ACCELERATOR MAGNET

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ANALYTIC MODELING OF 2D MULTIPOLE MAGNETS

To design a magnet, we utilize the concept of following complex potential.

$$F = \vec{A} + iV \tag{1}$$

By inserting the complex potential into Maxwell's equations and applying the Cauchy-Riemann conditions, we can expand F as a multipole series and derivce the equipotential lines. The magnetic field for a general 2n-pole magnet can be expressed as:

$$B_{nx} = -nC_n|z|^{n-1}sin(n-1)\theta \tag{2}$$

$$B_{ny} = -nC_n|z|^{n-1}cos(n-1)\theta \tag{3}$$

where the coefficients C_n are given by:

$$C_n = \frac{-1}{n\pi r_0^{n-1}} \oint B_x(\theta)|_{r=r_0} sin(n-1)\theta d\theta \tag{4}$$

$$C_n = \frac{-1}{n\pi r_0^{n-1}} \oint B_y(\theta)|_{r=r_0} cos(n-1)\theta d\theta \tag{5}$$

Using formula (2) and (3), skewed magnets can also be represented. We can evaluate the quality of the magnetic field using the computational method. By sampling the magnetic field B at fixed radius r over various angles. Smaller deviations indicate that there are fewer high-order components in the multipole field. Additionally, by performing a Fourier transform of the magnetic field, we can obtain the amplitude of each pole component.

MEASUREMENT OF THE REAL MAGNETIC FIELD

When using magnets, such as electromagnets or based on ferromagnetic materials, fabrication errors are inevitable during the manufacturing process. Additionally, because of the finite size of the magnets, fringe fields can also appear. Thus, it is essential to measure the magnetic field in order to evaluate how precisely the magnet has been fabricated. In the following, I will describe the methods for measuring the magnetic field. Before measuring the magnetic field, several calibrations should be performed. After magnet installation, the magnet must be allowed to stabilize for 24 hours before any measurement is performed, to ensure magnetic field stability and measurement accuracy. In addition, background magnetic fields, such as the Earth's magnetic field, must be calibrated inside the laboratory.

A. Rotating Coil Method

When a coil rotates inside a magnetic field, a time-varying magnetic flux is induced through the coil. By Faraday's law of electromagnetic induction, this change in flux generates an induced voltage. As the coil rotates at a constant angular speed, the induced voltage becomes a periodic function that Fourier components correspond to the magnetic multipole field components. For a uniform dipole field, the total flux linked to the coil over one rotation is proportional to the integrated magnetic field. So, with known coil geometry and rotation parameters, we can compute the integrated field. Also, the focusing strength of a quadrupole field is characterized by its integrated gradient.

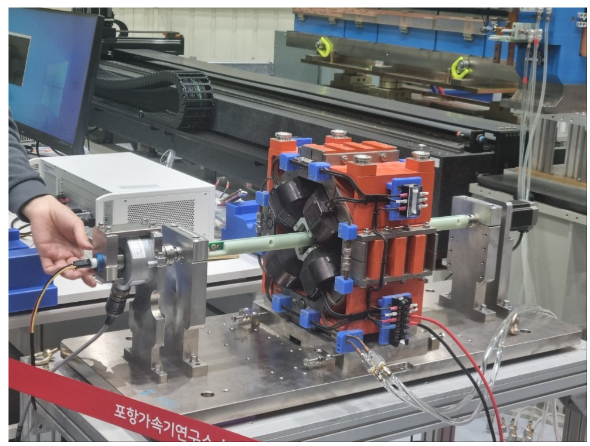

Figure 1: Rotating Coil Method for magnetic field measurement

B. Stretched Wire Method

Stretched wire method is based on the principle that a current-carrying wire experiences a Lorentz force within a magnetic field. The known current pulse is applied to the wire, causing it to vibrate under the influence of the magnetic field. We can measure the tension applied to the wire. By analyzing the amplitude and frequency response of the wire's vibration, fine details of the magnetic field distribution can be extracted. This method provides extremely high precision in determining the magnetic center and integrated magnetic field. But, it is highly sensitive to external disturbances such as mechanical vibrations and thermal drifts, which can affect measurement accuracy if not properly controlled.

C. Hall Probe Method

When an electric current flows through a thin conducting or semiconducting material inside a magnetic field, the Lorentz force deflects the charge carriers, leading to an accumulation of charges on the sides. This phenomenon generates voltage

$$V_H = \frac{IB}{qnd} \tag{6}$$

A Hall sensor is mounted onto a precision mechanical stage capable of controlled translation. The sensor is scanned along predefined paths through the magnet aperture or across specific planes. At each measurement point, the local magnetic field(B_x, B_y, B_z) are recorded. The collected data subsequently assembled into a detailed magnetic field map. Alignment of the Hall sensor is critical in magnetic field measurements, so minor deviations can significantly affect the results. Calibration of the Hall sensor is typically conducted using a standard reference magnet, which provides a well-made magnetic field for precise sensor calibration.

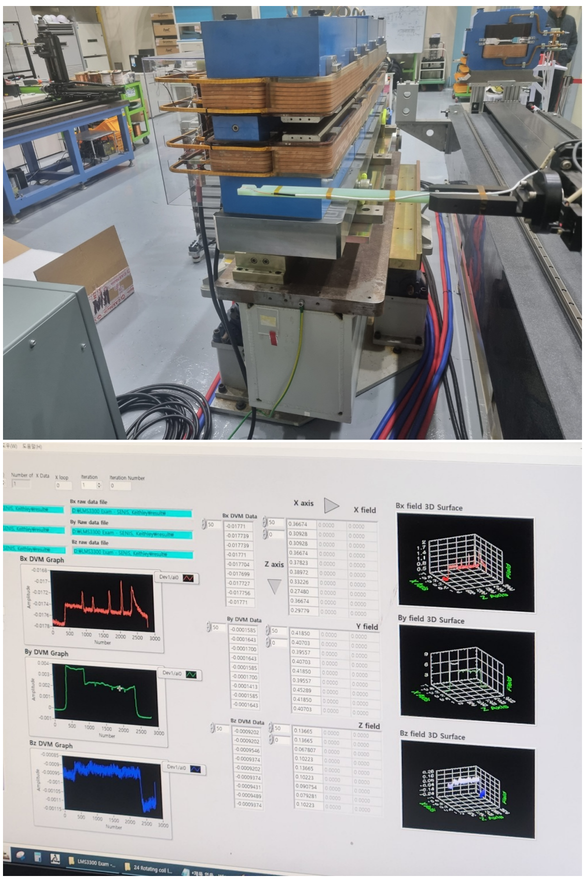

Figure 2: Hall Probe Method for magnetic field measurement

SADDLE SHAPED MAGNETS

In conventional non-vacuum magnet systems, magnets were precisely aligned relative to the beam pipe to ensure that the well-constructed magnetic field overlapped with the beam trajectory. Pole shims with micrometer-scale thicknesses have been used to reduce fringe field and improve field uniformity. By carefully shaping the shims, the magnetic field near the magnet ends can be better controlled, minimizing distortions and enlarging the effective well-made field area.

There is ongoing research on the insertion of saddle-shaped magnets to optimize space utilization. By adopting a saddle-shaped design, it becomes possible to efficiently guide particle beams along curved trajectories while minimizing the footprint of the magnetic system. This approach aims to achieve high field quality within confined geometries, which is critical for compact accelerator and beamline applications. The installation of a saddle-type magnet requires carefully wrapping a thin, flat electromagnetic sheet onto the supporting structure without tears. Accurate control of the wrapping process is needed to ensure the mechanical stability and magnetic field quality.


Figure 3: Saddle-shaped magnet

CONCLUSION

In this paper, we explored both the theoretical foundation and practical implementation of accelerator magnets. Starting from the analytic modeling magnets, we demonstrated how multipole magnetic fields can be described and evaluated through mathematical expansions and Fourier analysis. The importance of high-precision magnetic field quality was emphasized, as even small fabrication errors or fringe effects can significantly impact accelerator performance.

To ensure field accuracy, various measurement techniques were reviewed, including the rotating coil method, stretched wire method, and Hall probe mapping. Each method offers unique advantages in evaluating field strength and uniformity. Furthermore, recent advancements such as the implementation of saddle-shaped magnets highlight ongoing efforts to optimize space utilization while maintaining high field quality in compact accelerator environments.

REFERENCES

[1] Garam Hahn, *Lecture Notes on NUCE719P*, DIVISION OF ADVANCED NUCLEAR ENGINEERING, POSTECH, PowerPoint presentation, 2025.