The 23rd JKPS title:

5D tomographic phase-space reconstruction of particle bunches

May. 30, 2025

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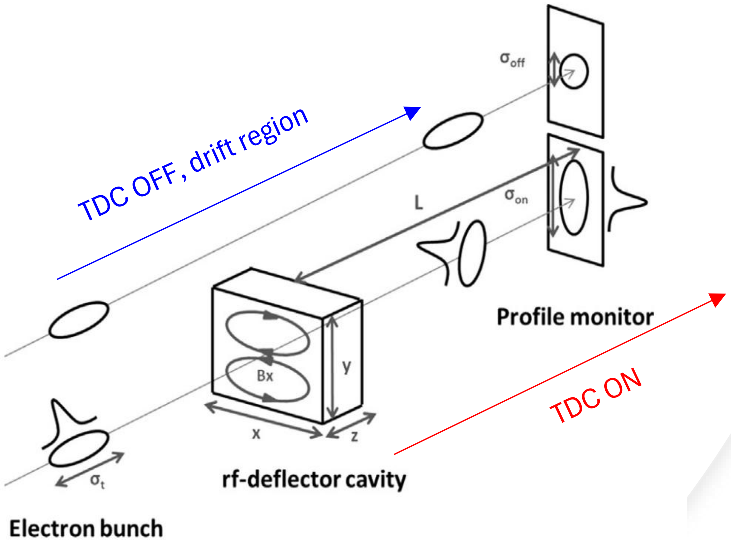
POSTECH

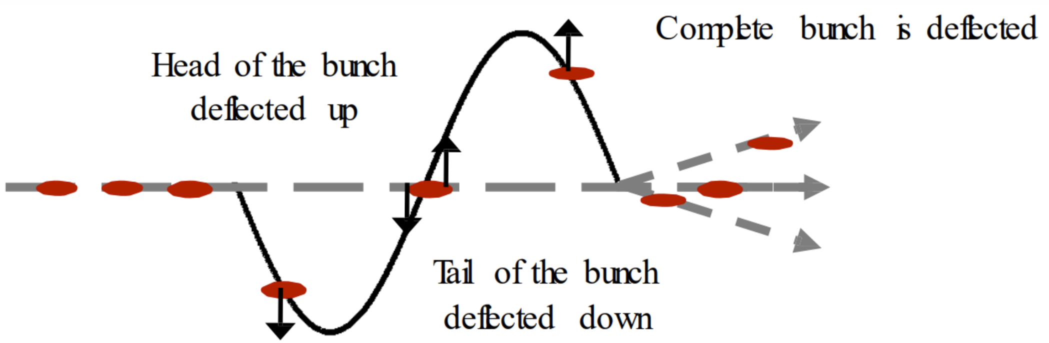
Background

Transverse Deflecting structure (TDS)

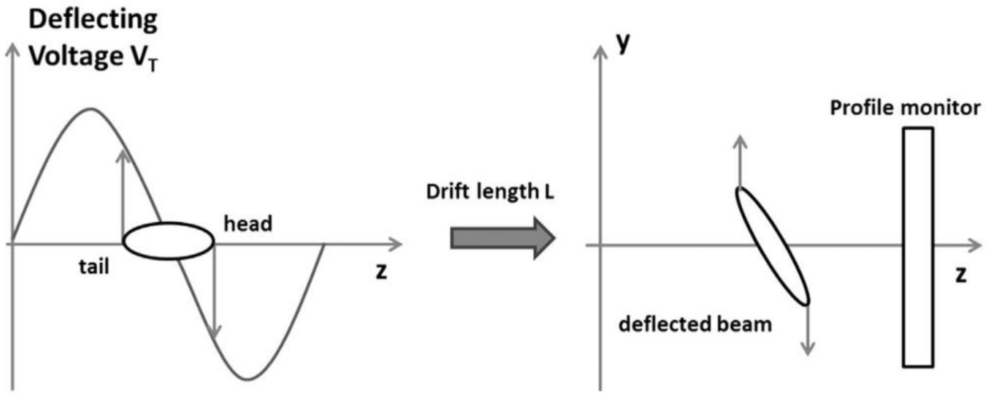
- The cavity kicks a charged particle in a transverse direction. ($\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$)
- One of bunch longitudinal profile monitor



- **Bunch motion in TDS w.r.t TDS phase**



- **Matched phase near zero-crossing phase**

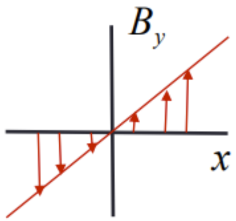


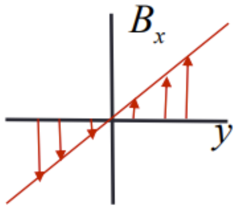
Background

Quadrupole Magnet (QM)

- Focusing in one plane, defocusing in the other

- $\vec{B} = \begin{bmatrix} K_y \\ K_x \\ 0 \end{bmatrix}, K = \text{const}$



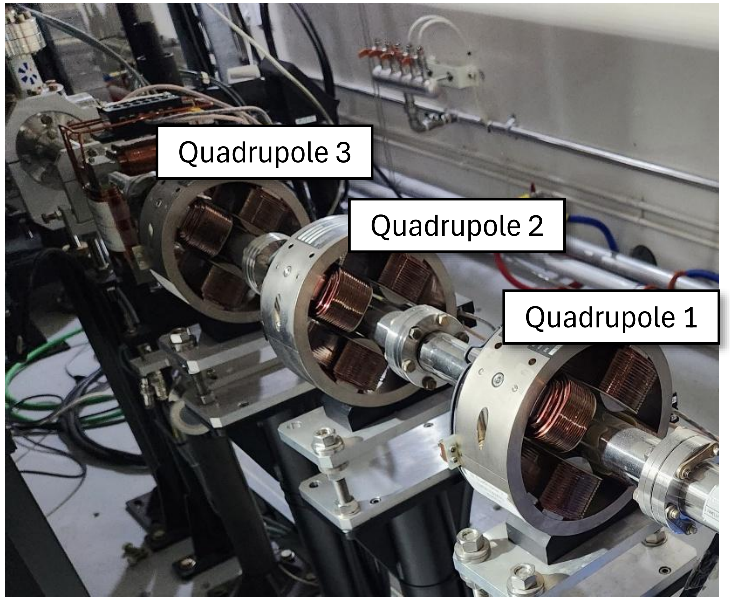


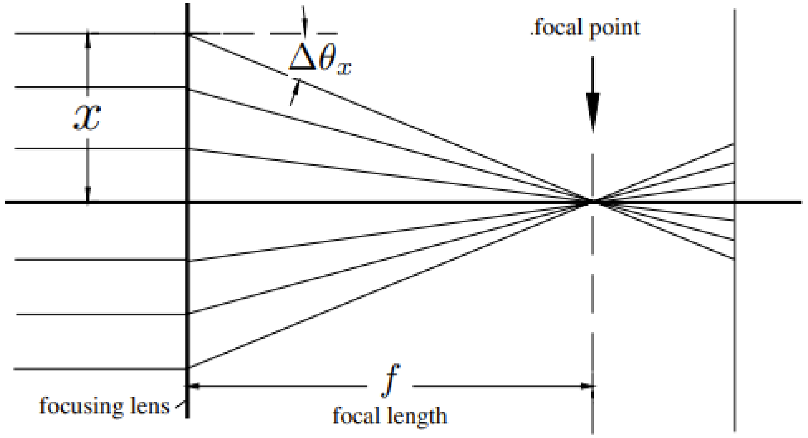
- $\vec{F} = q\vec{v} \times \vec{B} = qv_z K \begin{bmatrix} -x \\ y \\ 0 \end{bmatrix}$

Focusing in x

Defocusing in y

- Focal length $f = \dfrac{x}{-\Delta\theta_x} = \dfrac{y}{\Delta\theta_y} = \dfrac{B\rho}{Kl}$



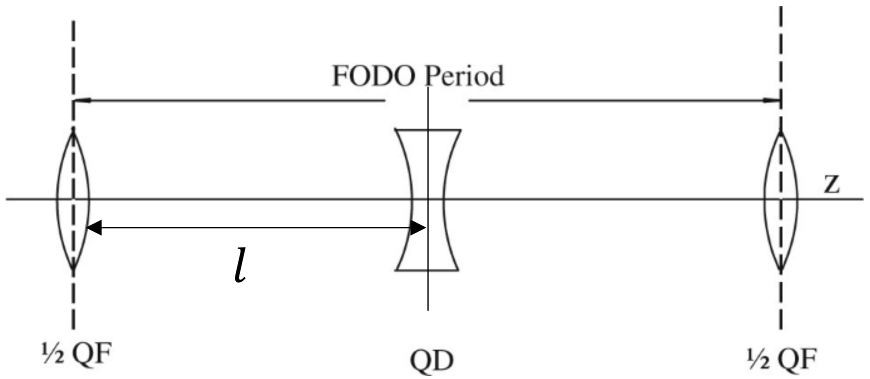


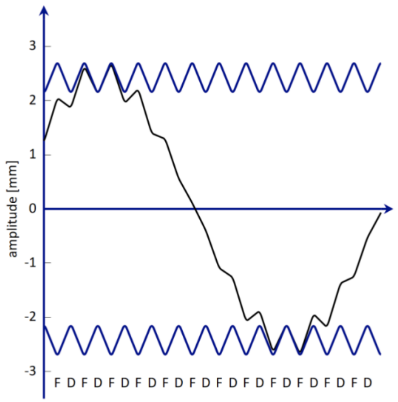
Background

FODO transfer matrix

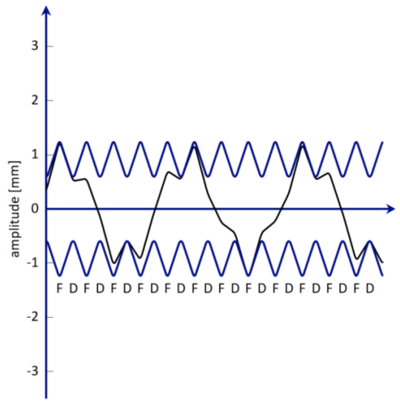
$$M_{FODO} = \begin{bmatrix} 1 - \dfrac{l^2}{2f^2} & 2l(1 + \dfrac{l}{2f}) \\ -\dfrac{l}{2f^2}(1 - \dfrac{l}{2f}) & 1 - \dfrac{l^2}{2f^2} \end{bmatrix} = \begin{bmatrix} \cos\psi & \beta \sin\psi \\ -\dfrac{1}{\beta}\sin\psi & \cos\psi \end{bmatrix}$$

- $\cos\psi = 1 - \frac{l^2}{2f^2}, \ \beta = \frac{2l\left(1+\frac{l}{2f}\right)}{\sin\psi}$ $\qquad f = \frac{B\rho}{Kl}$

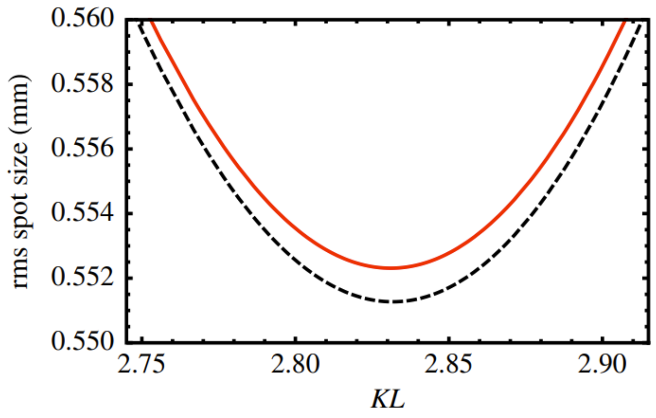




weak quads



strong quads

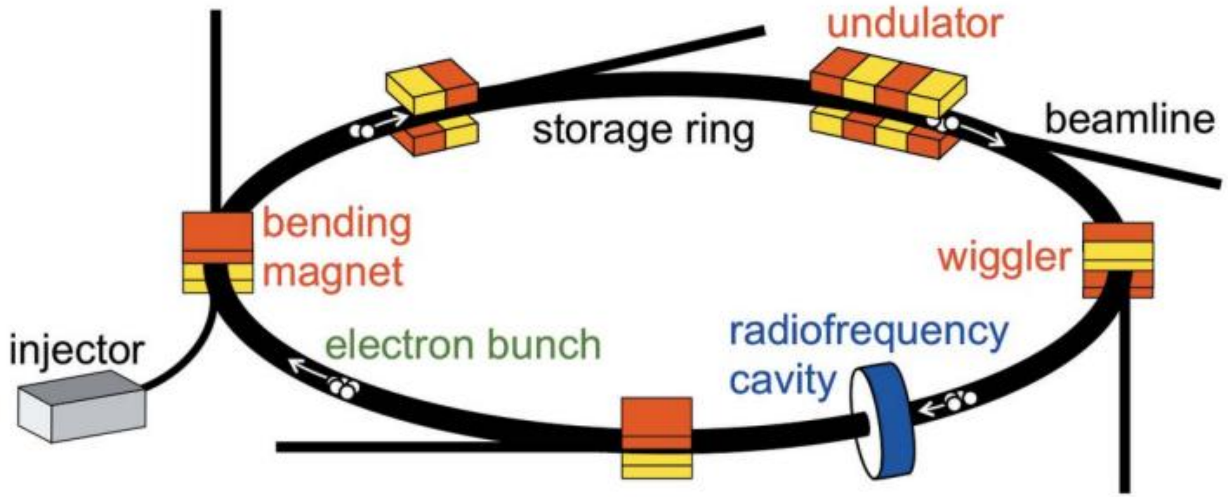


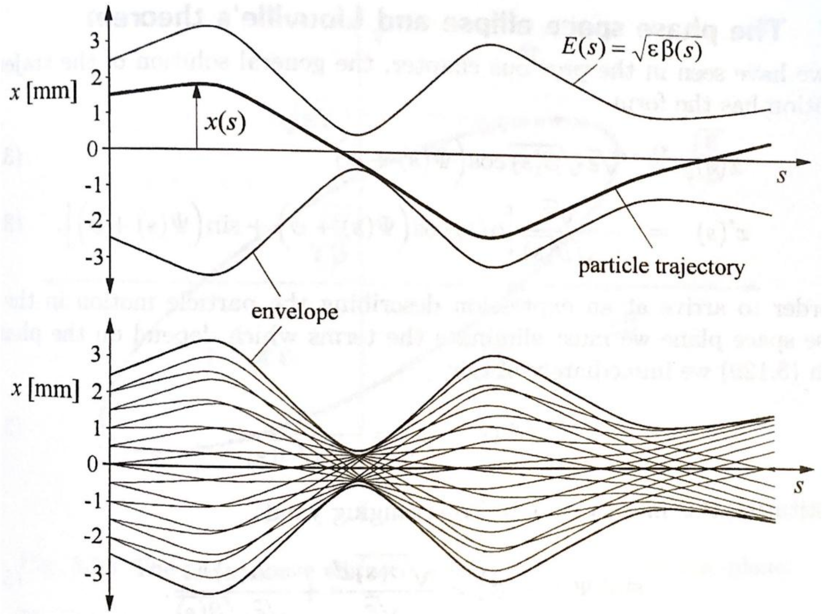
Background

The # of particles in storage ring

- 1.76 x 10^{12} electrons in storage ring

  (C = 281.82 m, I = 300 mA)



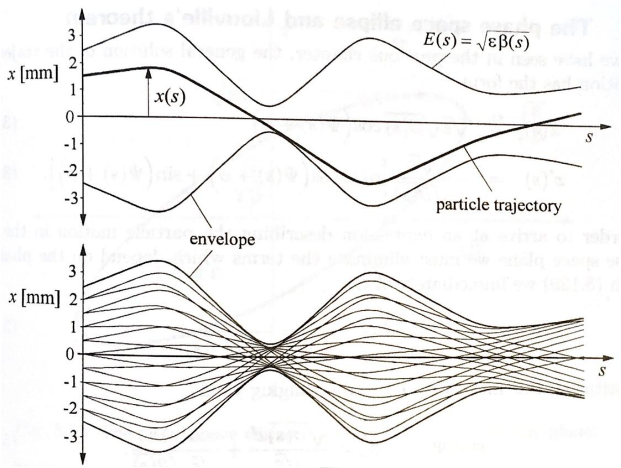


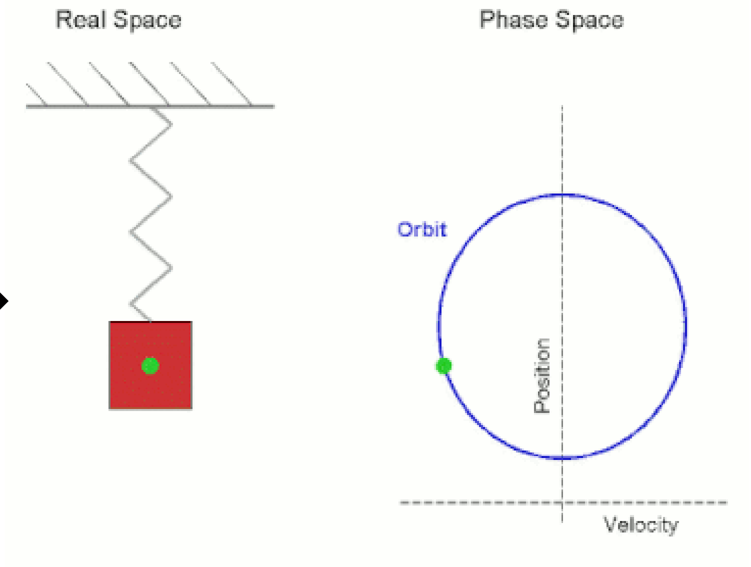
How can trillions of particles be properly analyzed?

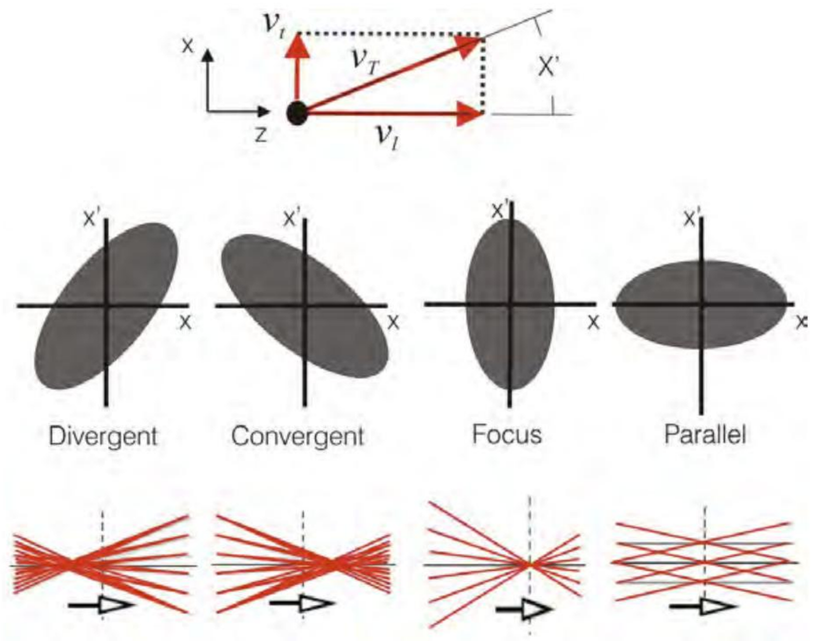
Background

Phase Space (PS)

- Phase space describes a particle's position and momentum in a combined framework.
- Beam dynamics are often analyzed in 2D phase space planes like (x, x'), (y, y') or (z, Δp/p).
  → 5D profile (x, x', y, y', t)
- ***Emittance***, the phase space area, **quantifies beam quality and spread.**







Why is this paper selected?

- **My current work list**

Covered in this paper

1. Transverse Deflecting Structure (TDS)
2. Phase space tomography (Longitudinal)
3. Longitudinal Feedback System (LFS)

1. A novel approach for reconstructing 5D phase space.
2. Understanding the tomography procedure
3. Investigation of a newly developed transverse deflecting structure
4. Presentation of related research

5D tomographic phase-space reconstruction of particle bunches

S. Jaster-Merz,[1,2,*] R. W. Assmann,[1,3,†] R. Brinkmann,[1] F. Burkart,[1] W. Hillert,[2] M. Stanitzki,[1] and T. Vinatier[1]

[1]*Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany*
[2]*Department of Physics, Universität Hamburg, Hamburg, Germany*
[3]*Laboratori Nazionali di Frascati, Frascati, Italy*

(Received 8 May 2023; revised 16 February 2024; accepted 16 May 2024; published 3 July 2024)

We propose a new beam diagnostics method to reconstruct the phase space of charged particle bunches in five dimensions, which consist of the horizontal and vertical positions and divergences as well as the time axis. This is achieved by combining a quadrupole-based transverse phase-space tomography with the adjustable streaking angle of a polarizable X-band transverse deflection structure. We demonstrate with detailed simulations that the method is able to reconstruct various complex phase-space distributions and that the quality of the reconstruction depends on the number of input projections. This method allows for the identification and visualization of previously unnoticed detailed features in the phase-space distribution and can thereby be used as a tool toward improving the performance of particle accelerators or performing more accurate simulation studies.

DOI: 10.1103/PhysRevAccelBeams.27.072801

Introduction

- Detailed information of beam properties is required.
    1. RF gun optimization
    2. FEL performance improvement
    3. Coupling terms investigation

- Low dimensional (2D) tomography give limited information.
    - Cannot analysis coupling terms from 2D.

- Higher tomography can reveal transverse-longitudinal coupling (e.g., space charge, CSR, etc.)

- PolariX TDS was developed in 2020.
  (by CERN, DESY, and PSI)

  → Can reconstruct 5D phase space! (x, x', y, y', t)

- **From a previous my JKPS paper,**

4D beam matrix: $\sigma^{4D} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy}^T & \sigma_{yy} \end{pmatrix}$

Coupling cause ε growth

Four-dimensional transverse beam matrix measurement using the multiple-quadrupole scan technique

Eduard Prat* and Masamitsu Aiba
Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland
(Received 10 July 2013; published 19 May 2014)

Accurate measurements of the transverse beam properties are essential to understand and optimize particle beams. We present an optimized method that uses three quadrupole magnets and one profile monitor to measure the full 4D transverse matrix of the beam. The method has been applied to the SwissFEL Injector Test Facility (SITF) at the Paul Scherrer Institute (Villigen). The SITF is the principal test bed and demonstration plant for the SwissFEL project, which aims at realizing a hard-x-ray free-electron laser in 2017. Simulations, measurements, and results of cross-plane coupling correction are presented.

DOI: 10.1103/PhysRevSTAB.17.052801 PACS numbers: 29.20.Ej, 41.60.Cr, 41.85.-p, 29.27.Fh

P. CRAIEVICH et al. (2020)

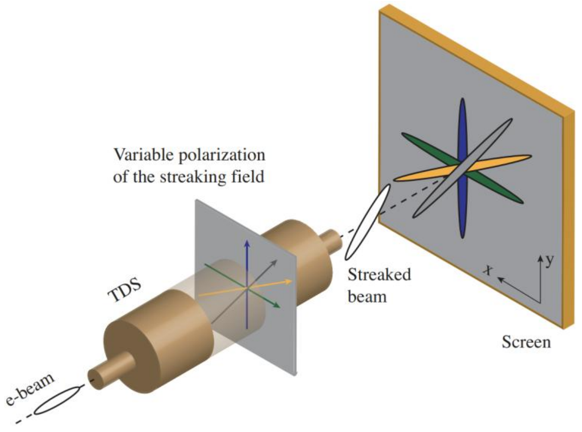
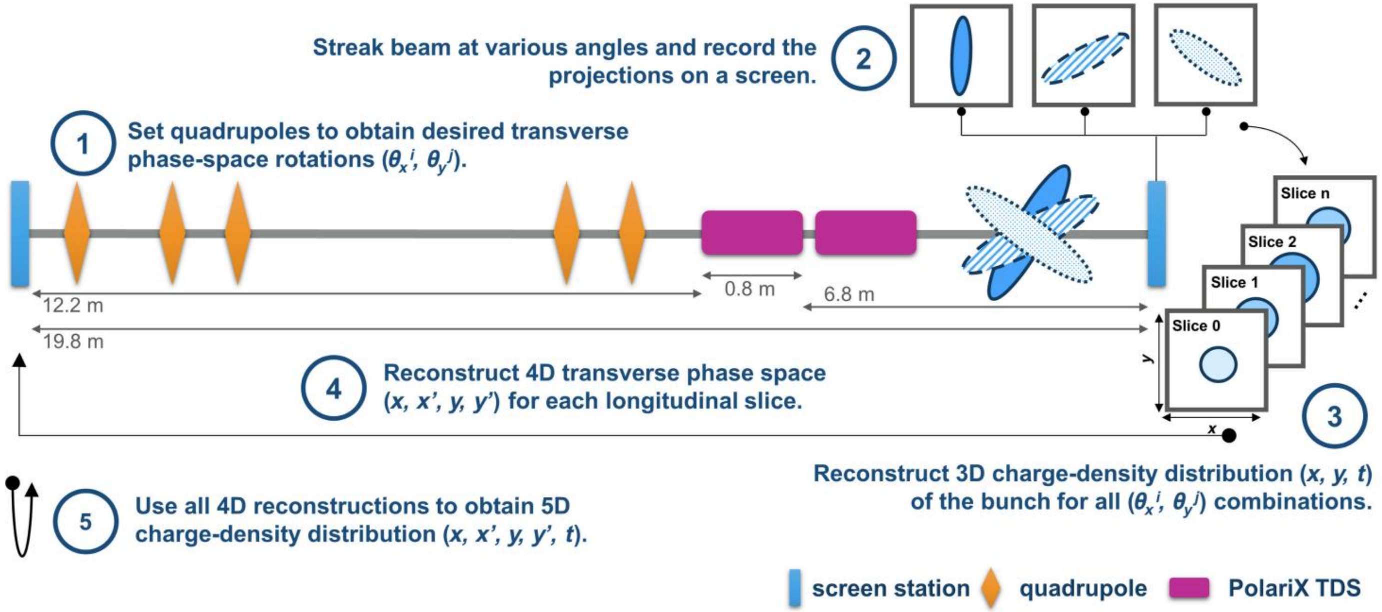


FIG. 1. Schematic of a longitudinal diagnostic system based on the variable polarization of the PolariX TDS.

ARES beamline

Accelerator Research Experiment at SINBAD (ARES)

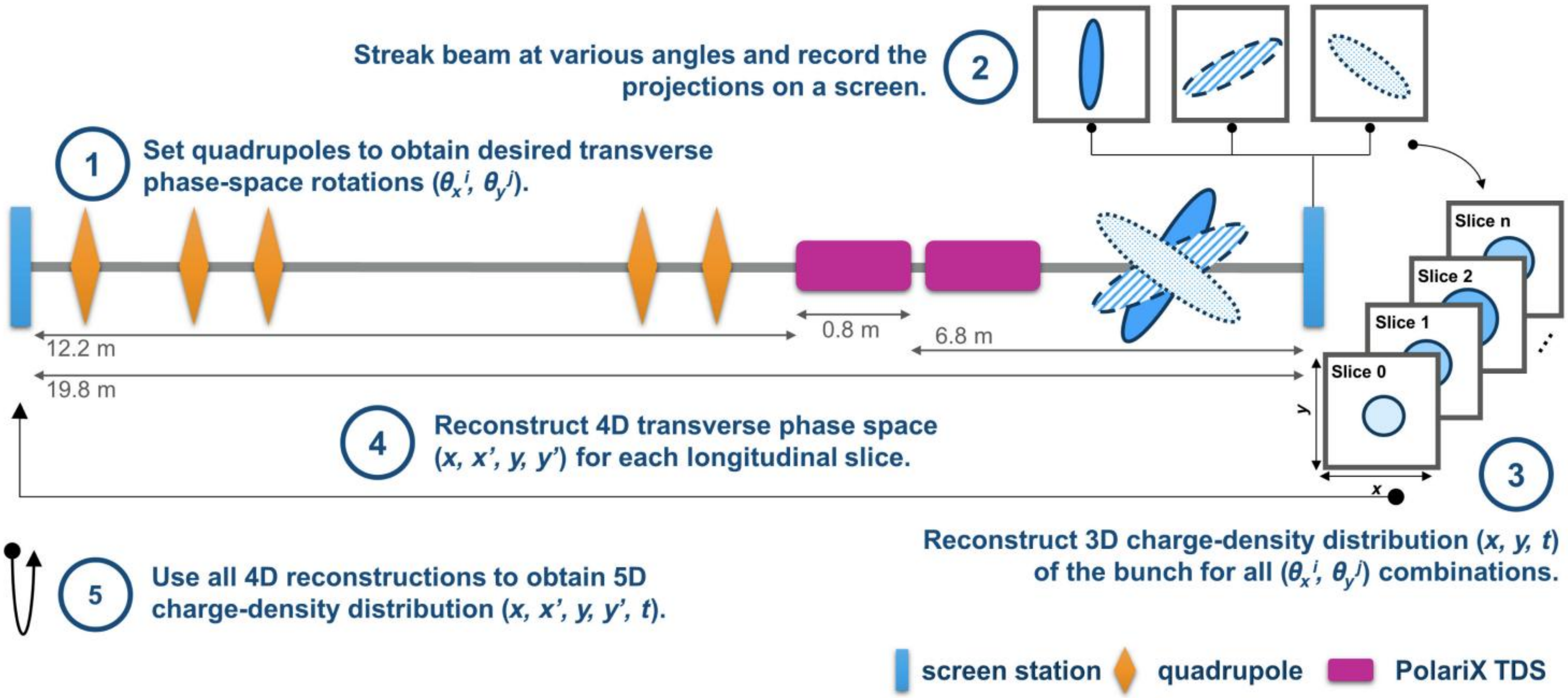
- Ultra-short bunched (fs to sub-fs), high brightness, 100-150 MeV energy range
- It located at the SINBAD facility at DESY.
- Includes a photoinjector



Applications:

- Dielectric Laser Acceleration (DLA)
- Laser-driven Plasma Acceleration (LPA)

Working principle



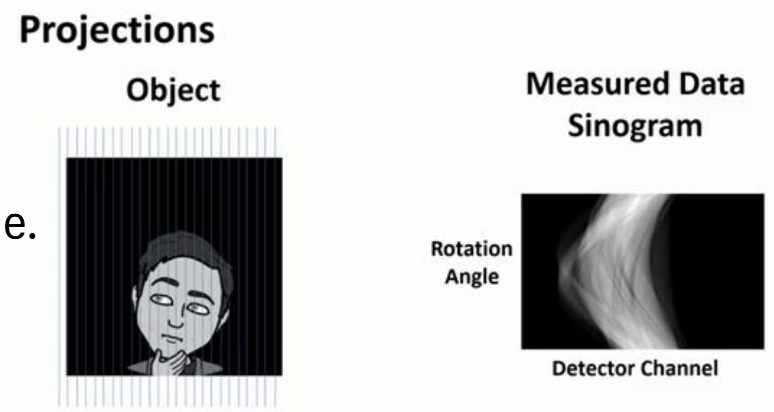
Principle of tomography

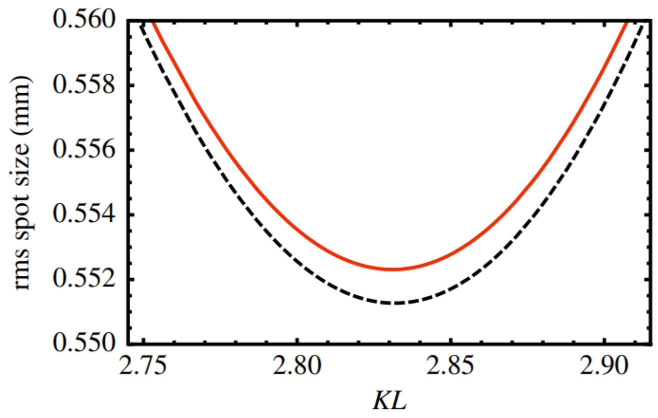
1. **1D Projection extraction**
   - One-dimensional (1D) projection data can be extracted from the 2D (x, y) plane.

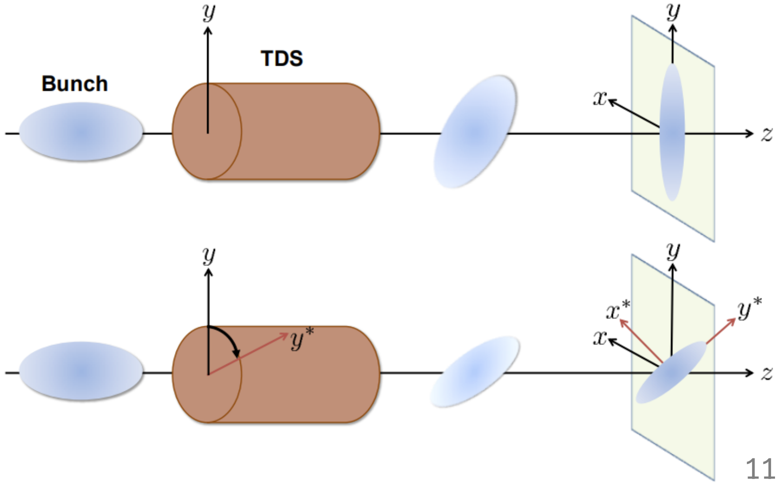
2. **Phase space representation**
   - This projection also accounts for the corresponding phase space distribution.

3. **Angular sampling requirement**
   - Data must be collected from multiple angles (e.g., QM scans).

4. **Tomographic reconstruction**
   - Reconstruction algorithms combine these to retrieve the original distribution.

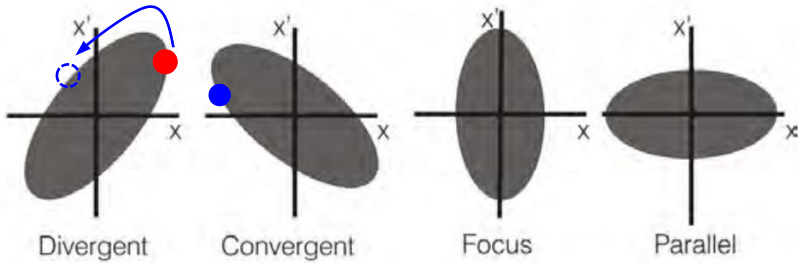






Important to obtain ***equally spaced angle data!***

Normalized phase space



K.M. Hock et al. "Beam tomography in transverse normalised phase space" (2011)

Normalized coordinate transformation

$$M(s_2|s_1) = \begin{pmatrix} \sqrt{\beta_2} & 0 \\ -\frac{\alpha_2}{\sqrt{\beta_2}} & \frac{1}{\sqrt{\beta_2}} \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{\beta_1}} & 0 \\ \frac{\alpha_1}{\sqrt{\beta_1}} & \sqrt{\beta_1} \end{pmatrix}$$

Rotation matrix

Normalization matrix

1. **Normalized phase space**
   eliminates β-dependence.

2. **Rotation angle = phase advance** μ
   provides clean rotation data.

3. **Equally spaced angles**
   minimize reconstruction error.

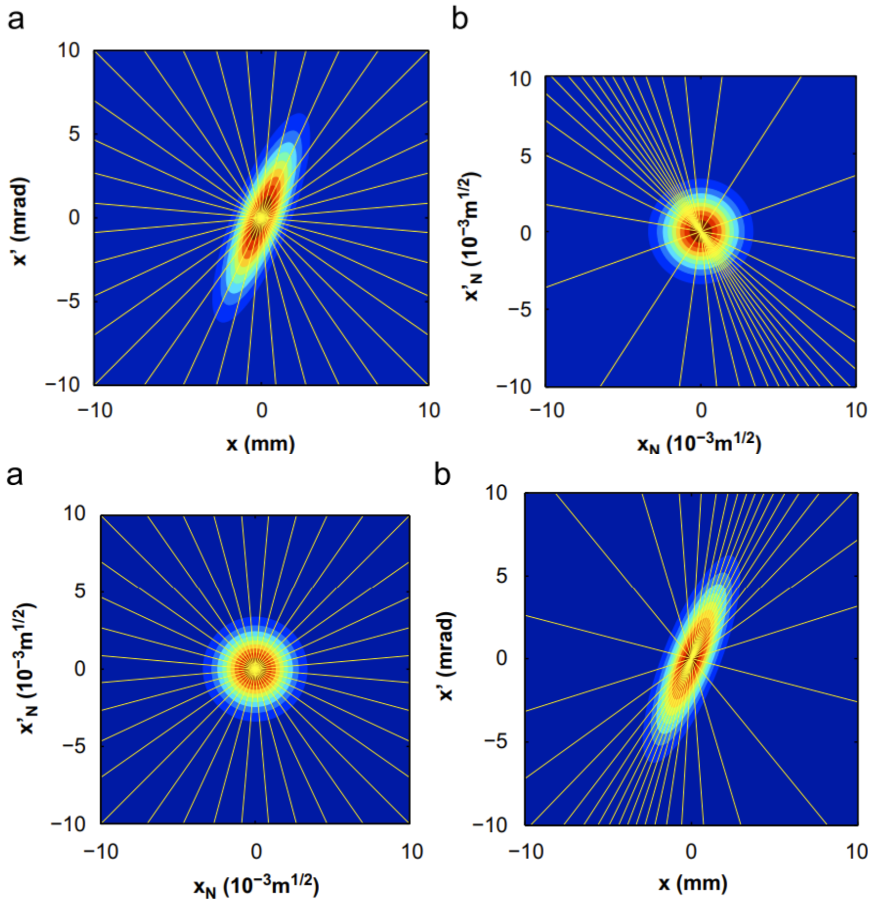


Fig. 3. (a) Normalised phase space with rays at uniform angular intervals. (b) Real phase space.

PolariX TDS

Polarizable X-Band Transverse Deflection Structure (PolariX TDS)

- $\vec{F} = q(\vec{E} + v \times \vec{B})$
- Rotate E-field direction by using both E-Rotator and phase shifter

P. CRAIEVICH et al. (2020)

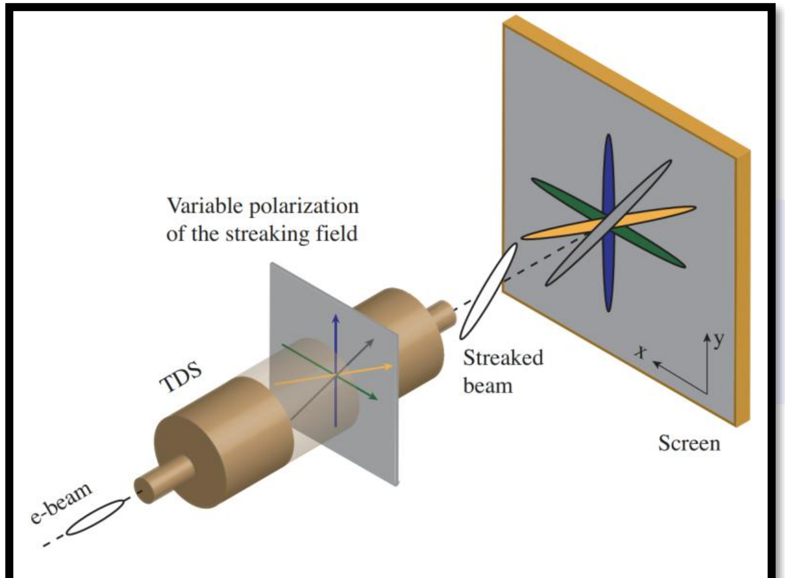


FIG. 1. Schematic of a longitudinal diagnostic system based on the variable polarization of the PolariX TDS.

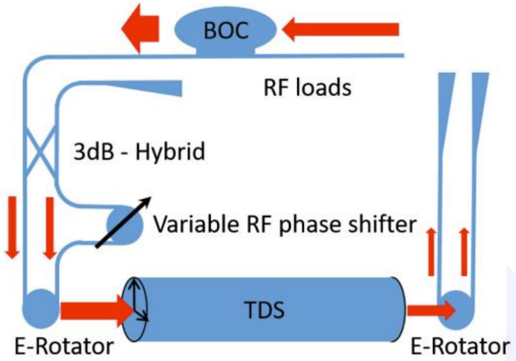


FIG. 2. Conceptual layout of the TDS with variable polarization.

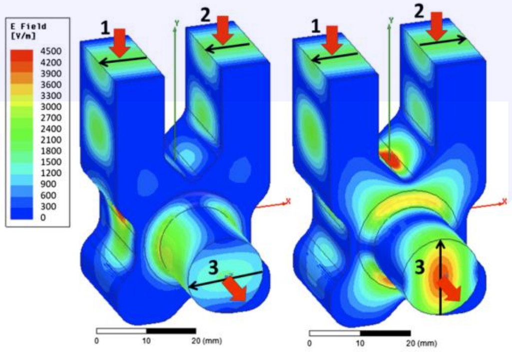
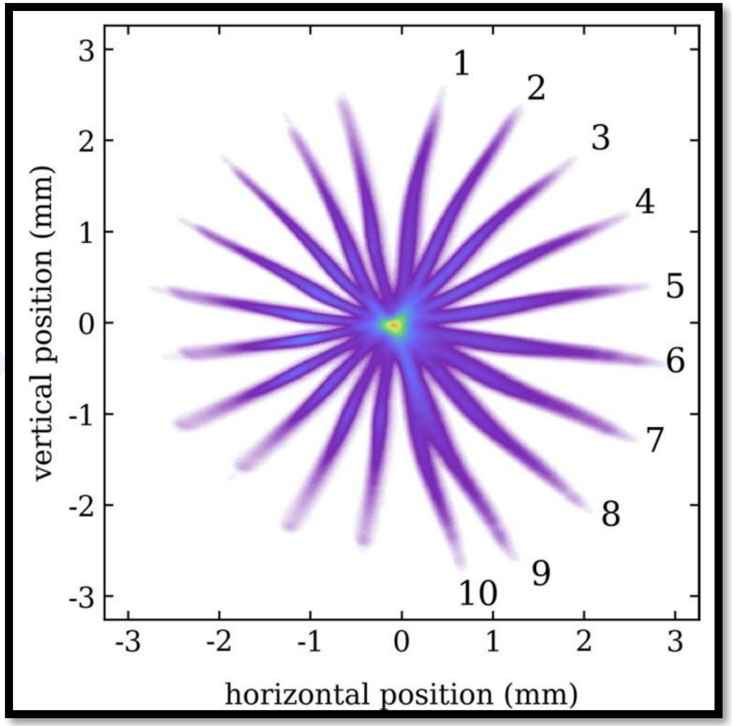


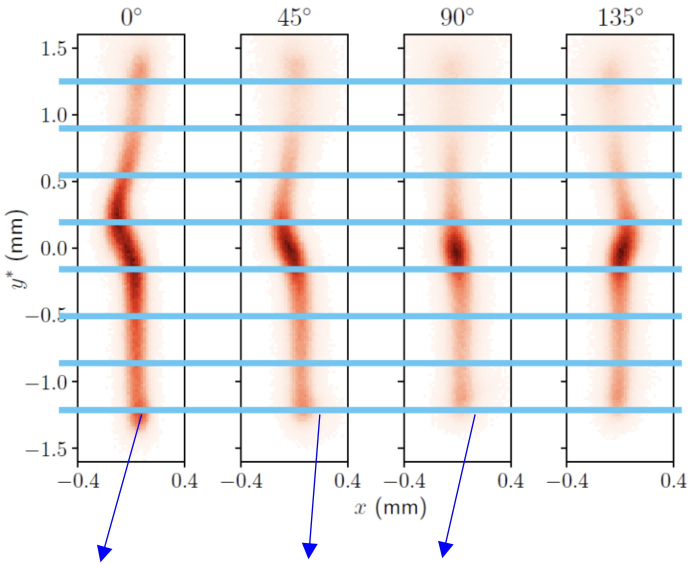
FIG. 6. E-rotator geometry and electric field distribution for 1 W of total input power into port 1 and 2 with the same (left) and with the opposite phase (right). The black arrows in the ports 1 and 2 indicate the rf phase. The black arrows in the output port show the polarization of the linear TE_{11} mode.



3D charged-density distribution

Application

- Reconstruction 3D bunch profile (x, y, t).



Projection $P_{n,1}$ $P_{n,2}$ $P_{n,3}$

Dejian Qiu et al. 2016

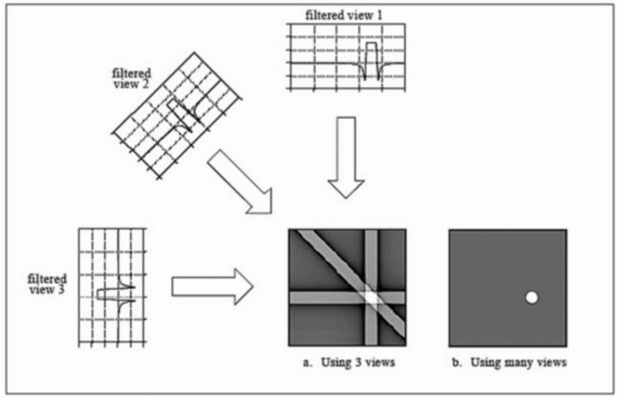
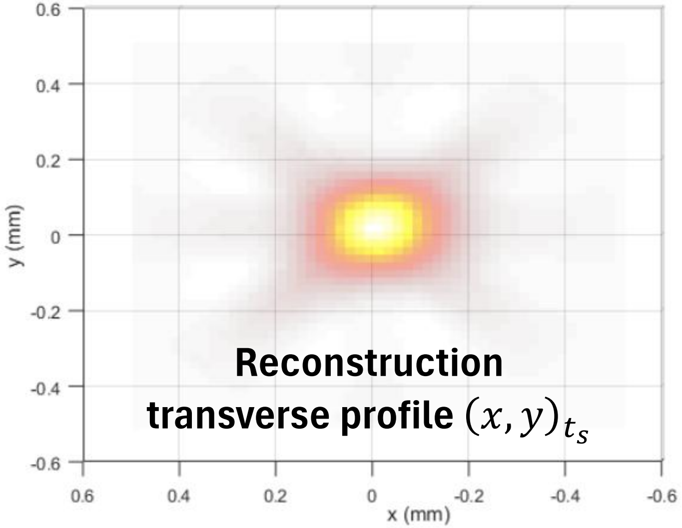
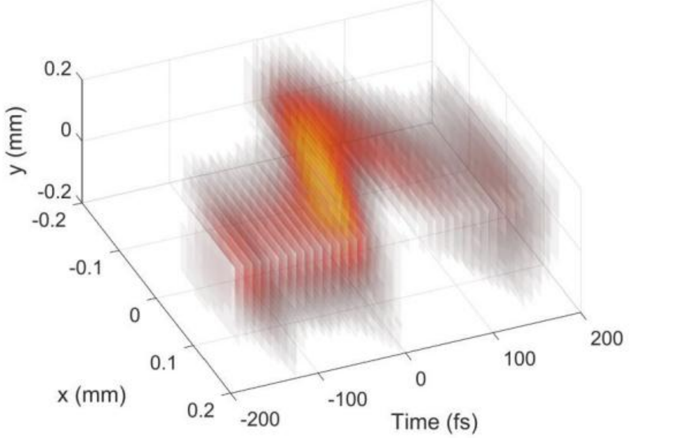


Figure 1: The image reconstruction process of the FBP algorithm. Each set of projection data, taken at different angles, undergoes convolution filtering before back-projection. This removes the blurring that would result from simple backprojection without filtering.[2]



Reconstruction 3D beam profile (x, y, t)



PolariX TDS

Application

- Reconstruction 3D bunch profile (x, y, t).

P. CRAIEVICH et al. (2020)

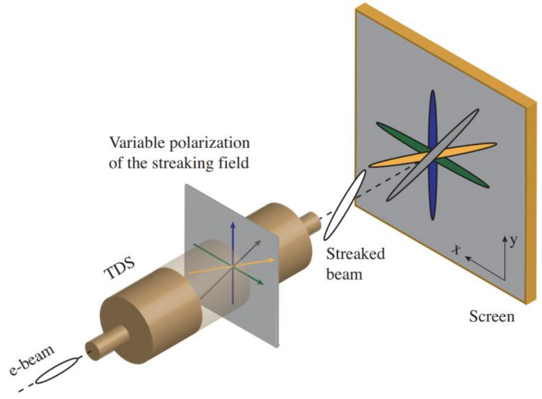


FIG. 1. Schematic of a longitudinal diagnostic system based on the variable polarization of the PolariX TDS.

B. Marchetti et al. (2021)

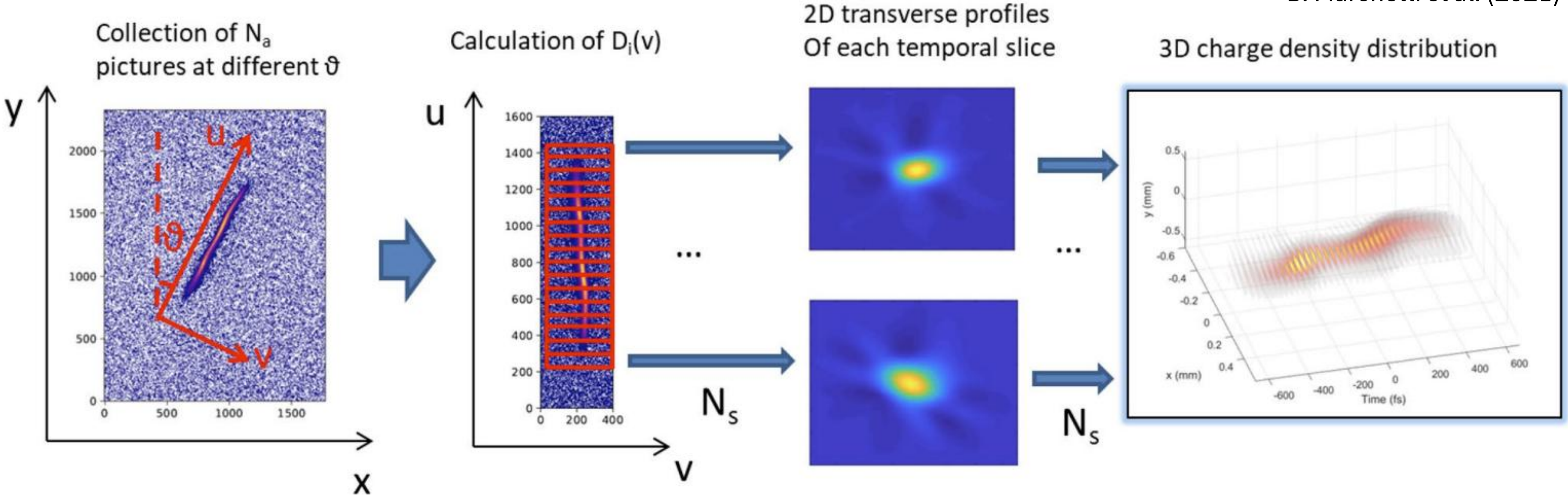
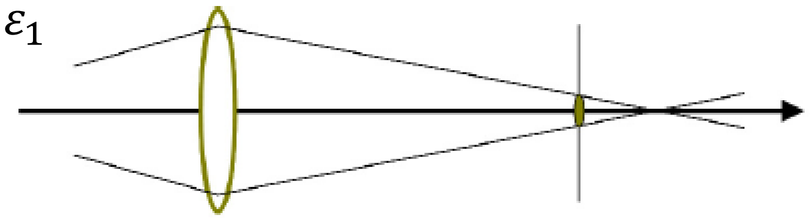


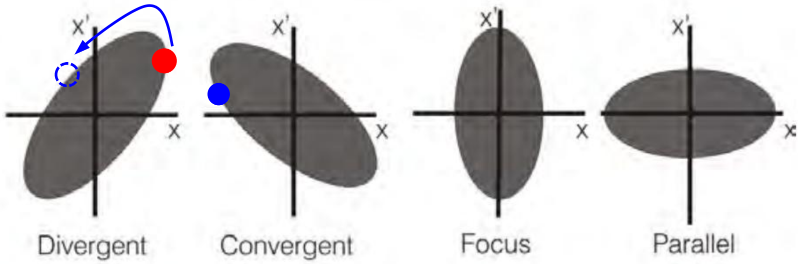
Figure 9. Algorithm for the tomographic reconstruction. The scheme illustrates schematically the conceptual steps performed in the data analysis of the images of the streaked beam to obtain a 3D reconstruction of the charge density distribution of the bunch.

Quadrupole scan method

$$M_{FODO} = \begin{bmatrix} 1 - \frac{l^2}{2f^2} & 2l(1 + \frac{l}{2f}) \\ -\frac{l}{2f^2}(1 - \frac{l}{2f}) & 1 - \frac{l^2}{2f^2} \end{bmatrix} = \begin{bmatrix} \cos\psi & \beta\sin\psi \\ -\frac{1}{\beta}\sin\psi & \cos\psi \end{bmatrix}$$

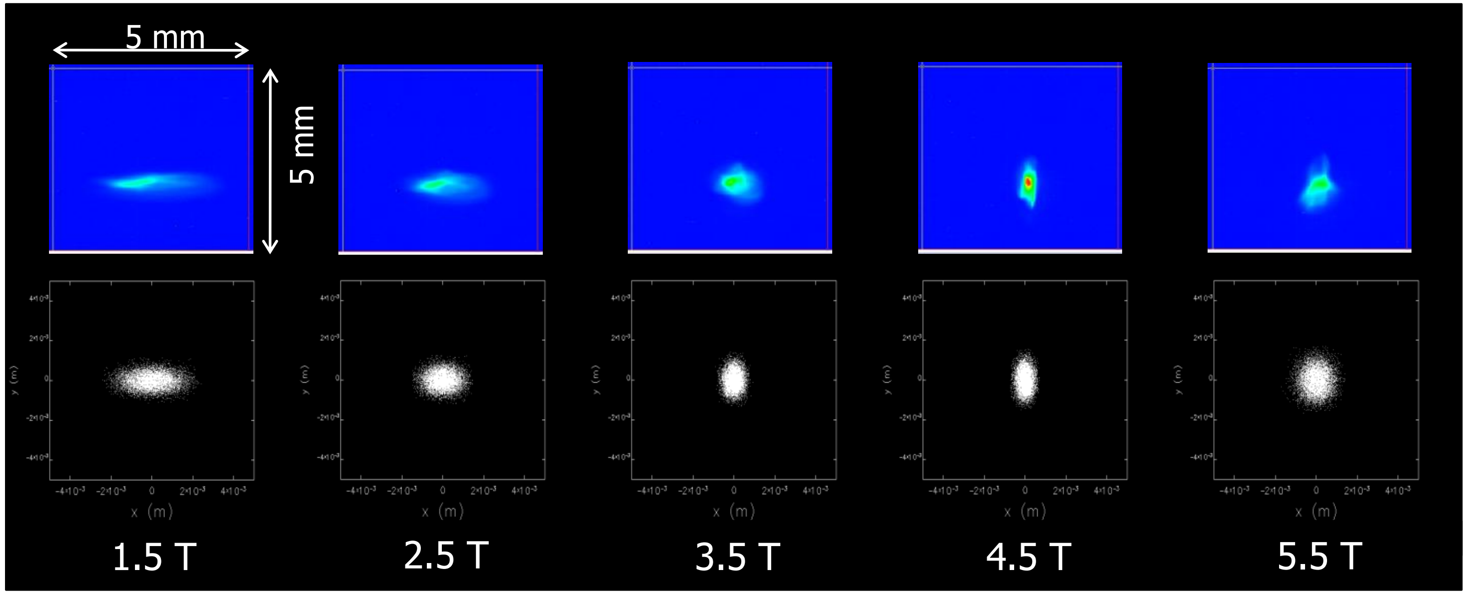
$$\sigma_2^2 = \beta_2\varepsilon_1$$

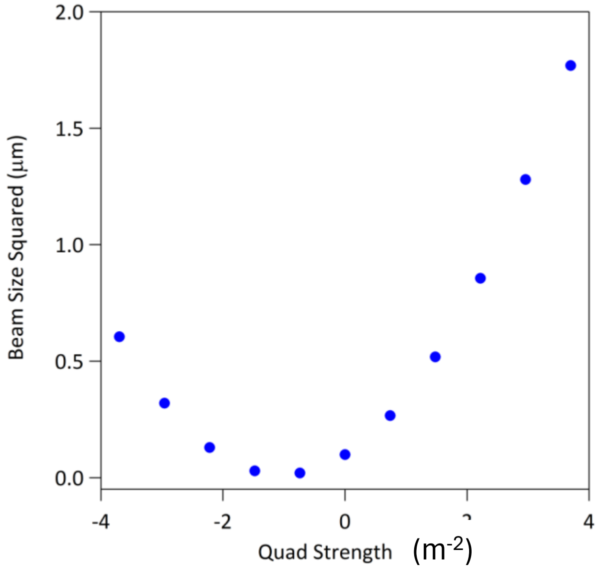




$$\begin{pmatrix} \beta_2 \\ \alpha_2 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} M_{11}^2 & -2M_{11}M_{12} & M_{12}^2 \\ -M_{11}M_{21} & M_{12}M_{21} + M_{11}M_{22} & -M_{12}M_{22} \\ M_{21}^2 & -2M_{21}M_{22} & M_{22}^2 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \alpha_1 \\ \gamma_1 \end{pmatrix}$$

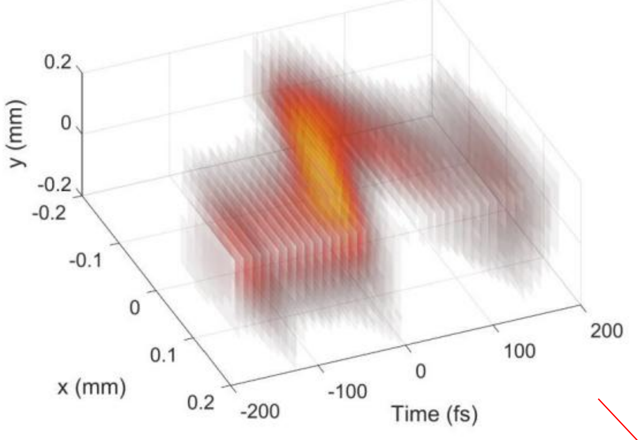
$$\sigma_2^2 = \beta_2\varepsilon_1 = \varepsilon_1[M_{11}^2 - 2M_{11}M_{12} + M_{12}^2]$$



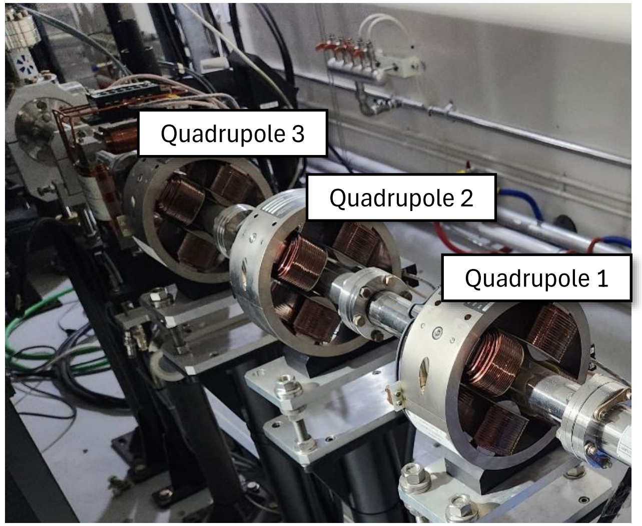


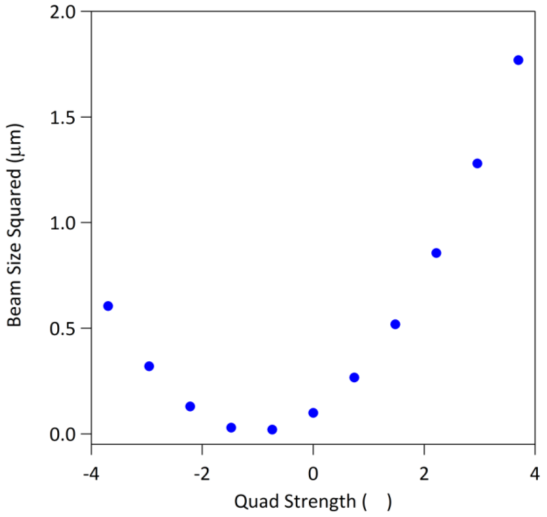
5D beam profile reconstruction

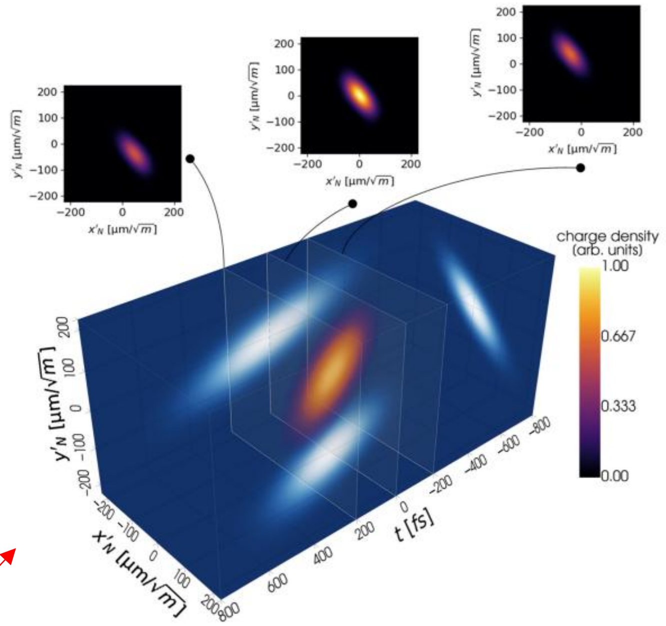
1. Using 3D beam profile (x, y, t) from TDS



2. Reconstruction sliced distribution
$(x, x', y, y')_{t_s}$







3. Combination the reconstructed data
$(x, x', y, y')_t$

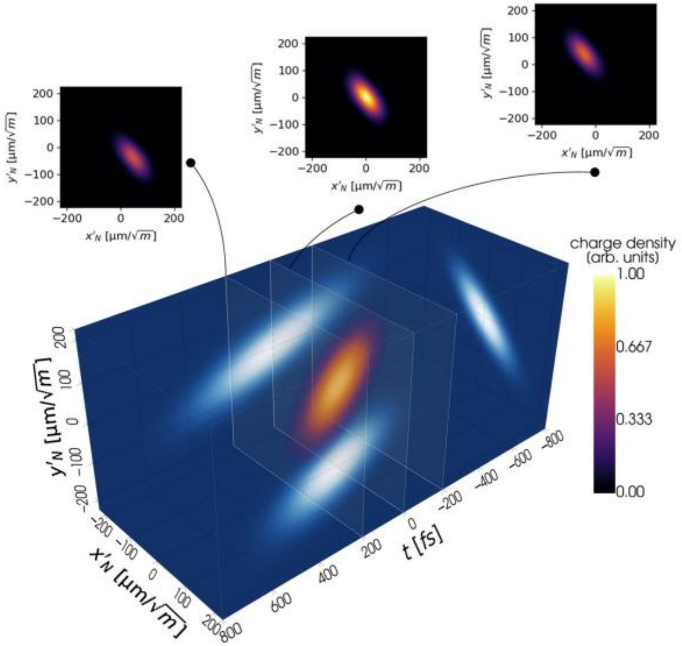
Results

- Single bunch reconstruction. (~ 42 h)

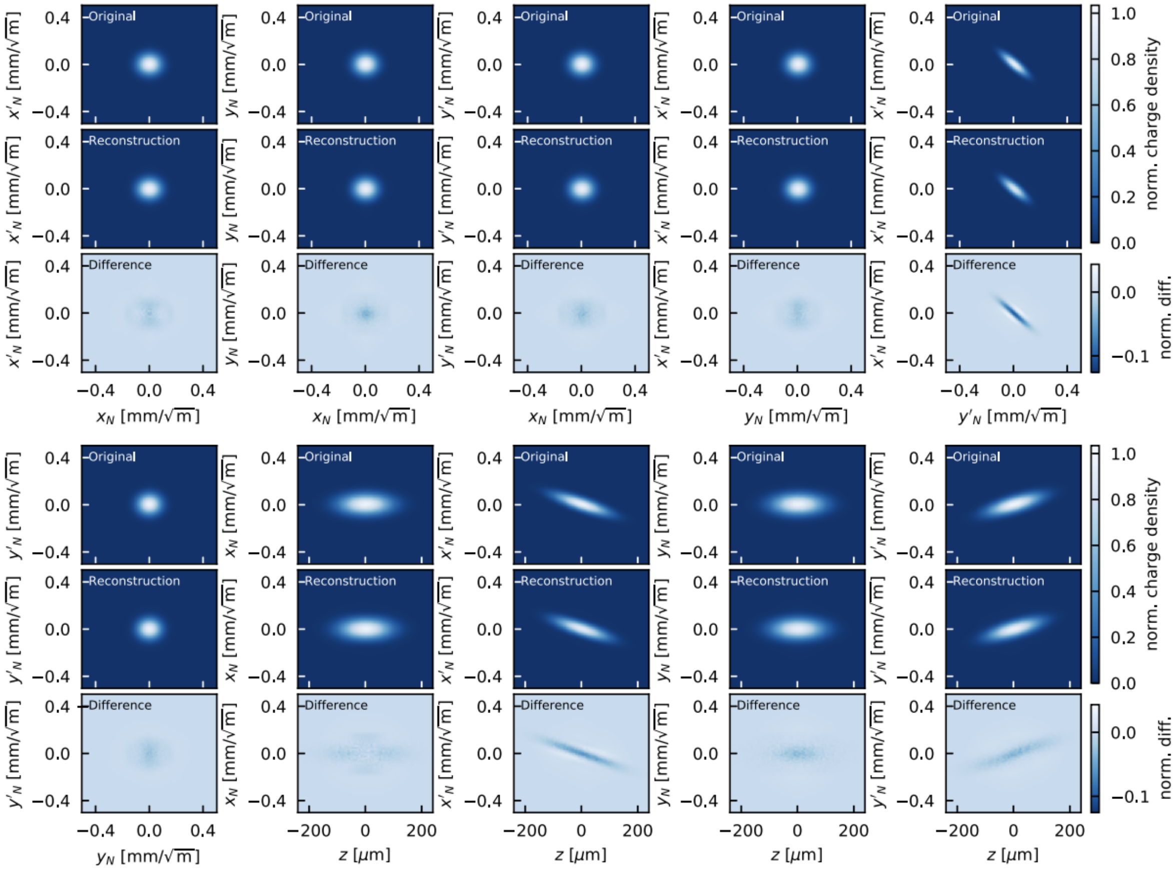
TABLE II. Original and reconstructed correlations of the Gaussian distribution with correlations.

Plane	Original	Reconstruction
(x'_N, y'_N)	-0.848	-0.814
(x'_N, z)	$-0.835\ \mathrm{m}^{1/2}$	$-0.793\ \mathrm{m}^{1/2}$
(y'_N, z)	$0.706\ \mathrm{m}^{1/2}$	$0.677\ \mathrm{m}^{1/2}$

- Can be neglect any transverse correlations.

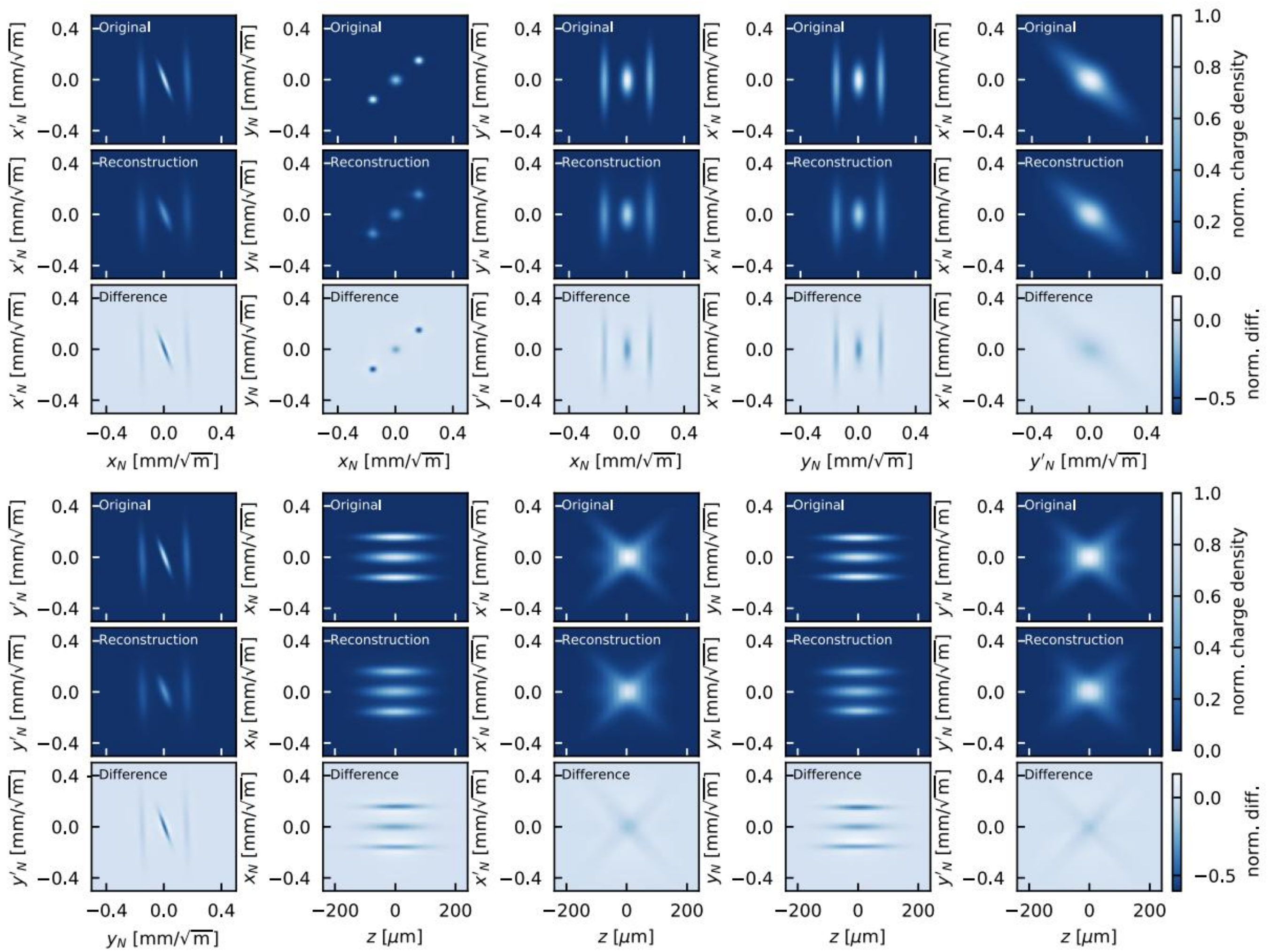


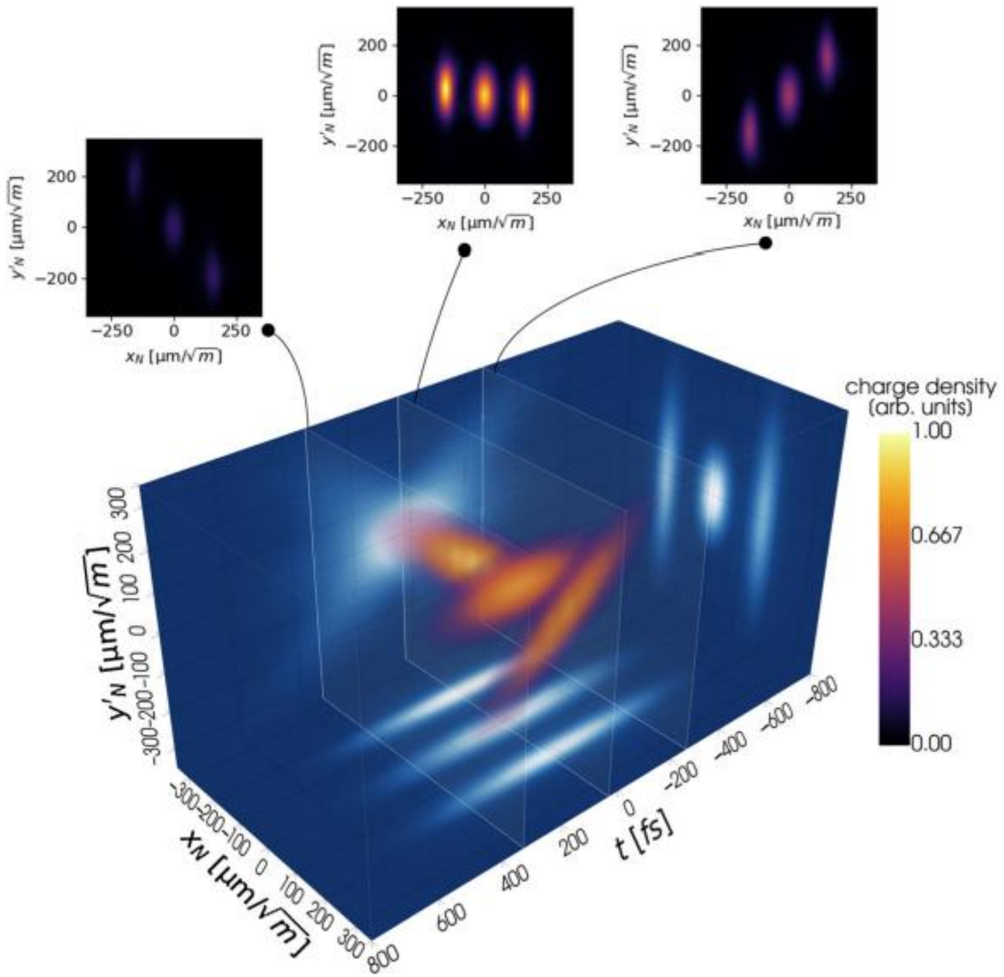
- If consider chromaticity effects of QM, ε_x, ε_y were increased 22 %, 11 %, respectively.



Results

- Multi-bunch reconstruction





Influence of rotation angles

1. **Angle number dependence**
   - More rotation and steaking angles improve accuracy
   - Accuracy saturates beyond ~50 streaking angles

2. **Error thresholds**
   - ~ 50x50 angles give <5% error; ~30x25 angles suffice for <10% in all planes.

3. **Correlation sensitivity**
   - Largest error appears in x'-plane due to strong $x' - t$ correlation

4. **Bunch length robustness**

5. **Algorithm Optimization (Alternative algorithm, AI)**

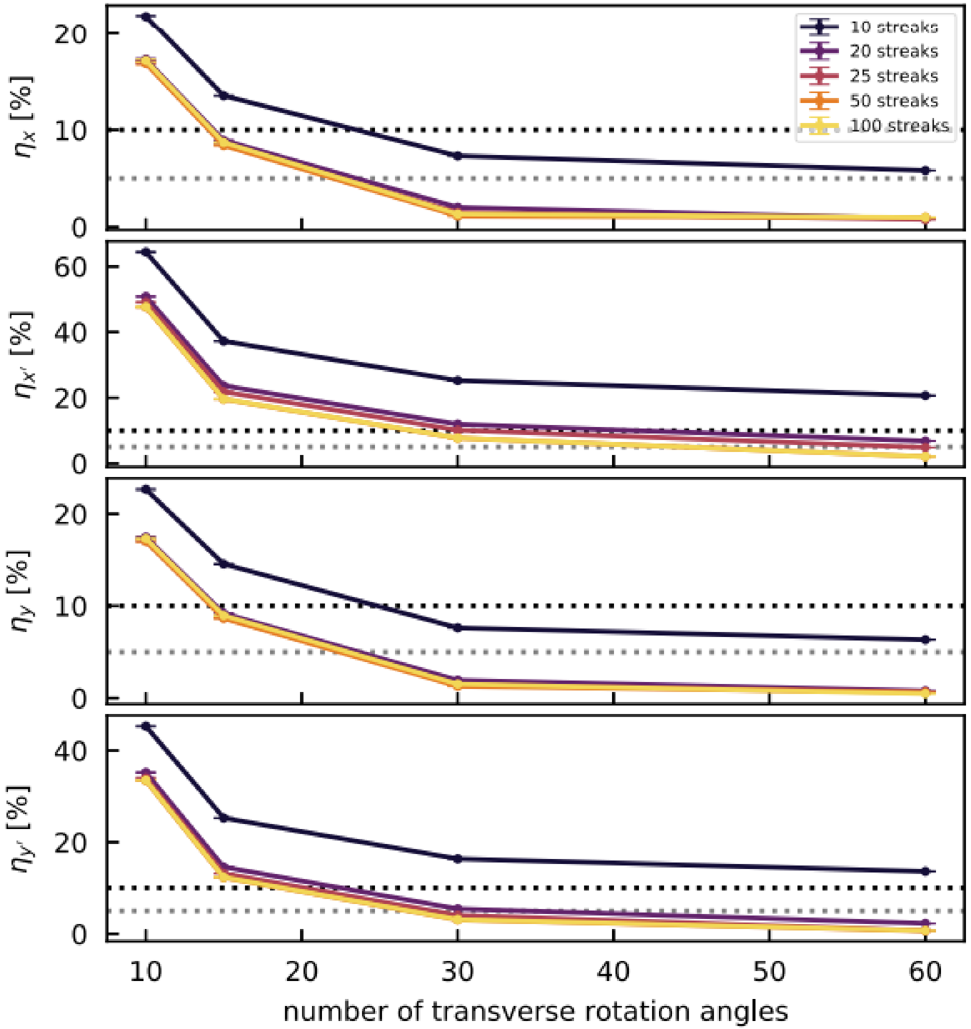


FIG. 6. Relative discrepancies, as defined in Eq. (5), between the reconstructed and input distribution of all transverse planes for various numbers of transverse rotation and streaking angles. The dotted gray and black lines show the 5% and 10% levels, respectively.

Additional information

- Experimental demonstration was done recently.

Experimental demonstration of a tomographic 5D phase-space reconstruction

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(Dated: May 21, 2025)

Detailed knowledge of particle-beam properties is of great importance to understand and push the performance of existing and next-generation particle accelerators. We recently proposed a new phase-space tomography method to reconstruct the five-dimensional (5D) phase space, i.e., the charge density distribution in all three spatial directions and the two transverse momenta. Here, we present the first experimental demonstration of the method at the FLASHForward facility at DESY. This includes the reconstruction of the 5D phase-space distribution of a GeV-class electron bunch, the use of this measured phase space to create a particle distribution for simulations, and the extraction of the transverse 4D slice emittance.

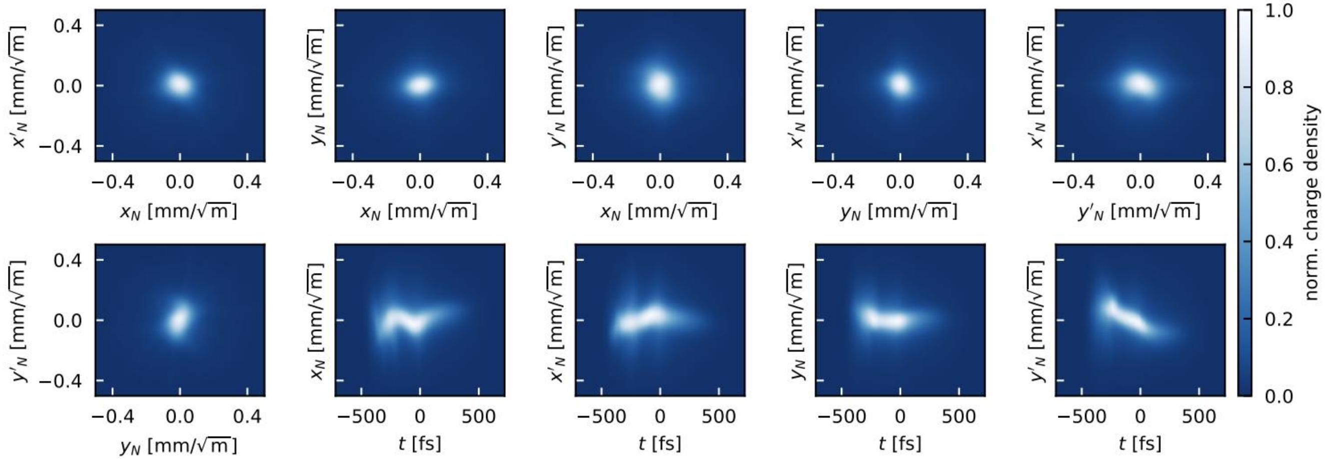

FIG. 2. 2D projections of the reconstructed 5D phase-space distribution normalized to their maximum value. The transverse projections are shown in normalized phase space. The head of the bunch is towards negative time values.

TABLE I. Reconstructed Beam Parameters

Param.	Units	Tomography	Quad. scan
ϵ_x^n	µm	6.63 ± 0.10	6.22 ± 0.68
ϵ_y^n	µm	8.25 ± 0.20	6.69 ± 0.62
α_x	-	0.09 ± 0.00	0.37 ± 0.08
α_y	-	-0.28 ± 0.01	0.05 ± 0.14
β_x	m	10.75 ± 0.04	11.74 ± 1.50
β_y	m	7.44 ± 0.05	7.04 ± 0.61
σ_t	fs	199^{+8}_{-7}	194 ± 16 [a]

[a] Obtained from streaked screen images at both zero crossings.

Conclusion

1. **Accurate 5D reconstruction**: Enables complete 5D (x, x', y, y', t) phase-space distribution visualization of accelerated bunches

2. **Gaussian validation**: Simulation shows excellent agreement with original beam parameters (≲5% discrepancy)

3. **Complex distribution handling**: All features reconstructed but smeared due to thin/sharp distribution characteristics

4. **Accuracy requirements**: ~30 rotation angles and 25 streaking angles needed for <10% discrepancy

5. **Future plans**: Exploring improved algorithms and experimental validation

Discussion

- Lack of clarity in normalized phase space effects
- Comparison: weighted projection vs normalized space reconstruction
- **Necessity:**
  1. Systematic evaluation.
  2. Potential advantages of adaptive weighting techniques.

K.M. Hock et al. “Beam tomography in transverse normalised phase space” (2011)

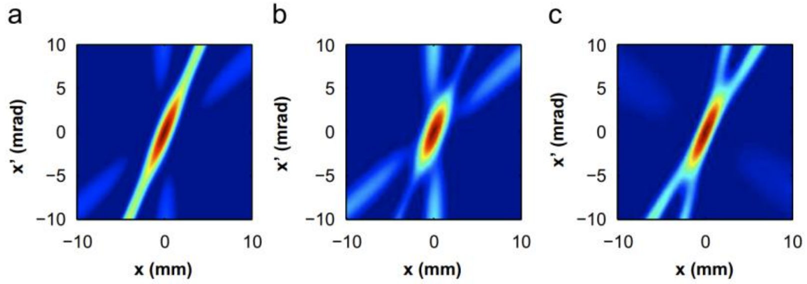

Fig. 5. Reconstruction in real phase space, using projections from the three screens in the ALICE tomography section: (a) projections are not weighted, (b) projections are weighted, (c) projections are weighted and the reconstruction location shifted some distance before screen 1.

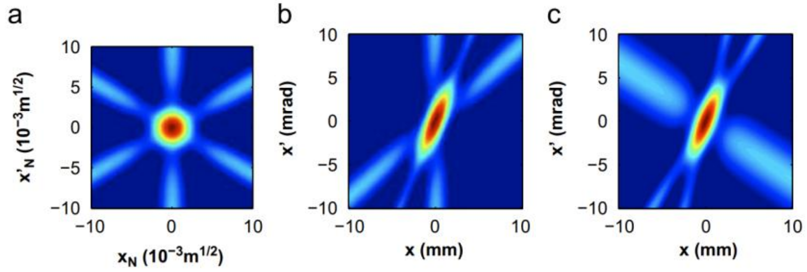

Fig. 6. (a) Reconstruction in normalised phase space; (b) the co-ordinates are transformed to real phase space; (c) the same, but with the reconstruction location shifted some distance before screen 1.

Discussion

- **Tomography of 3D charge density (x, y, t)** has already been confirmed using conventional TDS and does ***not require PolariX TDS.***

Reconstruction of the three-dimensional bunch profile by tomography technique with RF deflecting cavity

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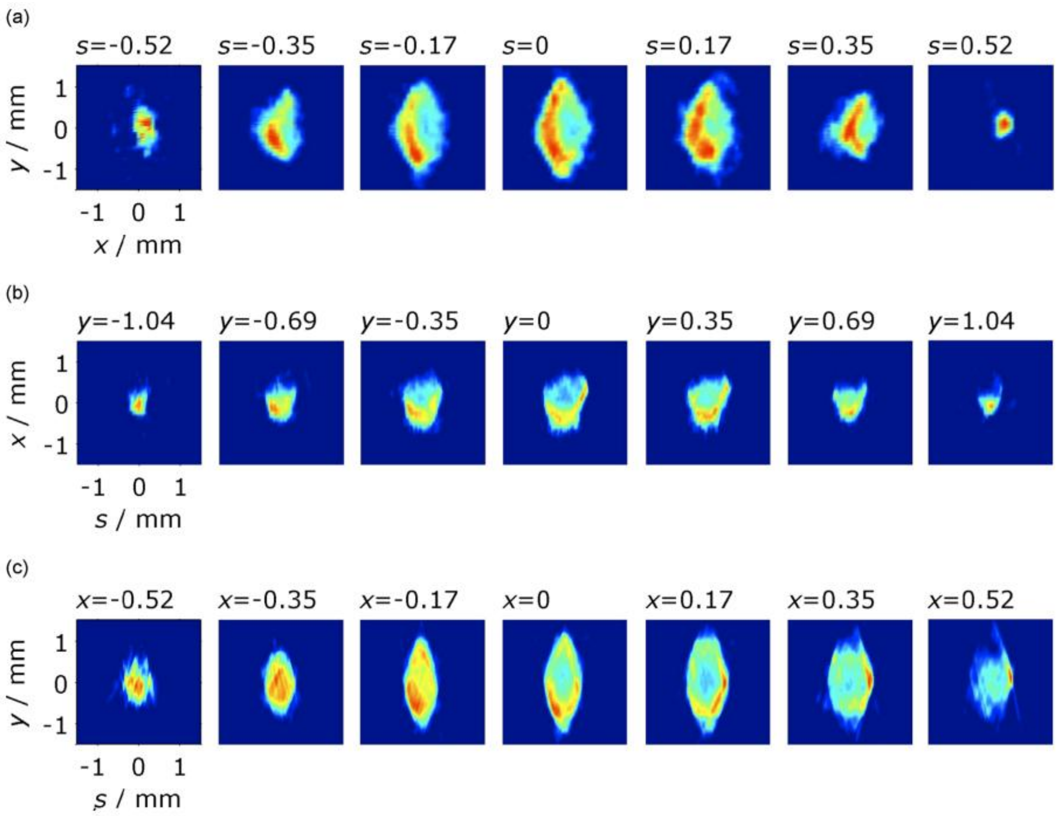

Fig. 9. Profiles of bunch slices.