Homework #1

2025 Accelerator Summer School

Due Aug. 6 (Wed.), 9:30 AM, 2025

1. Please read carefully Section [1.2.5 Diagnostics] of the textbook, and answer the following questions.
   (a) Why and how do we suppress the secondary electron emission in **Faraday cup**?
   (b) Within a storage ring, charge measurements are routinely made using either a **DC current transformer (DCCT)**, or a **wall-current monitor**. Please explain the principle of each device briefly.
   (c) What are the differences between **'button'-type beam position monitor (BPM)** and **'strip lines'-type BPM**?
   (d) What are the pros. and cons. of **yttrium aluminium garnet (YAG)** screen and **transition radiation** screen, respectively?
   (e) What is the common method used for **bunch length measurements** in an electron storage ring?
   (f) Please explain the **beam-based alignment (BBA)** brielfy.

2. The Hamiltonian in curved system with the longitudinal coordinate s (following the reference orbit) being an independent variable is given by

$$H_2 = x'\pi_x + y'\pi_y + t'\pi_t - L$$

where $\pi_{x,y}$ are the transverse canonical momenta, π_t is the longitudinal canonical momentum which is the negative of the old Hamiltonian (i.e. Hamiltonian with the time t being an independent variable). Then using the Lagrangian L of a particle moving in EM field and the corresponding canonical momenta with the independent variable s, show that this Hamiltonian leads to

$$H_2 = -(1+hx)\sqrt{\left(\frac{\pi_t + q\Phi}{c}\right)^2 - (\pi_x - qA_x)^2 - (\pi_y - qA_y)^2 - m^2c^2} - q(1+hx)A_s$$

where $h(s) = \frac{1}{\rho(s)}$, $A_s = \mathbf{A}\cdot\mathbf{e}_s$, and Φ is the electric potential.

3. The two-dimensional field expressed in complex notation (US convention) is given by

$$B(z) = B_y + iB_x = \sum_{n=0}^{\infty} C_n(x+iy)^n = \sum_{n=0}^{\infty}(B_n + iA_n)(x+iy)^n$$

where C_n is a complex constant, $C_n = B_n + iA_n$ with $B_n,\ A_n$ being real. Show that this form of the field satisfies the Maxwell's equations in source-free region

$$\nabla\cdot\mathbf{B} = 0 \qquad \text{and} \qquad \nabla\times\mathbf{B} = 0$$

4. (a) Derive the 6×6 transfer matrix for one cell of a FODO structure (without bends) given in Eq. (1) of Lecture slide 5, Linear Dynamics. Assume that the cell begins in the middle of the focusing quadrupole.

   (b) Find the expressions for β_x and β_y in the beginning (and end) of the cell.

5. Based on the textbook and lecture slides on Synchrotron Radiaion, we would like to prove the following expression step by step.

$$\left.\frac{dU}{dE}\right|_{E=E_0} = j_z \frac{U_0}{E_0}. \tag{1}$$

   (a) By expressing $P_\gamma(E) = \frac{C_\gamma}{2\pi} q^2 c^3 E^2 B^2$ and $B = B_0 + x\frac{\partial B_y}{\partial x}$, show that

$$\left.\frac{dP_\gamma(E)}{dE}\right|_{E=E_0} = 2\frac{P_\gamma(E_0)}{E_0} + 2\frac{P_\gamma(E_0)}{E_0}\left(\frac{\partial B_y}{\partial x}\right)\frac{\eta_x}{B_0}.$$

   (b) Now evaluate the following differentiation:

$$\left.\frac{dU}{dE}\right|_{E=E_0} = \frac{d}{dE}\left[\oint P_\gamma(E)\left(1 + \frac{\eta_x}{\rho}\delta\right)\frac{ds}{c}\right]_{E=E_0}.$$

   (c) Using the result of (b) and the definitions of I_2, I_4, j_x, and U_0, prove Eq. (1) above.