THEORETICAL AND EXPERIMENTAL APPROACHES TO ACCELERATOR RF SYSTEM*

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Abstract

This work introduces the basic theory and experimental techniques related to RF systems used in accelerators. Key concepts such as cavity resonance and impedance matching are briefly reviewed. Experiments were conducted using a vector network analyzer (VNA) to measure resonant frequency, time-domain reflectometry (TDR) to analyze impedance mismatches, and a stripline BPM to identify beam position. The results demonstrate the practical aspects of RF diagnostics in accelerator environments.

INTRODUCTION

RF systems are widely used in modern accelerators, including synchrotrons, storage rings, and linear accelerators. Unlike DC systems, RF systems can accelerate particles to high energies without discharge issues. The main components include resonant cavities and transmission lines, where impedance matching and frequency tuning are essential for efficient operation.

Understanding how these systems work requires both theoretical knowledge and hands-on experience. In this study, we review basic RF principles such as cavity behavior and the use of Smith charts for impedance analysis. We also perform several experiments to measure key characteristics of RF components using standard diagnostic tools like VNA and TDR. Beam position is verified using a stripline BPM.

In the following sections, we describe the basic concepts of RF systems, focusing on the resonant properties of cavities, as well as impedance matching and signal reflection in transmission lines.

BASIC THEORY OF RF SYSTEM

RF systems are designed to generate and control time-varying electromagnetic fields that can accelerate charged particles. These systems typically consist of RF sources, transmission lines, and resonant cavities. Before exploring the detailed behavior of cavities and impedance characteristics, we briefly review the general structure and purpose of RF systems used in particle accelerators.

Cavity

An RF cavity is a resonant structure that stores standing electromagnetic waves. It can be modeled as a bounded region of conducting walls in which Maxwell's equations are solved with specific boundary conditions. The solutions result in discrete resonant modes, primarily classified as transverse electric (TE) and transverse magnetic (TM) modes.

In cylindrical cavities, the resonant modes are labeled as $\mathrm{TE}_{mn\ell}$ and $\mathrm{TM}_{mn\ell}$, where m, n, and ℓ correspond to the number of variations in the azimuthal, radial, and longitudinal directions, respectively.

- **TE modes** (Transverse Electric): $E_z = 0$, i.e., no longitudinal electric field.
- **TM modes** (Transverse Magnetic): $B_z = 0$, supports a non-zero longitudinal electric field $E_z \neq 0$.

Since particle acceleration requires an electric field component along the beam direction, TM modes are suitable for acceleration. The most commonly used mode is the TM_{010} mode, which provides a uniform on-axis accelerating field. Its resonant frequency in a cylindrical pillbox cavity is given by:

$$\omega_{010} = \frac{2.405c}{a}, \tag{1}$$

where c is the speed of light and a is the cavity radius. The constant 2.405 is the first zero of the Bessel function J_0.

Two key parameters commonly used to evaluate the performance of an RF cavity are the **quality factor** and the **shunt impedance**. These parameters quantify how effectively the cavity stores and utilizes RF energy for particle acceleration.

The quality factor Q quantifies the energy loss per cycle relative to the energy stored in the cavity. It is defined as:

$$Q = \frac{\omega U}{P_{\mathrm{loss}}}, \tag{2}$$

where ω is the angular frequency, U is the stored electromagnetic energy, and P_{loss} is the power loss to the cavity walls per RF cycle. A higher Q indicates lower losses and a narrower bandwidth.

The shunt impedance R_s represents the efficiency with which the cavity converts RF power into accelerating voltage:

$$R_s = \frac{V_{\mathrm{acc}}^2}{2P_{\mathrm{loss}}}, \tag{3}$$

where V_{acc} is the accelerating voltage and P_{loss} is the RF power dissipated in the walls. In many cases, the effective shunt impedance per unit length R_s/L is used to compare different cavity designs. A higher shunt impedance means more effective acceleration for a given amount of power loss.

An RF cavity can be modeled as a parallel RLC resonant circuit, where the cavity behaves like an inductor and capacitor that store magnetic and electric energy, respectively, while resistive losses in the cavity walls are modeled by a resistor.

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In this model, the impedance of the cavity as a function of angular frequency ω is given by:

$$Z(\omega) = \frac{R}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}, \tag{4}$$

where R is the shunt impedance, ω_0 is the resonant frequency, Q is the quality factor, j is $-\sqrt{-1}$.

At resonance ($\omega = \omega_0$), the reactive components cancel out and the impedance becomes purely real and equal to R. Away from resonance, the impedance acquires a reactive component, and its magnitude decreases.

Figure 1 shows a typical frequency response of the cavity impedance based on this RLC model. The peak of the curve corresponds to the resonant frequency, and the bandwidth of the peak is determined by the quality factor Q.

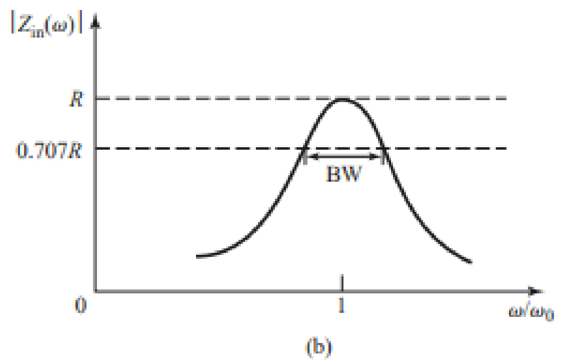

Figure 1: Impedance of a cavity modeled as a parallel RLC circuit [1].

Impedance Matching and Smith Chart

In an RF transmission system, power is transmitted by an incident wave along a transmission line. When the wave encounters a load whose impedance differs from the characteristic impedance of the line, a portion of the wave is reflected back toward the source. This occurs commonly at the interface between a transmission line and an RF cavity if they are not impedance-matched.

The reflection coefficient Γ describes the ratio of the reflected wave to the incident wave and is given by:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}, \tag{5}$$

where Z_L is the load impedance (e.g., cavity impedance) and Z_0 is the characteristic impedance of the transmission line. When $Z_L = Z_0$, $\Gamma = 0$, indicating perfect matching and no reflection. Conversely, large impedance mismatch leads to significant reflection ($|\Gamma| \to 1$) and inefficient power transfer.

Smith Chart Visualization

The Smith chart is a graphical tool used to represent complex impedance and the reflection coefficient on a normalized polar coordinate system. For any load impedance Z_L, the normalized form is defined as:

$$z_L = \frac{Z_L}{Z_0} = r_L + jx_L = \frac{(1+\Gamma_r) - \Gamma_i}{(1-\Gamma_r) - \Gamma_i}, \tag{6}$$

where r_L and x_L are the normalized resistance and reactance and $\Gamma = \Gamma_r + j\Gamma_i$. This normalized impedance corresponds to a unique point on the Smith chart.

Using the Smith chart, one can visualize the impedance trajectory over frequency, identify the degree of mismatch and resonant points.

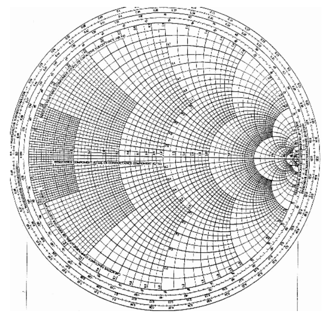

Figure 2: Example of smith chart showing the frequency-dependent impedance of an RF cavity [2].

As shown in Fig 2, the Smith chart contains two main sets of circles: constant-resistance circles and constant-reactance circles.

The resistance circles are centered along the horizontal axis and represent constant real parts of the normalized impedance. The reactance circles appear as arcs above and below the axis, indicating constant imaginary parts.

By observing the impedance trajectory on the Smith chart, one can identify how the cavity impedance varies with frequency and locate the resonance point where the impedance becomes purely real.

EXPERIMENT OF RF SYSTEM

VNA

The experimental setup for characterizing the RF cavity using a VNA is illustrated in Fig 3. A one-port reflection measurement was performed to evaluate the resonant properties of the cavity over a wide frequency range.

The measurement procedure consists of the following steps:

1. **Calibration:** A standard one-port calibration was performed using a commercial calibration kit. The kit includes open, short, and load standards, which allow for accurate removal of systematic errors such as directivity, source match, and reflection tracking. The calibration reference plane was set at the end of the coaxial cable to ensure precise measurement of the cavity input.

2. **Cavity Connection:** After calibration, the coaxial cable was connected to a pillbox-type cavity designed

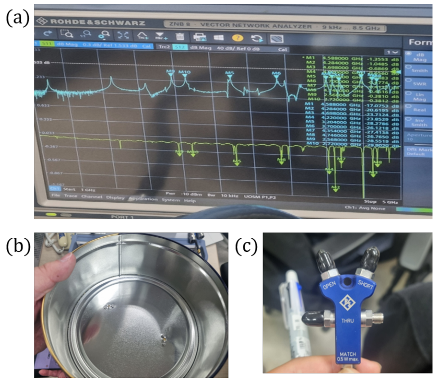

Figure 3: Devices of the VNA experiments: (a) vector network analyzer, (b) pillbox cavity, (c) calibration kit.

to support TM modes. The measurement covered a frequency range from several hundred MHz up to a few GHz to identify all possible resonant modes within this range.

3. **S-Parameter Measurement:** The VNA measured the reflection coefficient S_{11} over the defined frequency range. Resonances appear as dips in the magnitude of $|S_{11}|$, corresponding to frequencies where the cavity absorbs energy efficiently.

4. **Resonance Analysis:** From the S_{11} data, the resonant frequencies were identified, and the loaded quality factor Q_L was extracted by analyzing the 3-dB bandwidth around each resonance. The results provide insight into the energy storage and loss characteristics of the cavity.

TDR

Time-Domain Reflectometry (TDR) is a diagnostic technique used to detect impedance discontinuities along a transmission line. By sending a fast-rising step signal and analyzing the reflected waveform in the time domain, one can locate mismatches such as open circuits, short circuits.

A TDR was conducted to check impedance matching at the connection point between the transmission line and a stripline beam position monitor (BPM).

Instead of connecting the cavity, the transmission line was directly connected to the BPM (see Fig 4). A step signal was injected, and the reflected waveform was observed in the time domain. The result revealed a clear impedance mismatch at the BPM input, indicating a discontinuity at the connector interface.

BPM measurement

A experiment was conducted to verify the signal response of a stripline beam position monitor (BPM). Instead of using an actual particle beam, a signal generator was used to simulate the beam-induced signal.

As shown in Fig 5, a 500 MHz signal was sent through a wire placed along the beam axis inside the BPM structure. The signal was picked up by four electrodes located

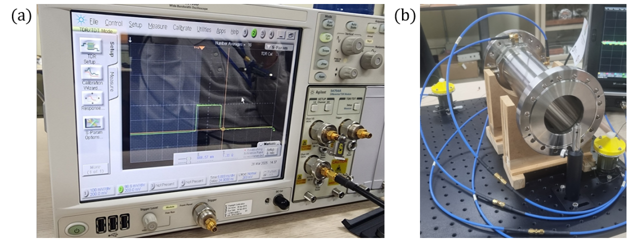

Figure 4: TDR measurement setup with BPM: (a) TDR devices, (b) stripline BPM.

symmetrically around the structure. Each electrode output was connected to a digital oscilloscope, which recorded the time-domain waveforms from all four ports.

The oscilloscope data were then transferred to a computer for post-processing and analysis. This setup allowed basic validation of the BPM's ability to detect and resolve transverse beam position based on signal asymmetry between the ports.

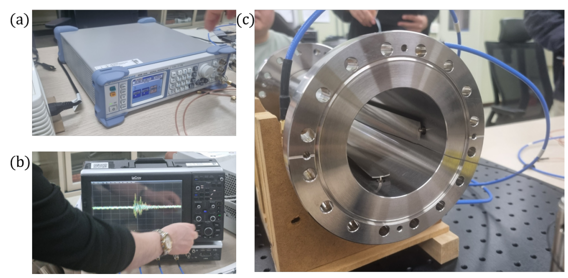

Figure 5: Experimental setup for BPM signal measurement: (a) signal generator, (b) oscilloscope, (C) wire inside BPM.

CONCLUSION

This work provided a basic overview of RF systems used in accelerators, focusing on the theoretical principles of cavity resonance and impedance matching. Key measurements were performed using a VNA and TDR to examine cavity response and connection mismatches. In addition, a simple BPM signal test was conducted to observe electrode responses. Through these experiments, we gained practical insight into the behavior and diagnostics of RF systems.

REFERENCES

[1] D. M. Pozar, *Microwave engineering /*, 4th ed. New Delhi :: Wiley,, c2012., includes index.

[2] F. Caspers, "Rf engineering basic concepts: the smith chart," 01 2012.