1. Wake and impedance
2. Ion effect, strong head-tail instability, head-tail instability
3. Coupled-bunch instability
4. Potential-well distortion and microwave instability
5. Beam-beam interaction
6. Space charge, Touschek lifetime, intrabeam scattering

Wake and Impedance

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Reference

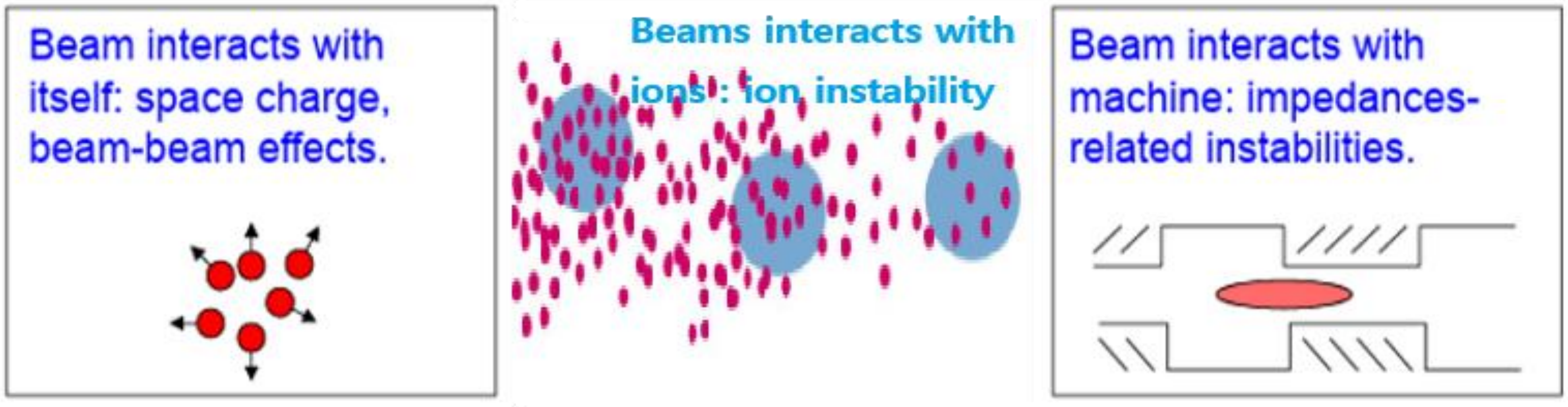
- Andy Wolski, wake field and impedance
- A. Chao, Physics of collective beam instabilities in high energy accelerator, 1993
- L. Palumbo, V.G. Vaccaro and M. Zobov, LNF-94/041 (P) 1994, Wake Fields and Impedance
- Lecture 4 wakefield in a bunch of particles, USPAS 2019

What are collective effects

■ Collective effects in general take into account the effects of the beam's own Coulomb force field on itself and it's environment (vacuum chambers).

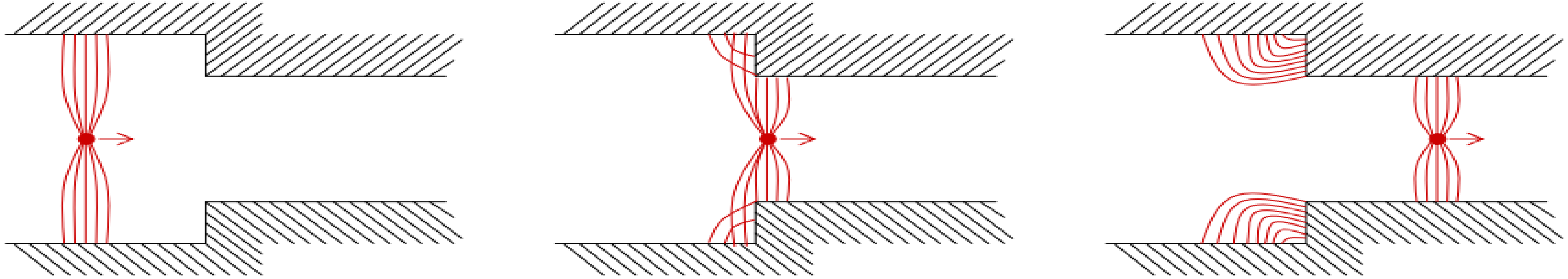
■ As the number of the charged particle increases, the particles' own fields (and fields induced by them) can affect particle's behavior, which is called the collective effects.

■ In a very general sense, we can break collective effects down into three categories: beam-self, beam-beam and beam-environment

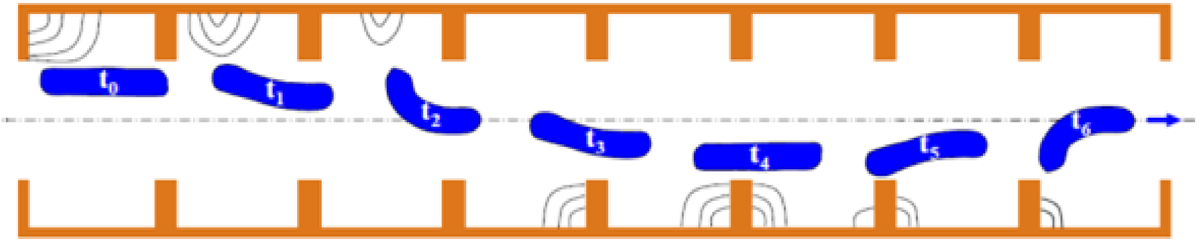


Collective instabilities

- Beam interacts with its surroundings to generate an electromagnetic field, known as wakefield. This field then acts back on the beam, perturbing its motion.



- Under some conditions, the perturbation on the beam are continously enhanced by the wakefield, leading to the collective instabilities.



Wakes are transient fields

■ Wake duration depends on the geometry & material of the structure.

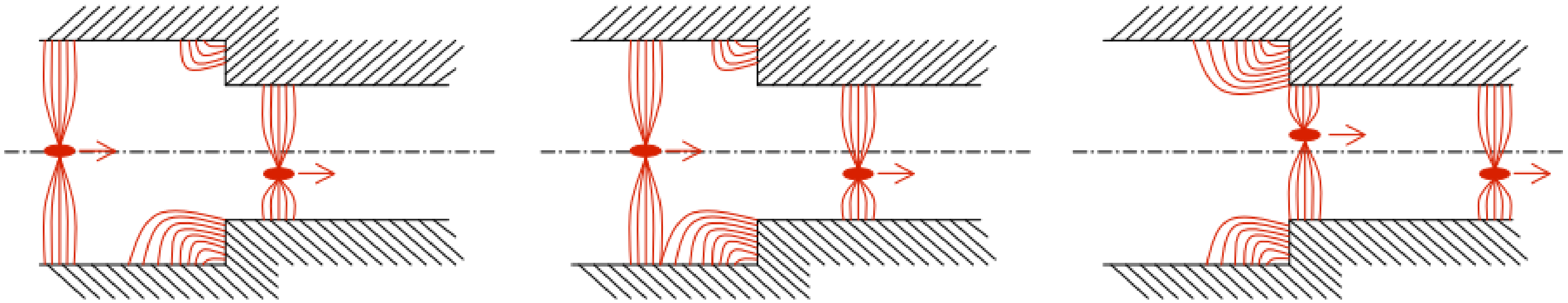
■ Wake persists for the duration of a bunch passage.

- Particles in the tail interact with wakes due to particles in the head.
- Single bunch instabilities can be triggered

(distortion of the longitudinal distribution, bunch lengthening, transverse instabilities)

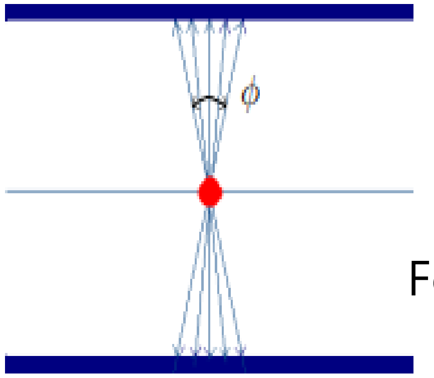
■ Wake field may last longer than the time between bunches.

- Trailing bunches interact with wakes from leading bunches to generate multi-bunch instabilities.



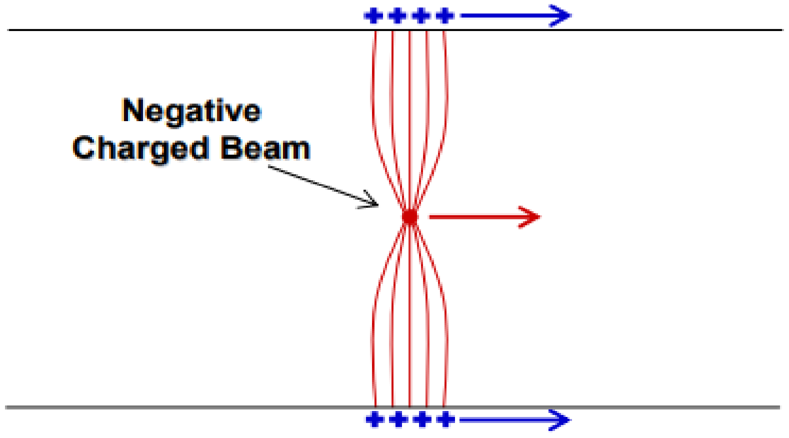
Vacuum Chamber Effects: Image Charge

- In the lab. System, beam electromagnetic field of a relativistic particle is transversely confined within an angle of ~ $1/\gamma$.
- Electric field associated with the particle beam must terminate perpendicularly on the chamber conductive walls.
- This boundary conditions requires that the same amount of charge but with opposite sign, travels on the vacuum chamber together with the beam. Such charge is referred as the image charge.

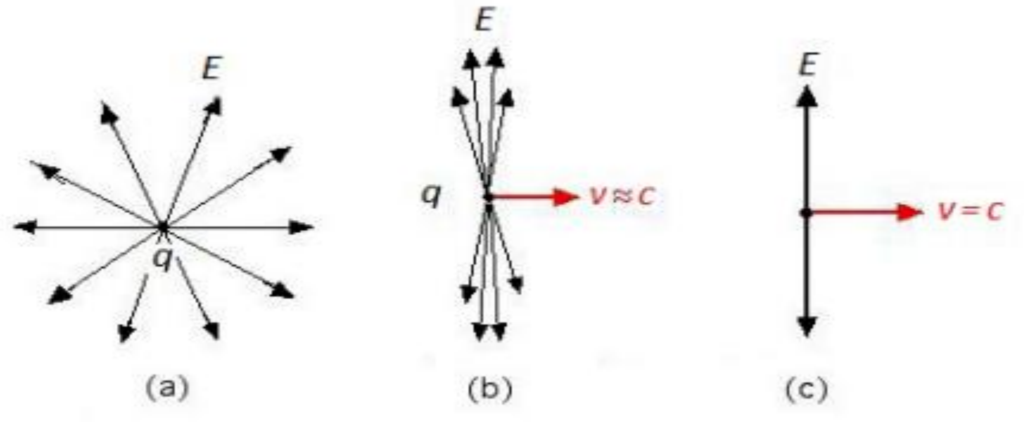


For 10 MeV

$\phi = 1/\gamma = 0.511$ MeV/E = 2.89 degree



A moving charge



Homework 1

$\nabla \bullet E = 4\pi\rho$

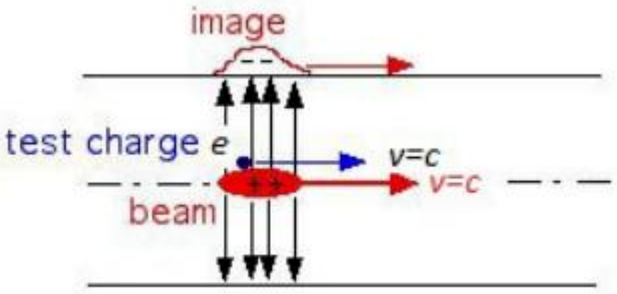
$\nabla \times B = \frac{1}{c}(4\pi j + \frac{\partial E}{\partial t})$

$E_r = \frac{2q}{r}\delta(z-ct)$

$B_\theta = \frac{2q}{r}\delta(z-ct)$

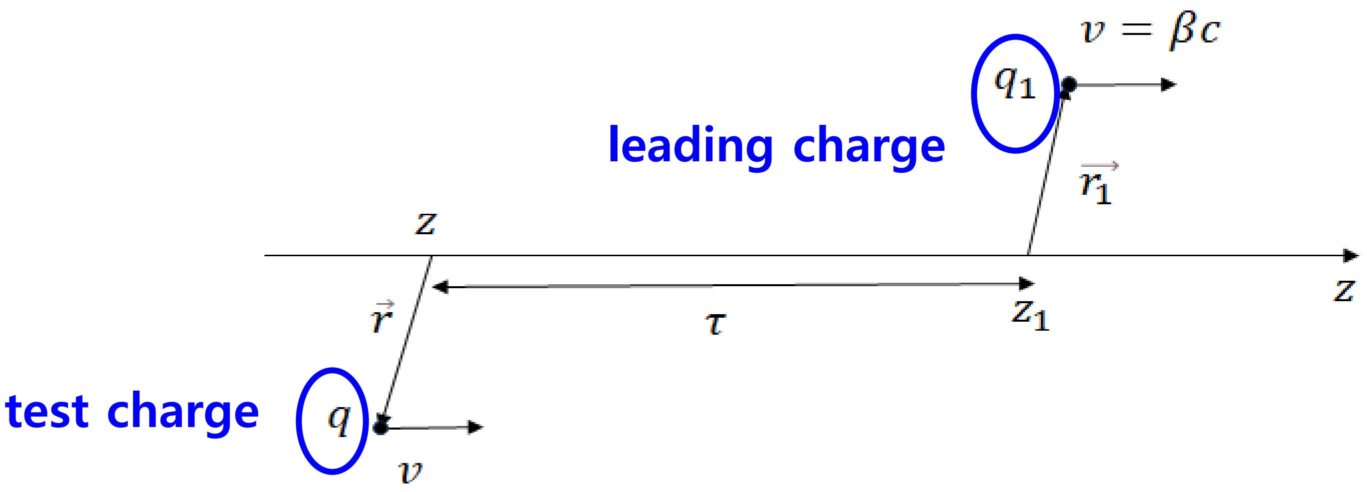
$B_\theta = E_r$ when v=c

Electric field lines of a charge: (a) stationary; (b) moving relativistically; (c) when v = c



Ultrarelativistic beam going down perfectly conducting smooth vacuum pipe

Longitudinal Wake Field



■ Time delay between two charges is τ and longitudinal coordinates at any instant t, are $z_1(t) = vt$, $z(t) = v(t - \tau)$

■ Lorentz force experienced by test charge q, due to fields created by leading charge q_1 is $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$.

■ Energy lost from leading charge q_1 is given by the work done against electromagnetic fields

$$\Delta U_{11}(r_1) = -\int_{-\infty}^{\infty} F(r_1, z_1, t)\, dz \;, \qquad t = \frac{z_1}{v}$$

for longitudinal component,

$$\Delta U_{11} = -q_1 \int_{-\infty}^{\infty} E_z(r_1, z_1, t) dz \;, \qquad t = \frac{z_1}{v}$$

Longitudinal Wake Field

■ Loss factor $k(r_1)$ is defined as energy loss to self-field per unit charge squared.

$$k(r_1) = \frac{\Delta U_{11}(r_1)}{q_1{}^2} \quad (V/C)$$

■ Test charge experiences an energy change due to fields produced by leading charge

$$\Delta U_{21} = -q \int_{-\infty}^{\infty} E_z(r, z, r_1, z_1, t) dz \ , \ t = \frac{z_1}{v} + \tau$$

■ Longitudinal wake function $W_z(r, r_1, \tau)$ is defined by as energy lost by trailing charge q per unit charge of both q_1 and q.

$$W_z(r, r_1, \tau) = \frac{\Delta U_{21}(r, r_1, \tau)}{q_1 \, q} \quad (V/C)$$

Longitudinal Wake Field

■ We need to know **wake function for a distribution of particles in a bunch**. Wake field of an arbitrary charge distribution $\mathrm{i_b}(\tau)$, $q_1 = \int_{-\infty}^{\infty} \mathrm{i_b}\, d\tau$, is obtained by convolution of δ-function wake function with bunch distribution.

Energy lost by a trailing charge q because of wake produced by slice at τ'

$$\mathrm{d}U(r,\, \tau - \tau') = q\, \mathrm{i_b}(\tau')\, wz(\tau - \tau')\, d\tau'$$

wake function of a bunch distribution $W_z(r,\tau) = \frac{U(r,\tau)}{q_1\, q} = \frac{\int_{-\infty}^{\infty} \mathrm{i_b}(\tau') w_z(\tau-\tau') d\tau'}{q_1}$ (1)

$$w_z(\tau - \tau') = 0 \quad for\ \ \tau > \tau'$$

(Test particle in bunch cannot influence on preceeding particles.)

For a unit test charge q_1=1, the wake function is known as wake potential V(z)

Loss factor for a bunch becomes

$$K(r) = \frac{U(r)}{q^2{}_1} = \frac{\int_{-\infty}^{\infty} \mathrm{i_b}(\tau) W_z(r,\tau)\, d\tau}{q_1}$$

Longitudinal Beam Impedance

■ Beam impedance is defined as Fourier transform of wake function.

$$\boldsymbol{Z_{||}(\omega) = \int_{-\infty}^{\infty} d\tau \, e^{-i\omega\tau} W_z(\tau), \qquad W_z(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \, e^{i\omega\tau} Z_{||}(\omega)}$$

■Fourier spectrum of charge distribution, $I(\omega)$, is

$$I(\omega) = \int_{-\infty}^{\infty} d\tau \, e^{-i\omega\tau} \, i_b(\tau)$$

from (1), $W_z(\tau) = \frac{\int_{-\infty}^{\infty} Z_{||}(\omega) I(\omega) e^{i\omega\tau} d\omega}{2\pi q_1}$

Longitudinal Beam Impedance

$$K(r) = \frac{\int_0^\infty Z(\omega)|I(\omega)|^2\, d\omega}{\pi q_1^2}$$

For a Gaussian charge distribution

$$I(\omega) = q_1 e^{-\frac{(\omega\sigma)^2}{2}}, \qquad (\sigma\text{: } bunch\ length)$$

$$K(r) = \frac{\int_0^\infty Z(\omega) e^{-(\omega\sigma)^2}\, d\omega}{\pi}$$

Wake Potential

$$V_z(\omega) = I(\omega) Z(\omega)$$

wake and impedance

- The e.m. fields induced by the beam are referred to as wake fields due to the fact that they are left mainly behind the traveling charge.
- Longitudinal wake potential (volts) is the voltage gain of a unit trailing charge due to the fields created by a leading charge.
- Transverse wake potential (volts) is the transverse momentum kick experienced by the beam because of the deflecting fields.
- Wake functions are defined as the wake potentials per unit charge (volt /coulomb)
- The frequency Fourier transform of the wake function is called coupling impedance.

Longitudinal wake of a resonant cavity

■ Impedance of RLC circuit is

$$\frac{1}{Z_{||}} = \frac{1}{R_s} + \frac{1}{i\omega L} + i\omega C, \quad Z_{||} = \frac{R_s}{1+iQ(\frac{\omega_R}{\omega}-\frac{\omega}{\omega_R})} \quad (2) \qquad (\omega_R = \sqrt{\frac{1}{LC}}, \ \ Q = R_s\sqrt{\frac{C}{L}})$$

■ Charge q induces a voltage

$$\frac{q}{C} = q\frac{\omega_R R_s}{Q}$$

■ Energy stored in capacitor

$$\Delta U = \frac{q^2}{2C} = \frac{\omega_R R_s}{2Q}q^2 = kq^2, \qquad \text{loss factor } k = \frac{\omega_R R_s}{2Q}$$

■ Wake function is given by inverse Fourier transform of (2) (Homework 2)

$$W(z) = 2\alpha Rs e^{\frac{\alpha z}{c}}\left(\cos\frac{\bar{\omega}z}{c} + \frac{\alpha}{\bar{\omega}}\sin\frac{\bar{\omega}z}{c}\right), \qquad (\bar{\omega} = \sqrt{\omega_R^2 - \alpha^2}, \ \alpha = \frac{\omega_R}{2Q})$$

Transverse wake fields

■ **Consider leading particle, q_1, transversely displaced from the axis.**
From small displacements, dipole component is dominant.

Test charge q receives a momentum (kick) from the fields

$$\Delta P_{21} = q \int_{-\infty}^{\infty} (E + v \times B)_{\perp} \, \mathrm{d}z \, , \qquad t = \frac{z_1}{v} + \tau$$

■ Transverse wake function is defined as kick per unit of both charges

$$W_{\perp}(z) = \frac{\Delta p_{21}(z)}{q_1 q} = \frac{\int_{-\infty}^{\infty} (E + v \times B)_{\perp} \, \mathrm{d}z}{q_1}$$

■ Transverse loss factor is transverse kick given to the charge by its own wake per unit charge squared $\quad k_{\perp} = \frac{\Delta p_{11}}{q_1^2}$

Transverse wake and impedance

Transverse impedance and wake are

$$Z_\perp(\omega) = \mathrm{i}\int_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{-i\omega\tau}\, W_\perp(\tau\,)$$

$$W_\perp(\tau) = -\frac{\mathrm{i}}{2\pi}\int_{-\infty}^{\infty} d\omega\, \mathrm{e}^{i\omega\tau} Z_\perp(\omega)$$

Wake function은 실수이므로

$$Z_{||}^*(\omega) = Z_{||}\,(-\omega)$$

(Homework 3)

$$Z_\perp^*(\omega) = -Z_\perp\,(-\omega)$$

Wake field in a bunch

- Consider a beam that consists of N particles with distribution function $\lambda(z)$. $\int dz\,\lambda(z) = 1$
  (positive z in beam head and negative z in tail)

- A particle at z interacts with other particles of beam through wake

$$\Delta p_z(z) = -\frac{Ne^2}{c}\int_z^{\infty} dz'\,\lambda(z')\,w_l(z'-z)$$

- Energy change is $\Delta\varepsilon(z) = c\Delta p_z$

$$\Delta\varepsilon(z) = -Ne^2\int_z^{\infty} dz'\,\lambda(z')w_l(z'-z)$$: (beam induced voltage)

(Negative sign means that a positive wake results in energy loss.)

- **Wake field of bunch**

$$W_l(z) = \int_z^{\infty} dz'\,\lambda(z')\,w_l(z'-z) \qquad \frac{\Delta\varepsilon(z)}{Ne^2} = -W_l(z)$$

- Average value of energy loss $\Delta\varepsilon_{av}$ (per particle)

$$\Delta\varepsilon_{av} = \int_{-\infty}^{\infty} dz\,\Delta\varepsilon(z)\lambda(z)$$, Energy loss for whole bunch is $N\,\Delta\varepsilon_{av}$

- rms energy spread

$$\Delta\varepsilon_{rms} = \left[\int_{-\infty}^{\infty} dz\,(\Delta\varepsilon(z) - \Delta\varepsilon_{av})^2\lambda(z)\,\right]^{1/2}$$

Loss factor

$K_{Loss} = -\frac{\Delta\varepsilon_{av}}{Ne^2}$ (minus sign is chosen to make loss factor positive.)

$$K_{Loss} = -\frac{1}{Ne^2}\int_{-\infty}^{\infty} dz'\,\Delta\varepsilon(z)\,\lambda(z')$$

$$= \int_{-\infty}^{\infty} dz'\,\lambda(z')\int_{z'}^{\infty} dz\,\lambda(z)\,\mathrm{w}_l(z'-z)$$

For constant wake $\mathrm{w}_l = \mathrm{w}_0$ (for short bunch)

$$= \mathrm{w}_0\int_{-\infty}^{\infty} dz'\;\lambda(z')\int_{z'}^{\infty} dz\,\lambda\,(z)$$

By change of variable from z to u,

$$u(z') = \int_{z'}^{\infty}\lambda(z)\,dz\,,\quad du = -\lambda(z')\mathrm{d}z'$$

$$u(-\infty) = \int_{-\infty}^{\infty}\lambda(z)\,dz = 1,\quad u(\infty) = 0$$

$$= \mathrm{w}_0\int_1^0\,(-)\,udu$$

$$= \frac{\mathrm{w}_0}{2}$$

Loss factor

Energy Loss $= -Ne^2 \frac{W_0}{2} = -Ne^2 * K_{Loss}$

For a very short bunch (ie, a point change), the energy lost in passing through an impedance is one-half of the product of total charge with the longitudinal wake field.

→ Fundamental theorem of beam loading

$$q^2 \frac{\mathrm{W}_0}{2} = q^2 \frac{R_S \omega_R}{2Q} = q^2 k \qquad \text{for} \quad \mathrm{w}_l(z) = \frac{R_S \omega_R}{Q} cos \frac{\omega_R z}{c} \; (z < 0)$$

$$k = \frac{R_S \omega_R}{2Q}$$

Transverse kick in a beam

- Consider a beam passing through an element with an offset y which has transverse wake w_t. What is deflection angle θ at exit?

$$\theta(\mathrm{z}) = \frac{\Delta p_{\perp}(z)}{p} = \frac{Ne^2}{cp}\int_z^{\infty} dz'\,\lambda(\mathrm{z}')\,\mathrm{y}\,\mathrm{w_t}(z'-z)$$

$$= y\frac{Ne^2}{\gamma mc^2}\int_z^{\infty} dz'\,\lambda(\mathrm{z}')\,\mathrm{w_t}(z'-z)$$

$$\theta_{av} = \langle\theta\rangle = \int_{-\infty}^{\infty} dz\,\theta(z)\,\lambda(\mathrm{z})$$

rms spread is $\Delta\theta_{rms} = \langle(\theta-\theta_{av})^2\rangle^{\frac{1}{2}}$

define kick factor

$$k_{kick} = \frac{\gamma mc^2}{yNe^2}\theta_{av} = \int_{-\infty}^{\infty} dz\,\lambda(\mathrm{z})\int_z^{\infty} dz'\,\lambda(\mathrm{z}')\mathrm{w_t}(z'-z)$$

Collective effects in multi-particle beams

① Longitudinal impedance

■ Broad-band resonator model

- Vacuum chamber of accelerator is not a perfectly smooth round pipe and shows changes in the dimensions of the vacuum chamber (diagnostics device, rf cavity, kickers …)
  → consider their equipments to be small resonant cavities.

: resonant frequency $\omega_R = \frac{c}{b}$ with radius of b

In travelling the cavity, beam wake fields are left behind as the beam exits the cavity : energy loss to the beam

In broad-band resonator model, Q=1, Impedance $Z_0^{\parallel}(\omega) = \frac{R_S}{1+iQ(\frac{\omega_R}{\omega}-\frac{\omega}{\omega_R})}$

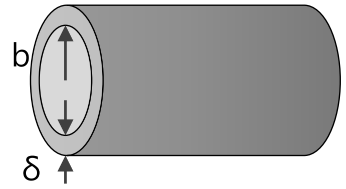
Collective effects in multi-particle beams

■ Impedance of resistive wall

If vacuum chamber walls have a finite conductivity, then energy will be dissipated by beam's induced current, and a wake field will be produced.

- wall conductivity : σ, pipe radius : b
- current flowing in a section of wall of length L passes through area A =2πbδ

  δ : skin depth(= $\sqrt{\frac{2}{\sigma\mu\omega}}$)



- Resistance per unit length

$$\frac{R_{wall}}{L} = \frac{1}{\sigma A} = \frac{1}{\sigma 2\pi b\delta} = \frac{1}{2\pi b}\sqrt{\frac{\mu\omega}{2\sigma}}$$

- Impedance

$$\frac{Z_0^{\parallel}(\omega)}{L} = \frac{1-i\,\mathrm{sgn}(\omega)}{2\pi b}\sqrt{\frac{\mu|\omega|}{2\sigma}}$$

wake function

$$\frac{W_0^{\parallel}(z)}{L} = -\frac{c}{4\pi b}\sqrt{\frac{c\mu}{\pi\sigma}}\frac{1}{\sqrt{|z|^3}}$$

Collective effects in multi-particle beams

② Transverse Impedance

■ Broad-band transverse impedance

$$Z_m^{\parallel}(\omega) = \frac{\omega}{c} Z_m^{\perp}(\omega)$$

- $Z_1^{\perp} = \frac{c}{\omega} Z_1^{\parallel}(\omega) = \frac{c}{\omega} \frac{R_s}{1+iQ(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R})}$
- wake function $W_1(z) = \frac{cR_s\omega_R}{Q\bar{\omega}} e^{\alpha z/c} sin \frac{\bar{\omega}z}{c}$ $\quad (\alpha = \frac{\omega_R}{2\theta}\ \bar{\omega} = \sqrt{\omega_R^2 - \alpha^2})$

■ Resistive wall transverse impedance

$$Z_1^{\perp}(\omega) \approx \frac{cZ_0^{\parallel}(\omega)}{\omega b^2}$$

- $Z_1^{\perp}(\omega) = \frac{1-isgn(\omega)}{2\pi b^3} \sqrt{\frac{\mu c^2}{2|\omega|\sigma}}$
- $W_1(z) = -\frac{c}{\pi b^3} \sqrt{\frac{c\mu}{\pi\sigma}} \frac{1}{\sqrt{|z|}}$

~~Homework 3~~

Collective effects in multi-particle beams

■ Resistive wake

- **longitudinal wake per unit length**

$$W_l(s) = -\frac{c}{4\pi^{3/2}b}\sqrt{\frac{Z_0}{\sigma s^3}} \qquad (s \gg s_0) \qquad : \text{long-range}$$

$$W_l(s) = \frac{Z_0 c\, 16}{4\pi b^2}\left(\frac{1}{3}e^{\frac{-s}{s_0}}cos\frac{\sqrt{3}s}{s_0} - \frac{\sqrt{2}}{\pi}\int_0^\infty \frac{x^2 e^{-\frac{x^2 s}{s_0}}}{x^6+8}dx\right) \qquad (s>0) : \text{short−range} \quad s_0 = (\frac{2b^2}{Z_0\sigma})^{1/3}$$

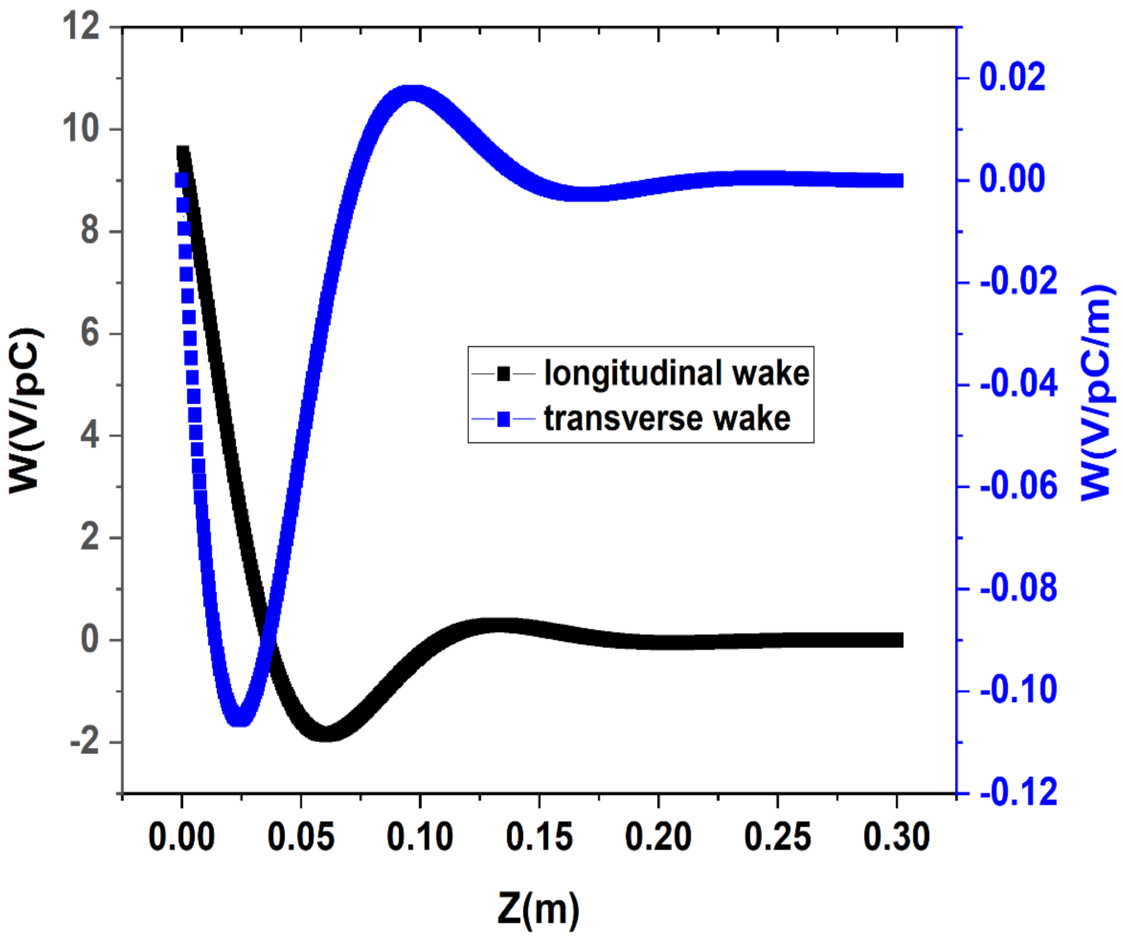
- **Transverse wake per unit length**

$$W_t(s) = \frac{c}{\pi^{3/2}b^3}\sqrt{\frac{Z_0}{\sigma s}} \qquad (s \gg s_0) : \text{long-range}$$

$$W_t(s) = \frac{8Z_0\, c\, s_0}{\pi b^4}\left\{\frac{1}{12}\left[-e^{-\frac{s}{s_0}}\cos\left(\frac{\sqrt{3}s}{s_0}\right) + \sqrt{3}e^{-\frac{s}{s_0}}\sin\left(\frac{\sqrt{3}s}{s_0}\right)\right] - \frac{2}{\sqrt{\pi}}\int_0^\infty \frac{-e^{-\frac{x^2 s}{s_0}}}{x^6+8}dx\right\} \quad (s>0) \ \text{short−rang}$$

Example : Broad-band Wake

: Z/n=0.1 Ohm   b=2cm $R_s = C_R/(2\pi b)Z/n$



longitudinal wake function

$$w'_m(z) = \begin{cases} 0 & if\ z > 0, \\ \alpha R_s & if\ z = 0, \\ 2\alpha R_s e^{\alpha z/c}\left(\cos\frac{\overline{\omega} z}{c} + \frac{\alpha}{\overline{\omega}}\sin\frac{\overline{\omega} z}{c}\right) & if\ z < 0, \end{cases}$$

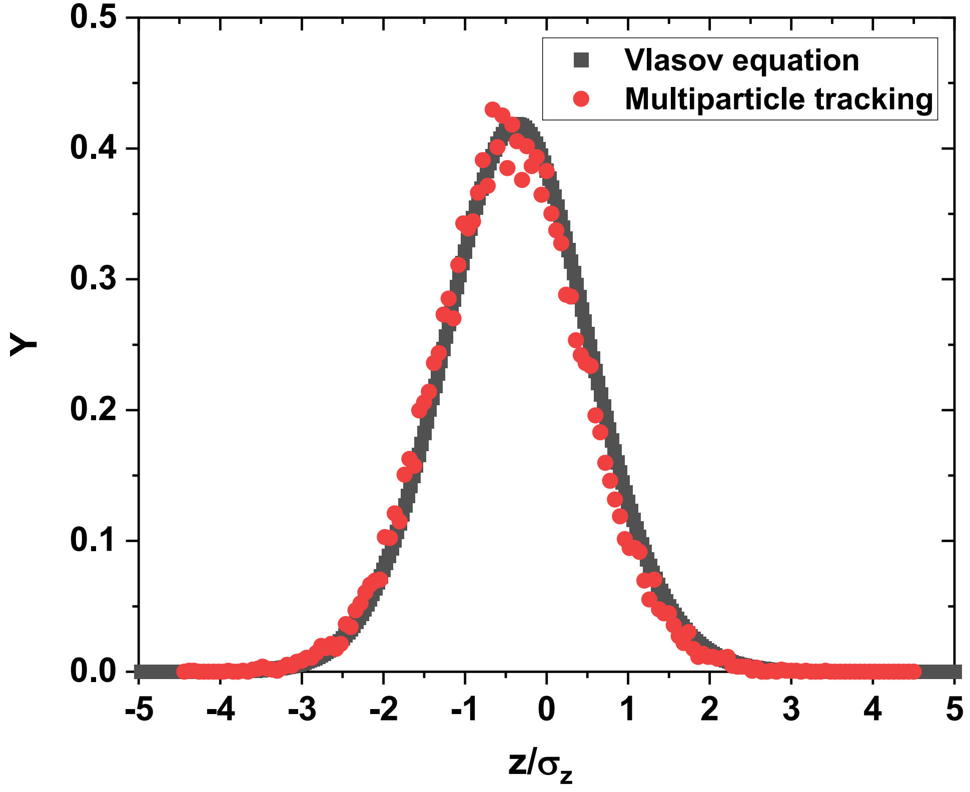
($\alpha = \omega_R/2\,Q$ and $\overline{\omega} = \sqrt{\omega_R^2 - \alpha^2}$)

Transverse wake function

$$w_m(z) = \frac{cR_S\omega_R}{Q\overline{\omega}} e^{\alpha z/c} \sin\frac{\overline{\omega} z}{c}$$

Example : Broad-band Wake

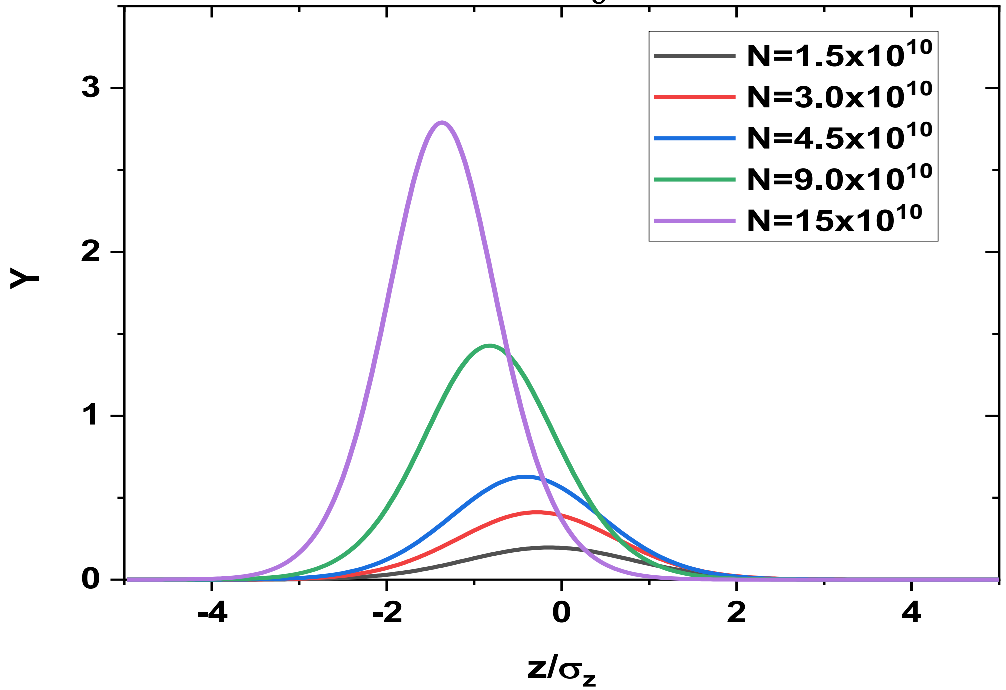
Potential well distortion for beam intensities



2.37 mA

4×10^{10}

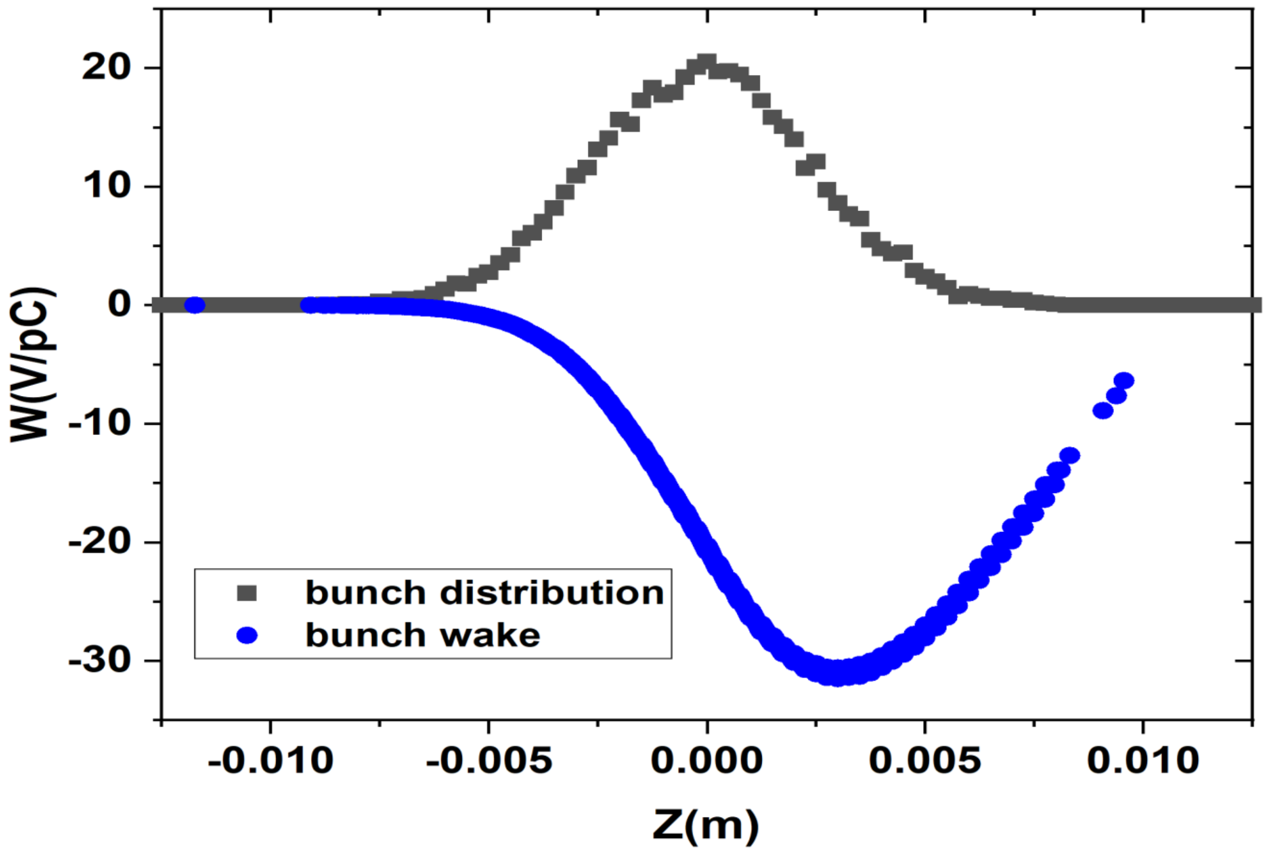
$$induced\ voltage\ V_0(t) = -\int_0^\infty W(t')I(t-t')dt'$$



beam current distribution below turbulent threshold

$$I(t) = K\exp\left[-\frac{t^2}{2\sigma_z^2} + \frac{\int_0^t V_0(t')dt'}{\dot{V}_{rf}\sigma_z^2}\right] \qquad \frac{\dot{I}}{I} = -\frac{t}{\sigma_z^2} + \frac{V_0(t)}{\dot{V}_{rf}\sigma_z^2}$$

Example : Broad-band Wake

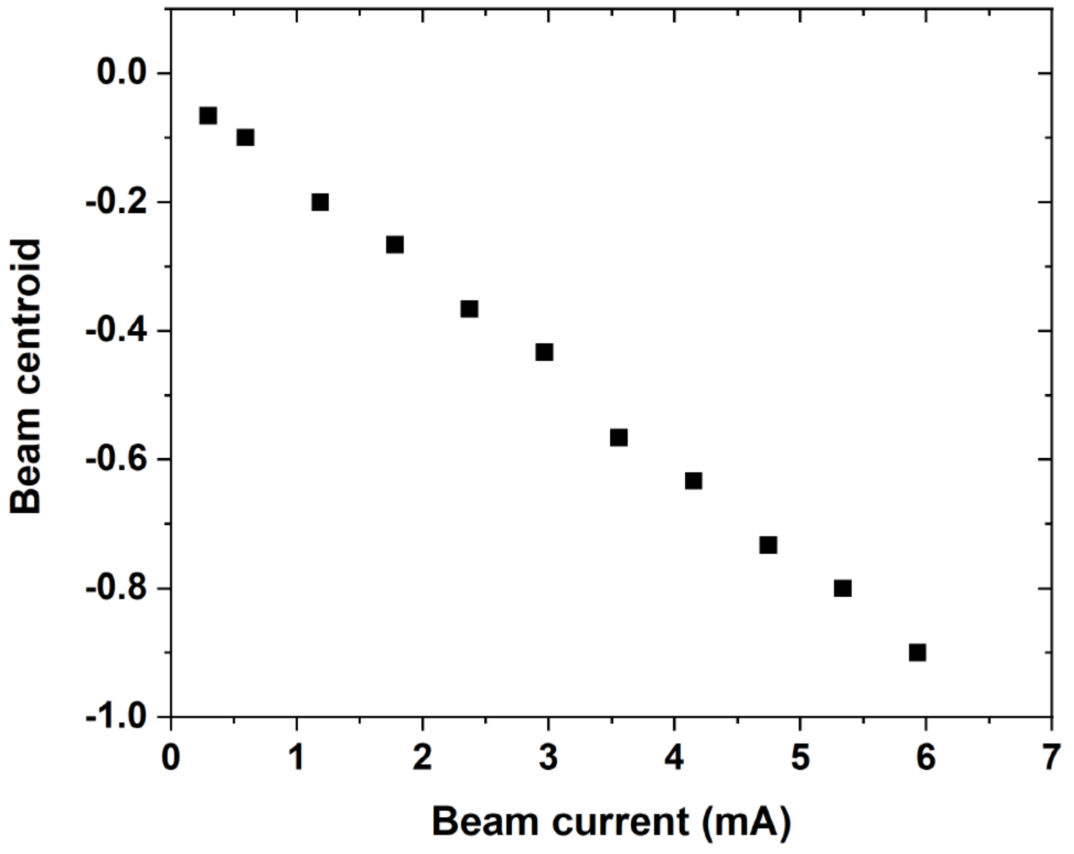


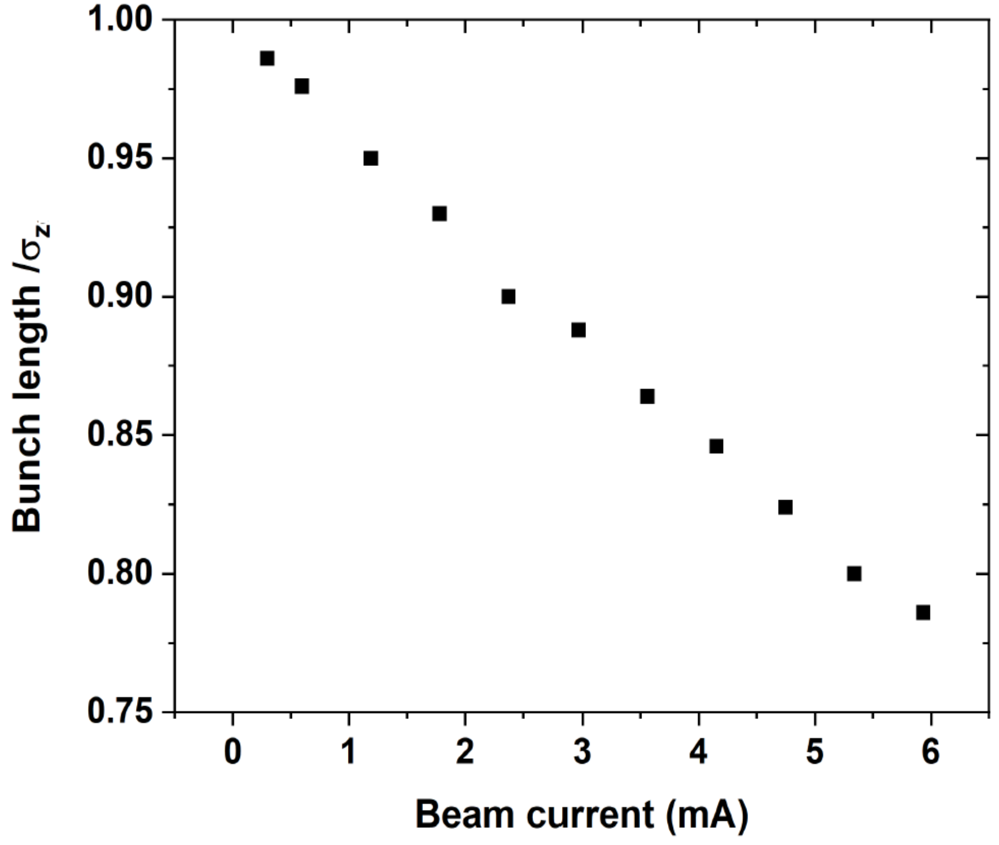
longitudinal bunch wake :
it shows capacitive aspect.

$$W_\lambda(s) = \int_0^\infty ds' w_l(s') \lambda(s - s').$$

Example : Broad-band Wake

Beam centroid and bunch length vs beam currents by PWD





Example : Broad-band Wake

Multi-particle beam trackings

$$\epsilon_i(n) = \epsilon_i(n-1) - \frac{2T_o}{\tau_d}\epsilon_i(n-1) + 2\sigma_{\epsilon o}\sqrt{\frac{T_o}{\tau_d}}r_{1i}(n) + V'_{rf}z_i(n-1) + W(z_i)(n),$$

$$z_i(n) = z_i(n-1) + \frac{\alpha c T_o}{E_o}\epsilon_i(n),$$

$$V'_{rf} = 2\pi\nu_{rf}\hat{V}_{rf}[1-(U_o/\hat{V}_{rf})^2]^{1/2}.$$

$$x_i(n) = M_{11}[\epsilon_i(n)]x_i(n-1) + M_{12}[\epsilon_i(n)]x'_i(n-1)(1-\frac{T_o}{\tau_x}) + \sqrt{\frac{2\epsilon_z\beta_x T_o}{\tau_x}}r_{1i}(n) + M_{12}[\epsilon_i(n)]\frac{W_i^x(n-1)}{E_o},$$

$$x'_i(n) = M_{21}[\epsilon_i(n)]x_i(n-1) + M_{22}[\epsilon_i(n)]x'_i(n-1)(1-\frac{T_o}{\tau_x}) + \sqrt{\frac{2\epsilon_z T_o}{\beta_x\tau_x}}r_{2i}(n) + M_{22}[\epsilon_i(n)]\frac{W_i^x(n-1)}{E_o},$$

$$W_o(z_i(n)) = -\frac{eN_b}{N_p}\sum_j^{z^i(n)<z^j(n)} N_j W'_o(z^i(n)-z^j(n)) - \frac{eN_b}{N_p}\sum_j^{z_i(n)<z_j(n)} W'_o(z_i(n)-z_j(n)) - \frac{eN_b}{N_p}W'_o(0),$$

$$W_o(0) = \frac{1}{2}\lim_{z\to 0^+} W'_o(z).$$

$$W_i^x(z_i(n)) = -\frac{eN_b}{N_p}\sum_i^{z^i(n)<z^j(n)} \bar{x}^j(n)N_j W_T(z^i(n)-z^j(n))$$

N_b: bunch population, N_p : number of macroparticles

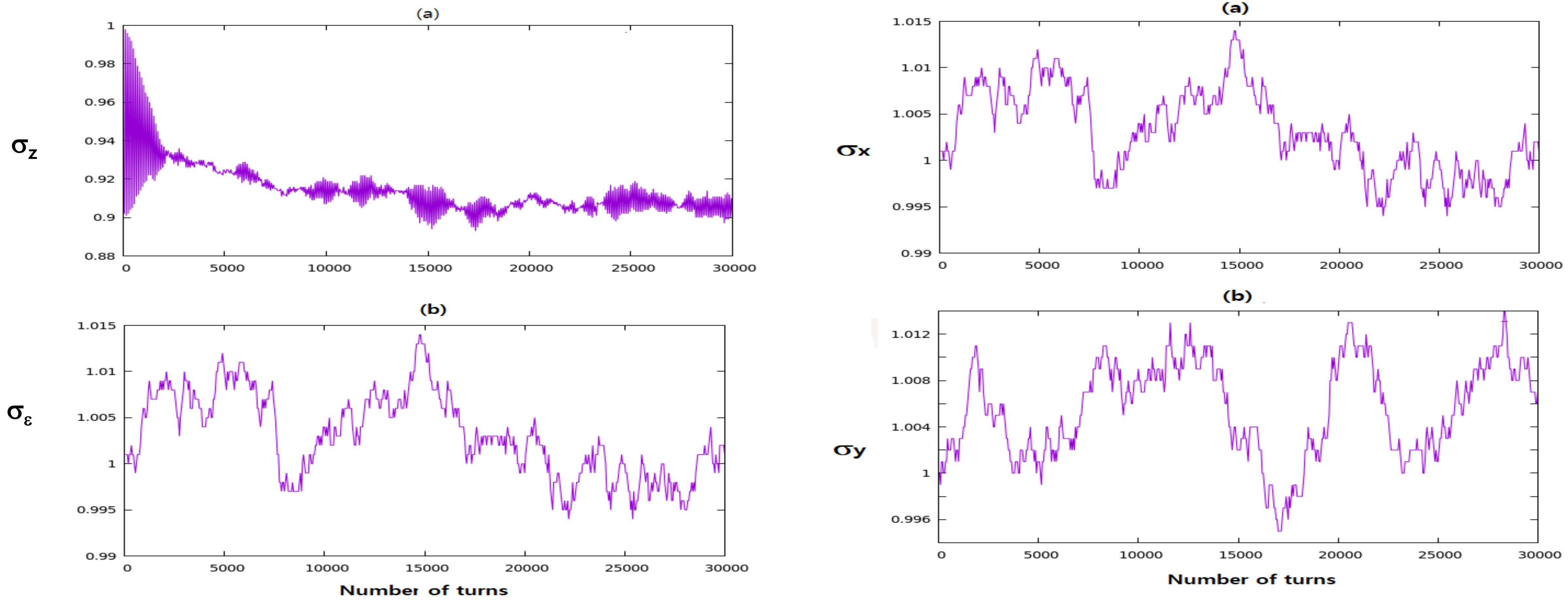
$\bar{x}^j$: average horizontal or vertical displacements of the particles in j^th bin

Example : Broad-band Wake

2.37 mA

4×10^{10}

Multiparticle beam tracking



Example : Resistive wall Wake

Short range longitudinal wake function

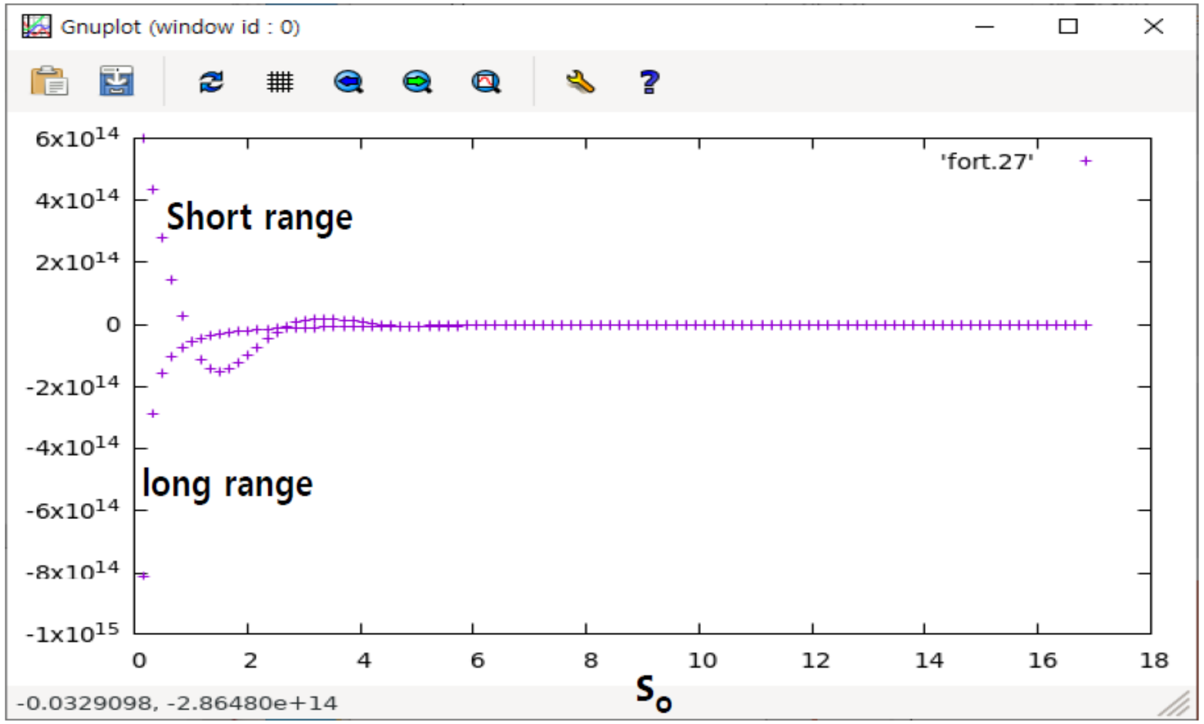
$$W(s) = \frac{16}{a^2}\left[\frac{e^{-s/s_0}}{3}\cos\frac{\sqrt{3}s}{s_0} - \frac{\sqrt{2}}{\pi}\int_0^\infty \frac{dx\, x^2 e^{-x^2 s/s_0}}{x^6+8}\right]$$

Long range longitudinal wake function

$$W(s) = -\sqrt{c/\sigma}/(2\pi a s^{3/2}).$$

Characteristic distance

$s_o = \left(\frac{2a^2}{Z_o\sigma}\right)^{1/3}$ = 18 μm



Example : Resistive wall Wake

Multiparticle beam trackings

$$\epsilon_i(n) = \epsilon_i(n-1) - \frac{2T_o}{\tau_d}\epsilon_i(n-1) + 2\sigma_{\epsilon o}$$
$$\sqrt{\frac{T_o}{\tau_d}} r_{1i}(n) + V'_{rf} z_i(n-1) + W(z_i)(n),$$

$$z_i(n) = z_i(n-1) + \frac{\alpha c T_o}{E_o}\epsilon_i(n),$$

$$V'_{rf} = 2\pi\nu_{rf}\hat{V}_{rf}[1-(U_o/\hat{V}_{rf})^2]^{1/2}.$$

$$x_i(n) = M_{11}[\epsilon_i(n)]x_i(n-1) + M_{12}[\epsilon_i(n)]x'_i(n-1)(1-\frac{T_o}{\tau_x})$$
$$+\sqrt{\frac{2\epsilon_z\beta_x T_o}{\tau_x}} r_{1i}(n) + M_{12}[\epsilon_i(n)]\frac{W_i^x(n-1)}{E_o},$$

$$x'_i(n) = M_{21}[\epsilon_i(n)]x_i(n-1) + M_{22}[\epsilon_i(n)]x'_i(n-1)(1-\frac{T_o}{\tau_x})$$
$$+\sqrt{\frac{2\epsilon_z T_o}{\beta_x\tau_x}} r_{2i}(n) + M_{22}[\epsilon_i(n)]\frac{W_i^x(n-1)}{E_o},$$

$$M(\epsilon) = \begin{bmatrix} cos2\pi Q_x(\epsilon) & \beta_x sin2\pi Q_x(\epsilon) \\ -1/\beta_x sin2\pi Q_x(\epsilon) & cos2\pi Q_x(\epsilon) \end{bmatrix},$$

$Qx(\varepsilon)=Qx(1+\varepsilon\xi/Eo)$

Example : Resistive wall Wake

Wake between different bin (with N_j macroparticles) and one macroparticle in a bin

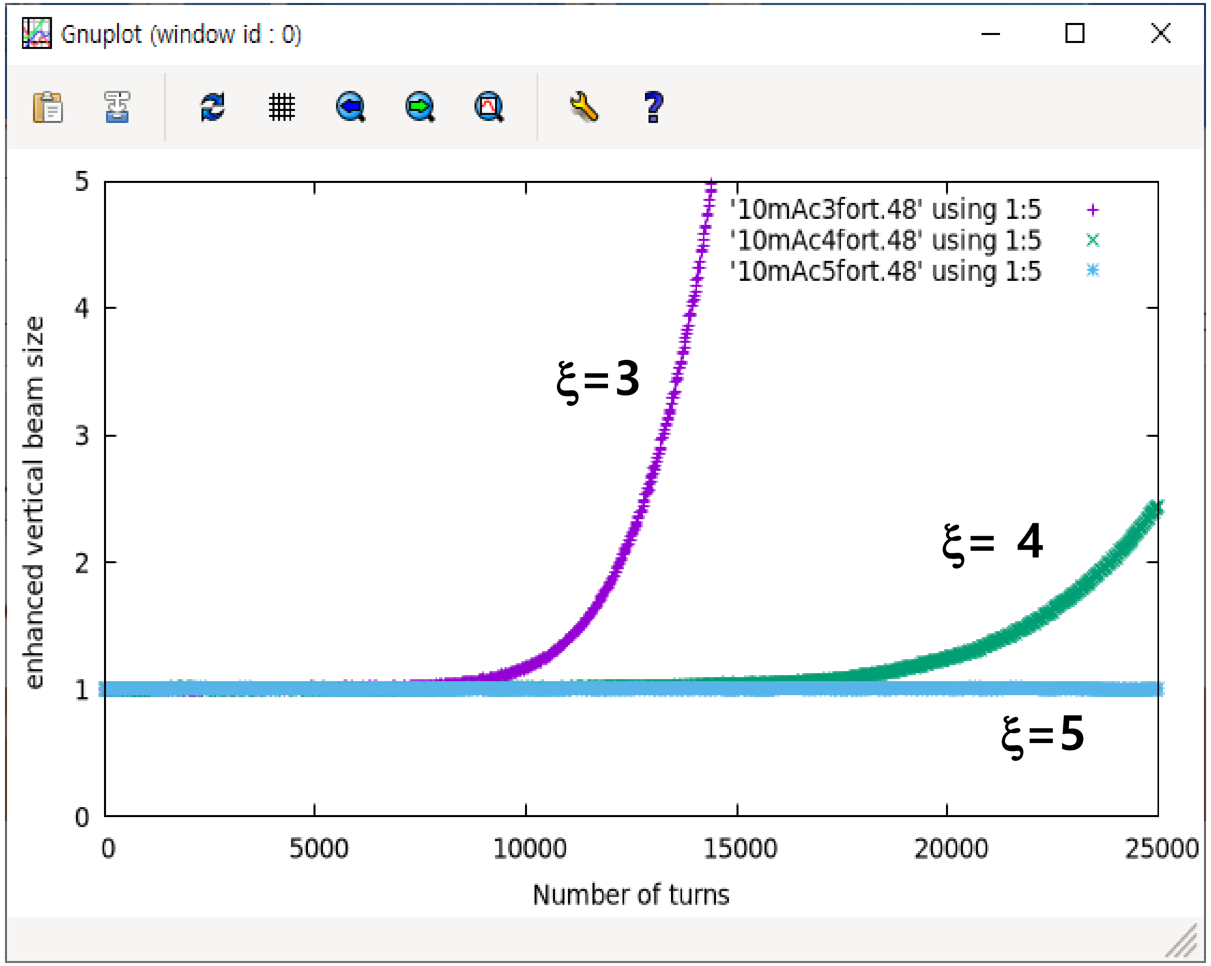
$$W_o\big(z_i(n)\big) = -\frac{eN_b}{N_p}\sum_{j}^{z^i(n)<z^j(n)} N_j W_o'(z^i\,(n)-z^j(n))$$

$$W_i^x\big(z_i(n)\big) = -\frac{eN_b}{N_p}\sum_{j}^{z^i(n)<z^j(n)} \bar{x}^j(n)N_j\,W_T(z^i\,(n)-z^j(n))$$

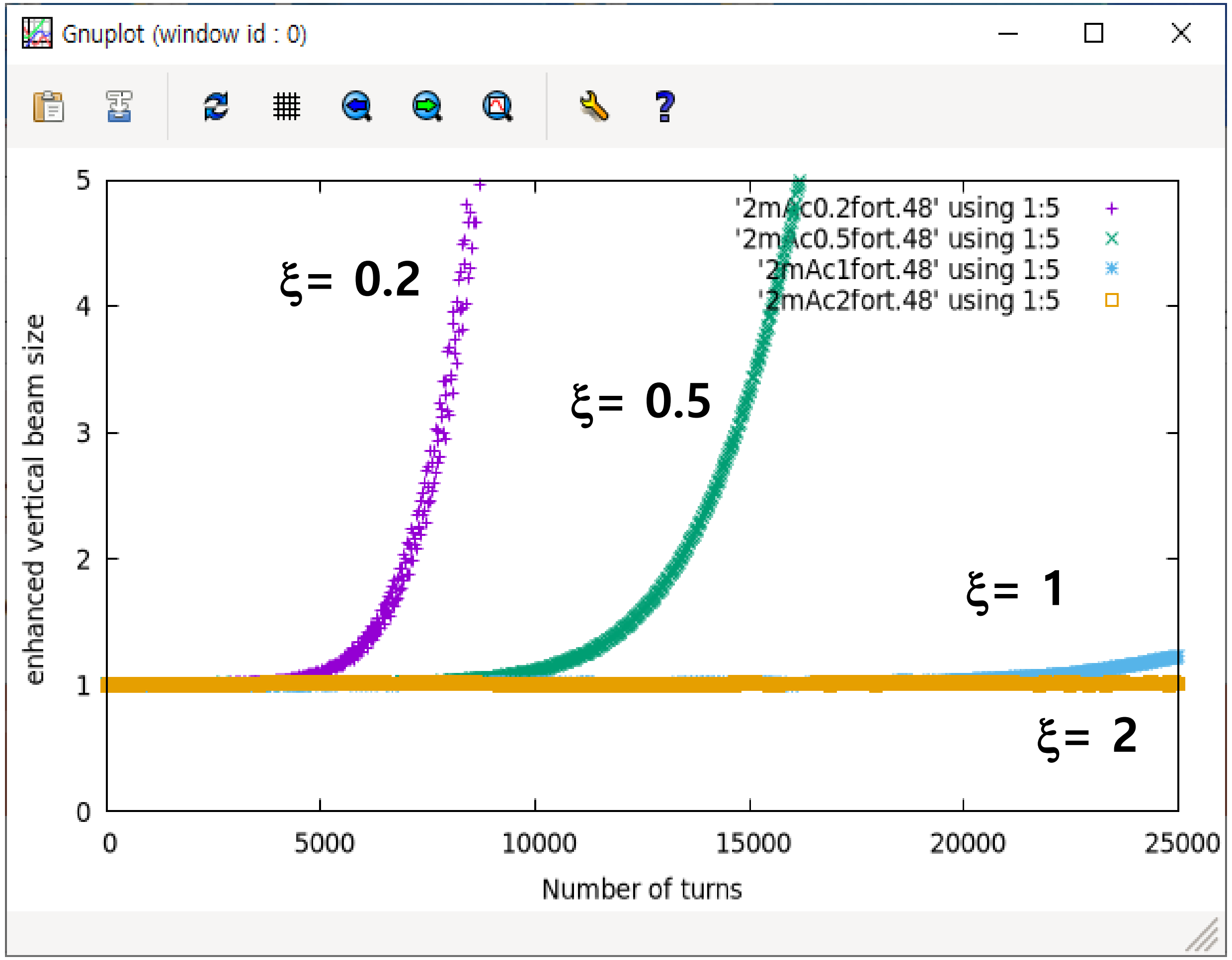
N_b: bunch population, N_p : number of macroparticles
$\bar{x}^j$: average horizontal or vertical displacements of the particles in j^{th} bin

Example : Resistive wall Wake

Transverse instability at chromaticities



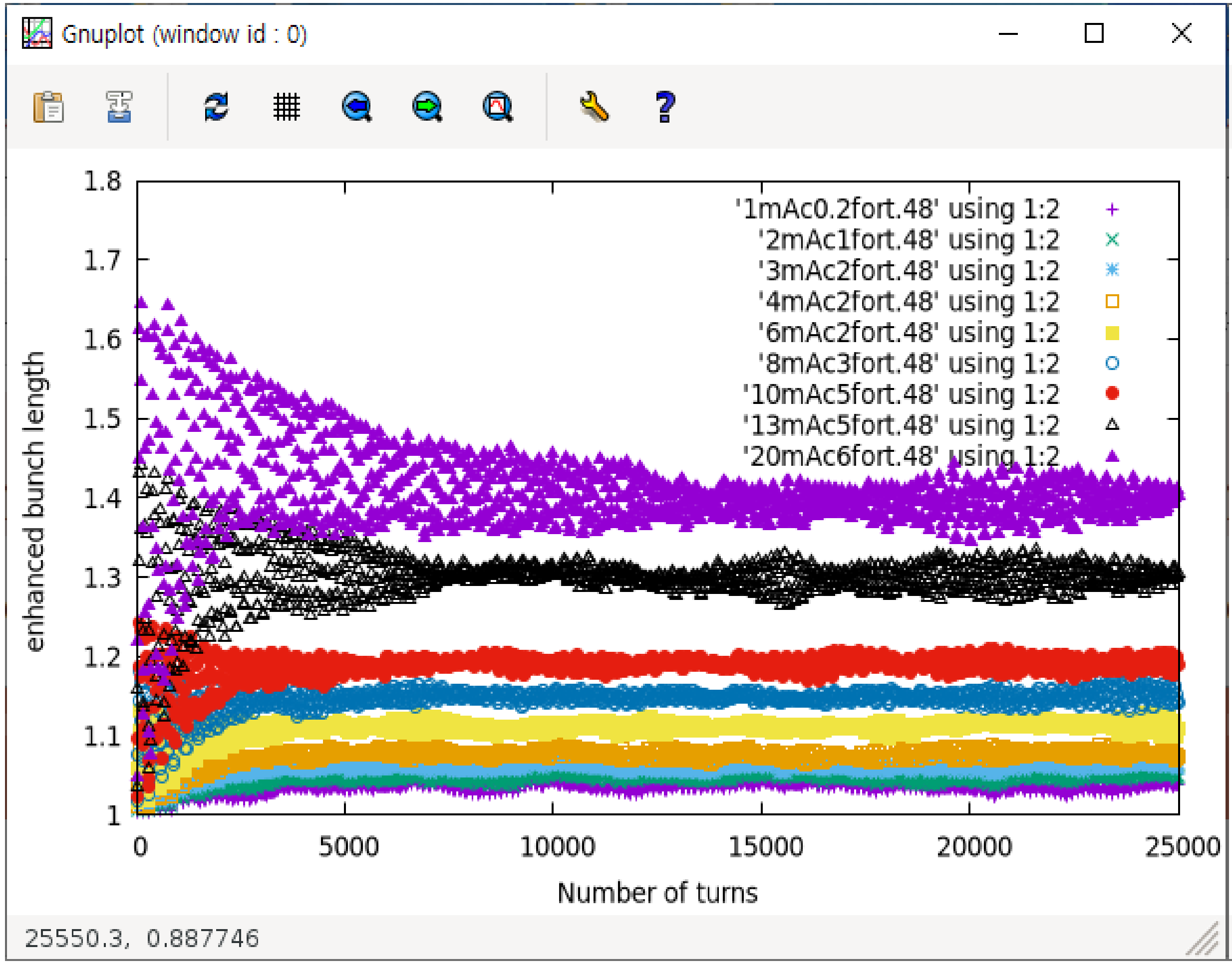
10 mA

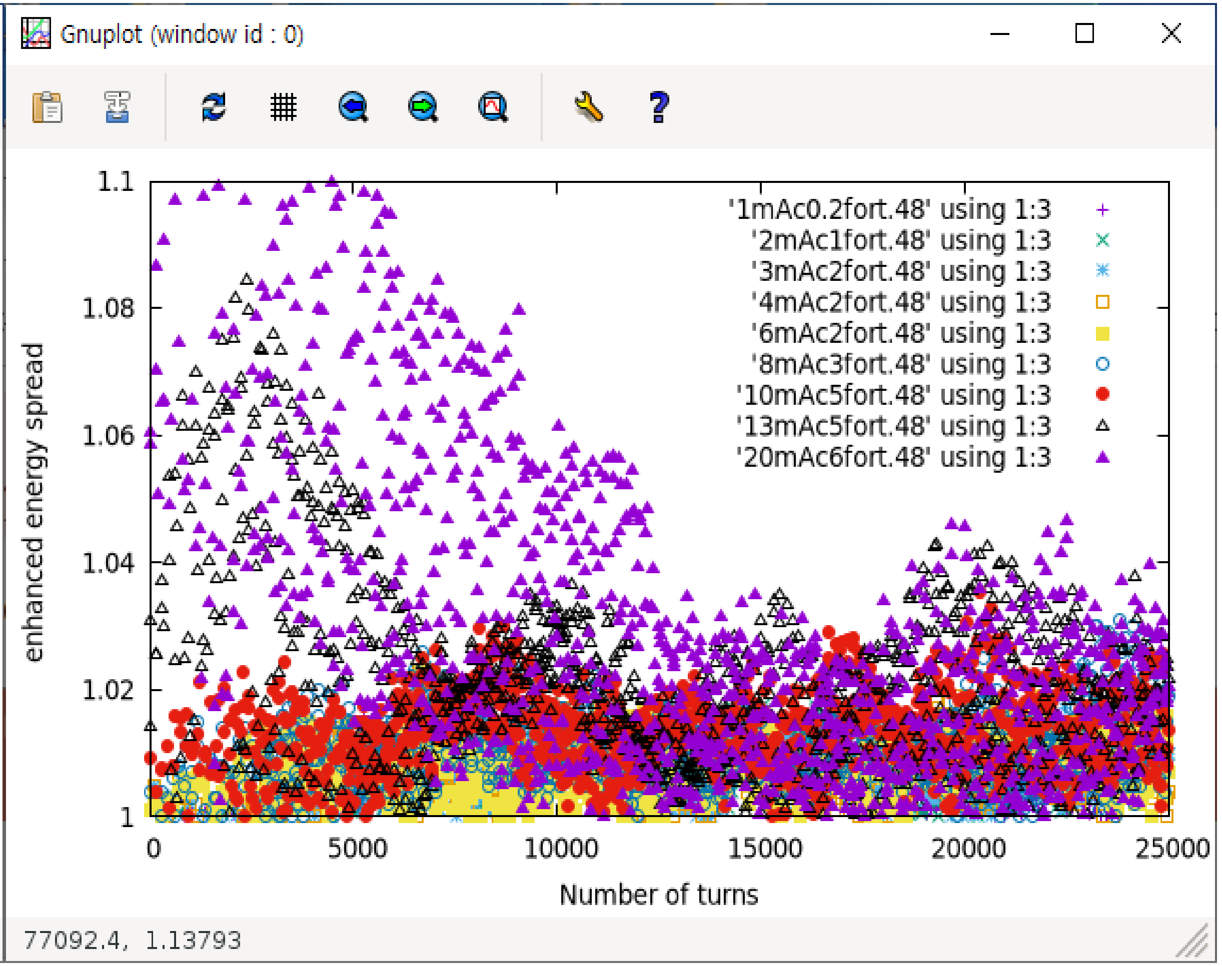


2 mA

Example : Resistive wall Wake

Bunch length and energy spread vs bunch currents





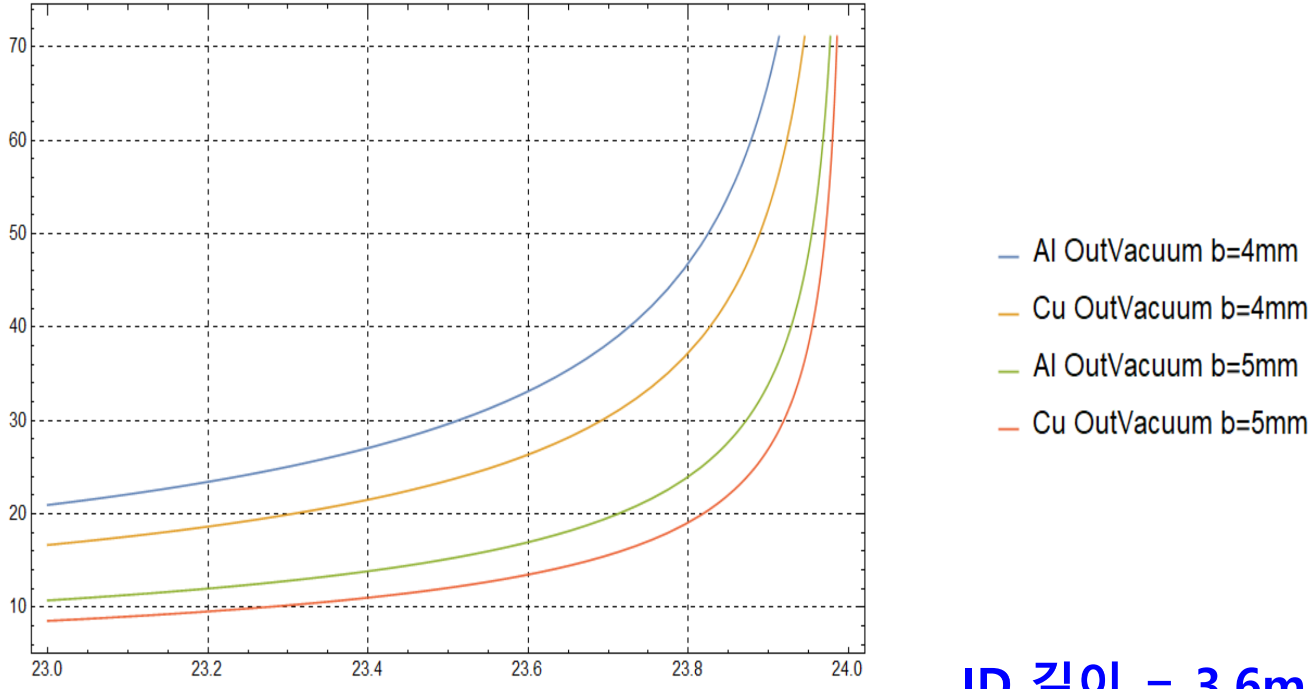
Resistive wall Wake

Transverse coupled-bunch instability

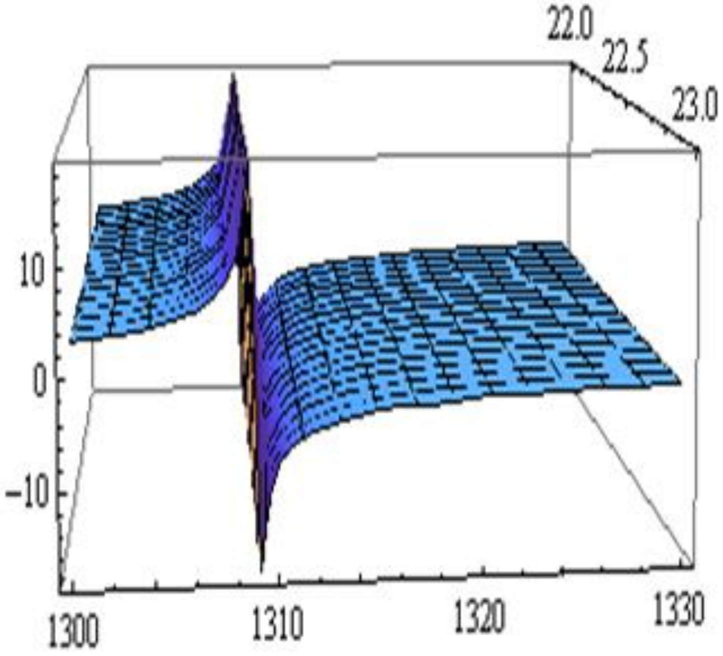
$$Z_t(\omega) = \frac{Z_0 L}{2\pi b^3}\sqrt{\frac{2}{\sigma_c \mu_0 |\omega|}}(sgn(\omega)-i)$$

$$[Z_t]^{\mu,a}_{eff} = \sum_{p=-\infty}^{+\infty} \left(\frac{\omega_p^t - a\omega_s - \omega_\xi}{\omega_0}\right)^{2a} \exp -\left(\frac{\omega_p^t - a\omega_s - \omega_\xi}{\omega_0}\right)^2 (c\sigma_t/R)^2 Z_t(\omega_p^t)$$

growth rate (Hz)



vertical tune



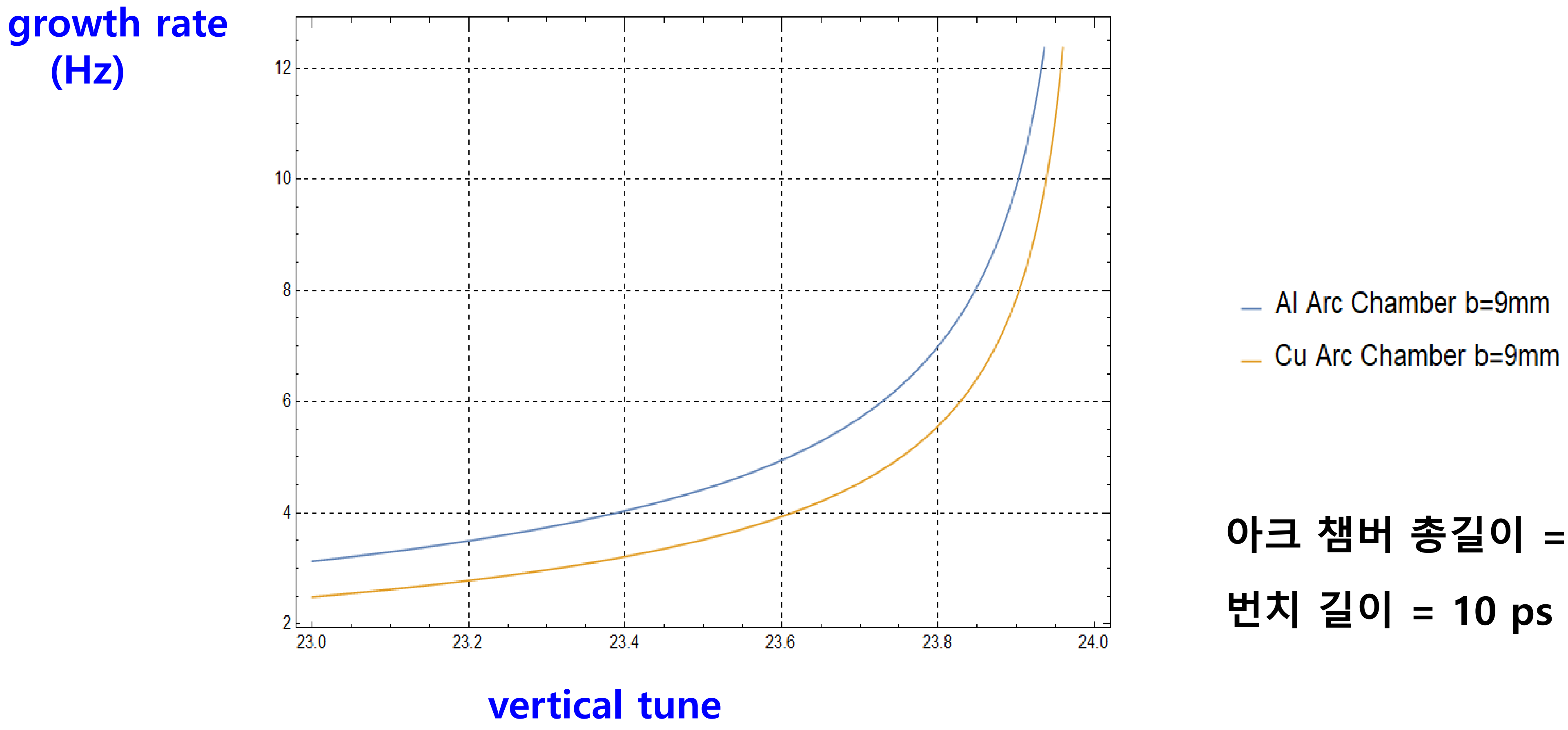
ID 길이 = 3.6m

번치 길이= 10 ps rms

Resistive wall Wake

Transverse coupled-bunch instability

Al and Cu, 400 mA



아크 챔버 총길이 = 5.6m

번치 길이 = 10 ps rms