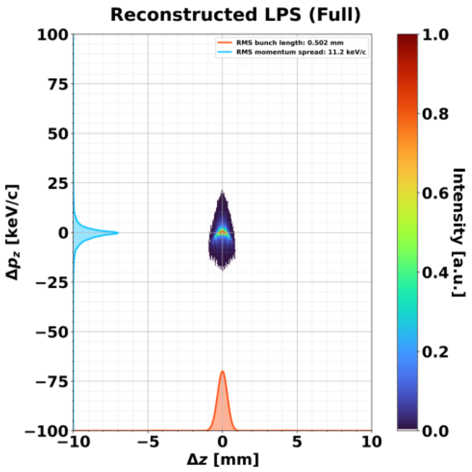
Finding issues

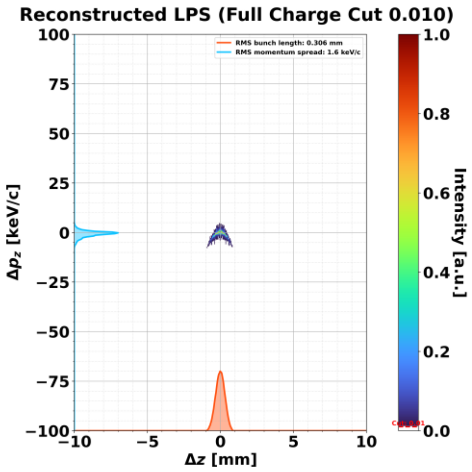


$$\vec{x}^{k+1} = \vec{x}^{k} \cdot \frac{\vec{A}^{T}\vec{m}}{\vec{A}^{T}\vec{A}\vec{x}^{k}} \Rightarrow \vec{x}^{k} \cdot \frac{\vec{A}^{T}\vec{A}x^{True}}{\vec{A}^{T}\vec{A}\vec{x}^{k}}$$

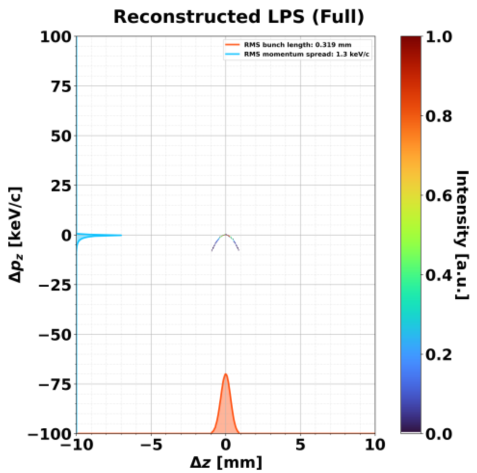


x^1: Uniform



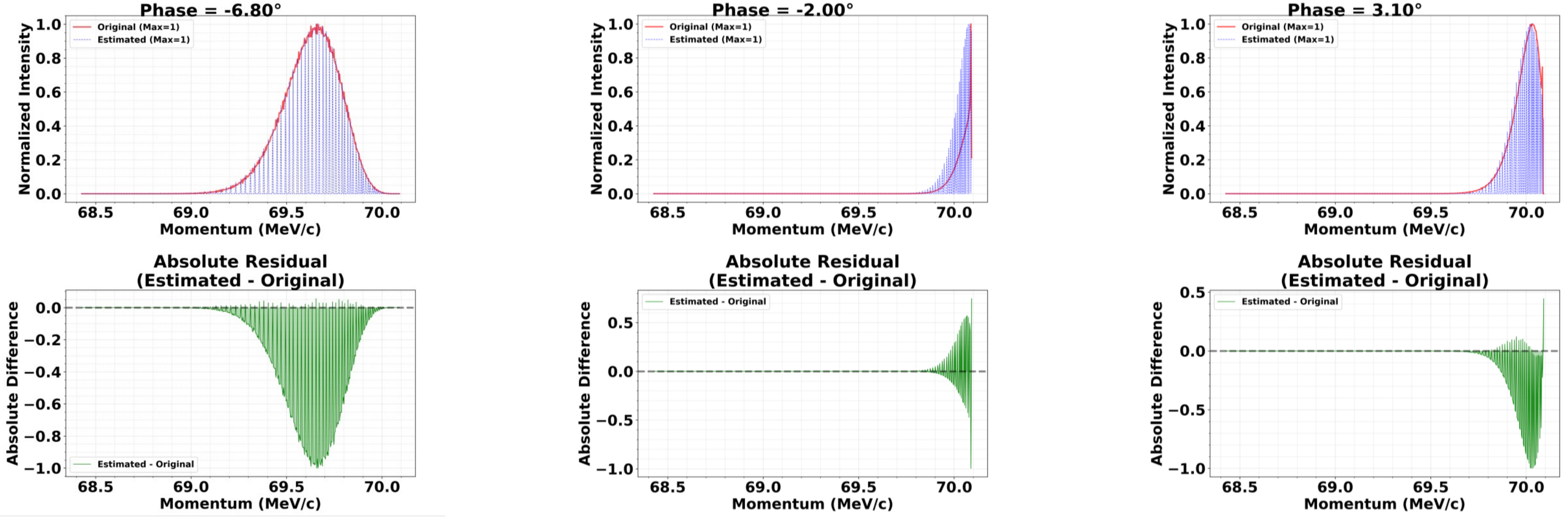


x^1: Ground truth



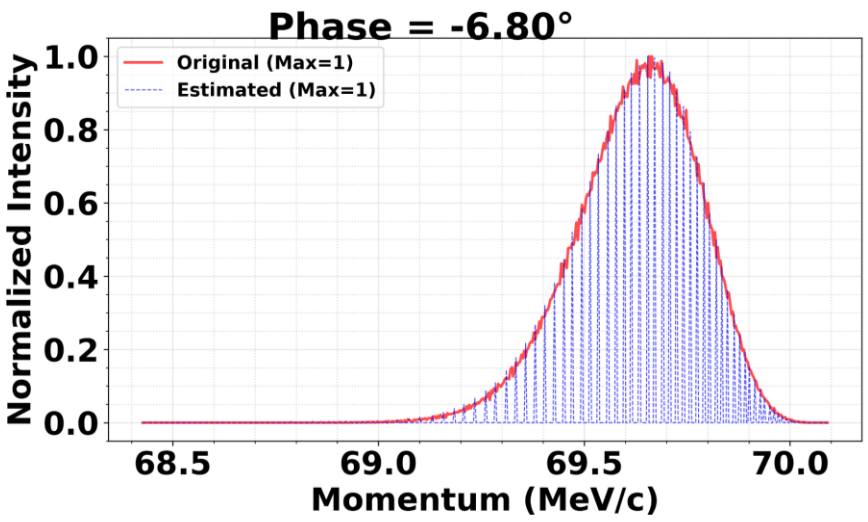
Bilinear interpolation

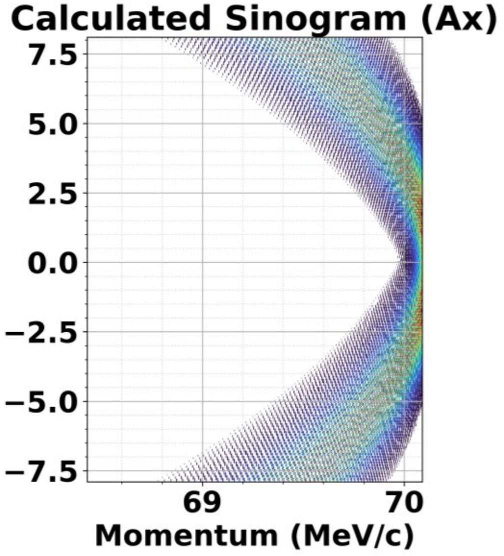
Constructed $\vec{A}$ (Simple binning) vs simulated momentum distribution



Bilinear interpolation

Simple binning





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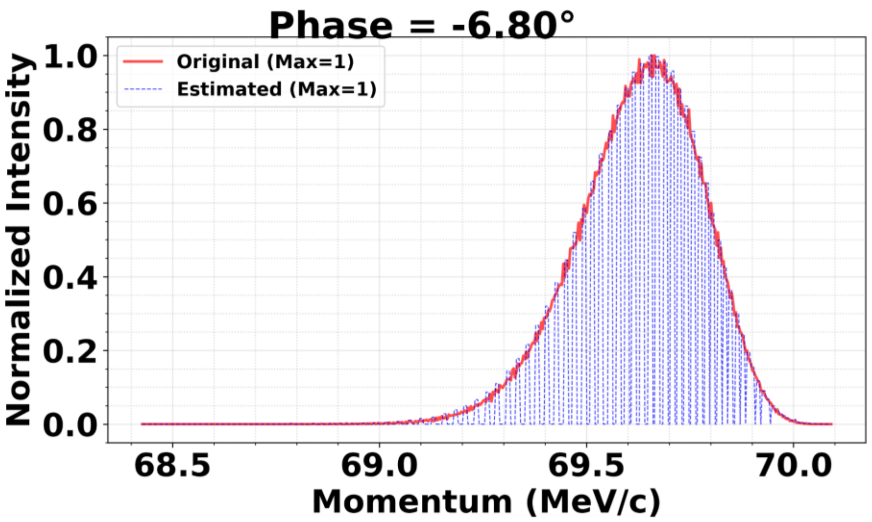
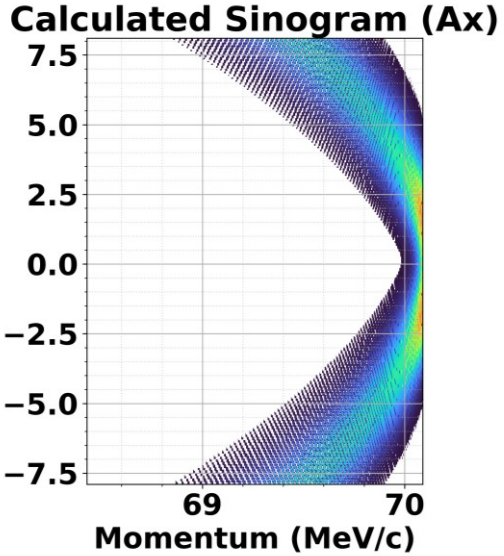


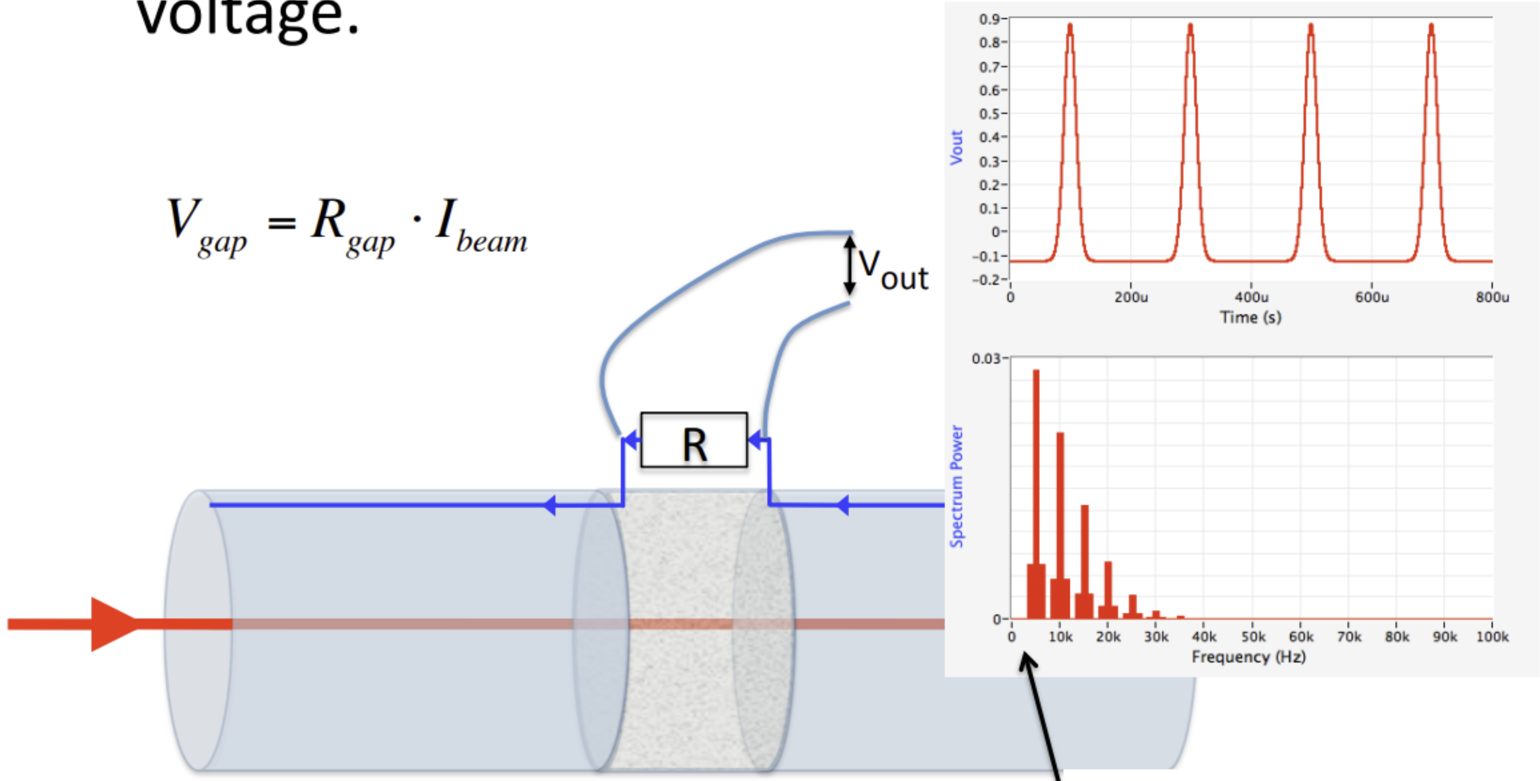


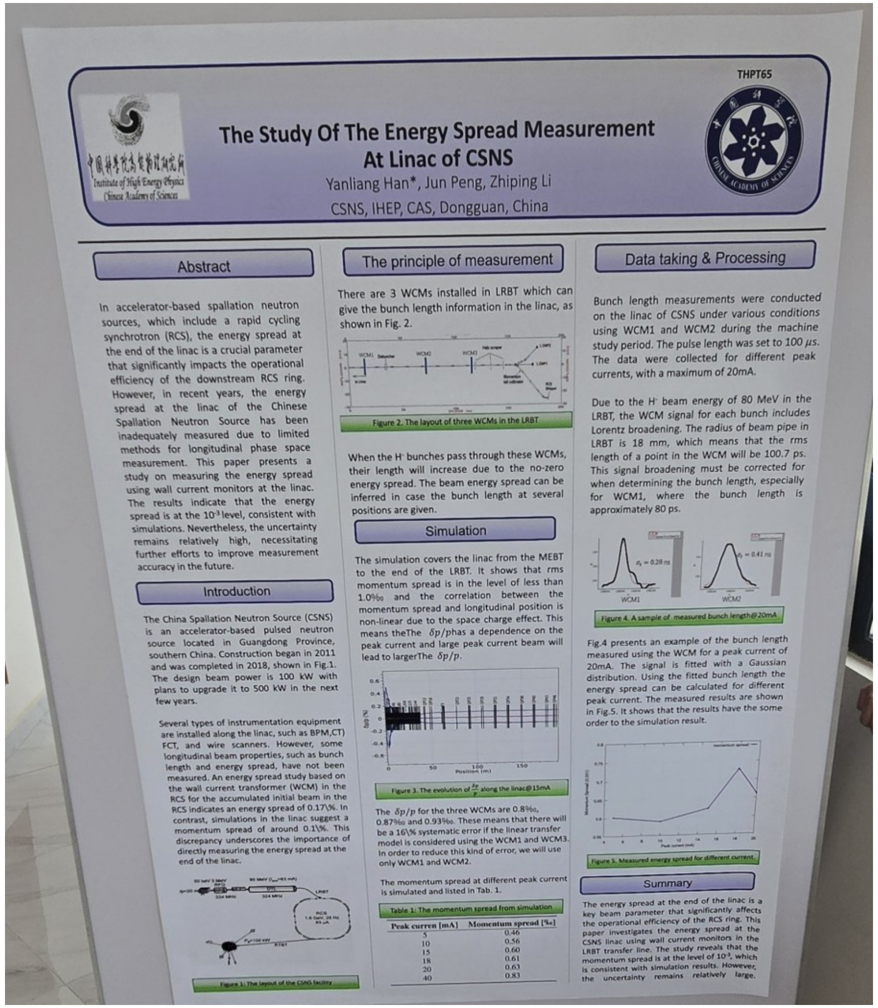
Energy spread measurement

Wall Current Monitor

- Put a resistor over the gap and measure its voltage.

$$V_{gap} = R_{gap} \cdot I_{beam}$$





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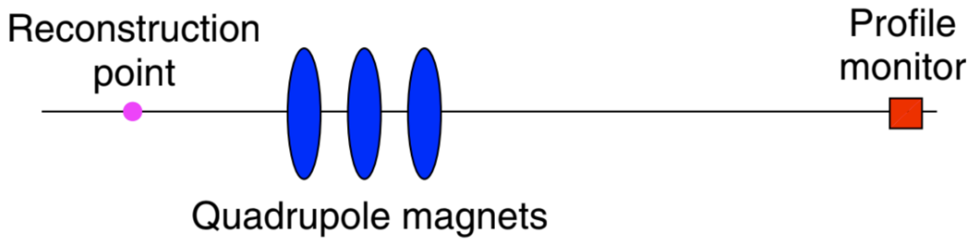
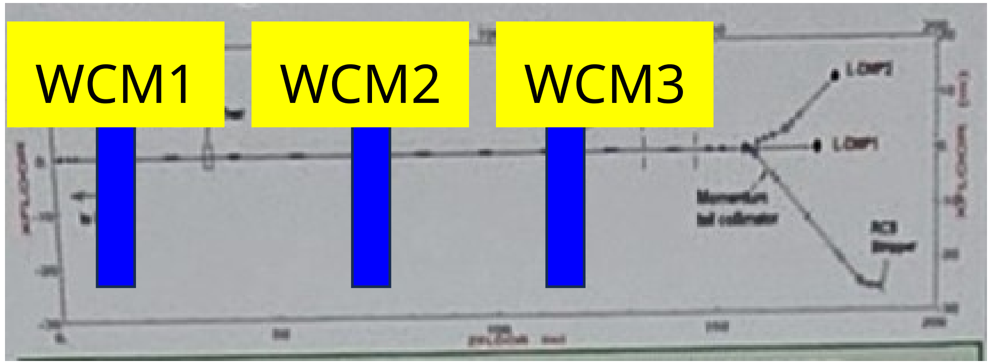




- Unknown: $\sigma_{z,0}^2, \sigma_{\delta,0}^2, \sigma_{z\delta,0}$
- $\Sigma_0 = \begin{pmatrix} \langle z^2 \rangle & \langle z\delta \rangle \\ \langle z\delta \rangle & \langle \delta^2 \rangle \end{pmatrix} = \begin{pmatrix} \sigma_{z,0}^2 & \sigma_{z\delta,0} \\ \sigma_{z\delta,0} & \sigma_{\delta,0}^2 \end{pmatrix}$
- $\Sigma_i = R_i \Sigma_0 R_i^T$
- $\sigma_{z,i}^2 = \left(R_{i,55}\right)^2 \cdot \sigma_{z,0}^2 + \left(R_{i,56}\right)^2 \cdot \sigma_{\delta,0}^2 + 2\left(R_{i,55} R_{i,56}\right) \cdot \sigma_{z\delta,0}$

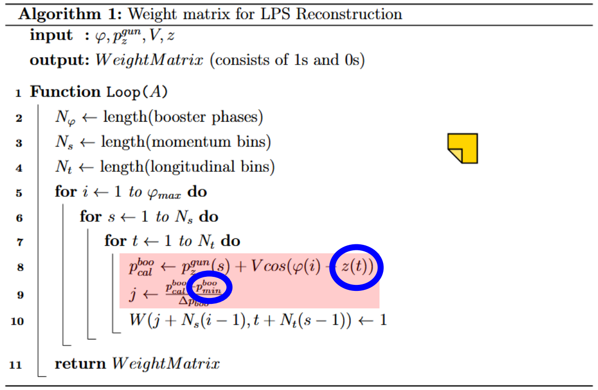
WCM 1: $\sigma_{z,1}^2 = \left(R_{1,55}\right)^2 \cdot \sigma_{z,0}^2 + \left(R_{1,56}\right)^2 \cdot \sigma_{\delta,0}^2 + 2\left(R_{1,55} R_{1,56}\right) \cdot \sigma_{z\delta,0}$

WCM 2: $\sigma_{z,2}^2 = \left(R_{2,55}\right)^2 \cdot \sigma_{z,0}^2 + \left(R_{2,56}\right)^2 \cdot \sigma_{\delta,0}^2 + 2\left(R_{2,55} R_{2,56}\right) \cdot \sigma_{z\delta,0}$

WCM 3: $\sigma_{z,3}^2 = \left(R_{3,55}\right)^2 \cdot \sigma_{z,0}^2 + \left(R_{3,56}\right)^2 \cdot \sigma_{\delta,0}^2 + 2\left(R_{3,55} R_{3,56}\right) \cdot \sigma_{z\delta,0}$

Summary

- **Checked Algorithm element ($z(t)$, and p_{min}^{boo})**



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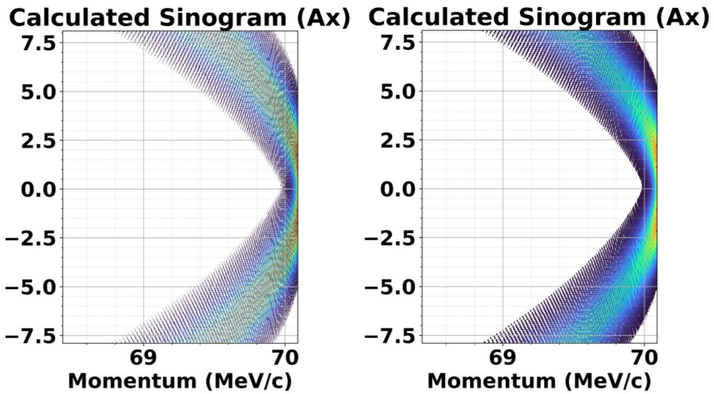
return $WeightMatrix$

- **$\vec{m}$ dependency**



- $\sigma_{z,i}^2 = (R_{i,55})^2 \cdot \sigma_{z,0}^2 + (R_{i,56})^2 \cdot \sigma_{\delta,0}^2 + 2(R_{i,55}R_{i,56}) \cdot \sigma_{z\delta,0}$