FUNDAMENTALS AND FIELD MEASUREMENTS OF ACCELERATOR MAGNETS

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Abstract

Particle accelerators rely on precise magnetic fields for effective beam guidance and focusing. This report presents the fundamental theory of accelerator magnets and introduces key measurement techniques, including the rotating coil and Hall probe methods, for magnetic field characterization. Laboratory exercises provided practical experience in evaluating field quality and mapping spatial distributions. The study also examined superconducting magnets, underscoring their essential role in achieving high magnetic field gradients for modern accelerators. These findings highlight the importance of accurate magnet design and measurement in advancing accelerator performance.

INTRODUCTION

Particle accelerators fundamentally rely on magnetic fields to guide, focus, and manipulate particle beams along well-defined trajectories. Magnets—ranging from dipoles and quadrupoles to higher-order multipole magnets—are essential components that determine beam stability, confinement, and transport efficiency. Precise control of magnetic fields is indispensable, as even slight deviations can lead to beam loss, increased emittance, or deterioration of accelerator performance.

Accordingly, the accurate measurement and characterization of magnetic fields are crucial in the design, commissioning, and maintenance of accelerator systems. Magnetic field measurements ensure that magnets meet strict performance specifications and enable necessary corrections to maintain optimal beam quality.

In recent accelerator developments, the need for high-gradient magnets has become increasingly prominent to achieve strong focusing within compact spatial constraints. High-gradient magnets contribute to higher luminosity, reduced accelerator footprints, and enhanced energy efficiency, but they impose more stringent demands on the precision of magnetic field characterization.

In this report, we conducted magnetic field measurements using two primary techniques: the rotating coil method and Hall probe measurements. These hands-on exercises provided practical insights into the strengths, limitations, and operational considerations of different metrology approaches. Additionally, we briefly observed a superconducting magnet and reviewed its basic characteristics.

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THEORY OF MAGNET

This section introduces the basic principles of magnetic fields, their expansion into multipole components for accelerator magnet performance.

Magnetic field basics

The motion of charged particles in a magnetic field is governed by the Lorentz force:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}), \tag{1}$$

where $\mathbf{F}$ is the force acting on a particle with charge q, $\mathbf{v}$ is the particle's velocity, and $\mathbf{B}$ is the magnetic flux density.

In vacuum regions without free currents, the magnetic field obeys two fundamental Maxwell's equations:

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}, \tag{3}$$

where μ_0 is the permeability of free space and $\mathbf{J}$ is the current density. In regions where no current flows ($\mathbf{J} = 0$), Eq. (3) simplifies to:

$$\nabla \times \mathbf{B} = 0. \tag{4}$$

This condition implies that the magnetic field can be derived from a scalar magnetic potential ψ:

$$\mathbf{B} = -\nabla \psi. \tag{5}$$

Substituting into Gauss's law for magnetism (Eq. (2)), we find that ψ satisfies the Laplace equation ($\nabla^2 \psi = 0$).

Thus, the study of magnetic fields in accelerator magnets reduces to solving Laplace's equation under appropriate boundary conditions, leading naturally to multipole expansions, as discussed in later sections.

Multipole expansion

In two-dimensional Cartesian coordinates (x, y), assuming no variation along the longitudinal z-direction (i.e., $\partial/\partial z = 0$), the general solution to Laplace's equation can be expressed as a series expansion:

$$\psi(x, y) = \sum_{n=1}^{\infty} \left(A_n r^n \cos(n\theta) + B_n r^n \sin(n\theta) \right), \tag{6}$$

where r and θ are the polar coordinates related to x and y by $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}(y/x)$.

Taking the gradient of ψ, the transverse magnetic field components (B_x, B_y) are given by:

$$B_x = -\frac{\partial \psi}{\partial x}, \quad B_y = -\frac{\partial \psi}{\partial y}. \tag{7}$$

Substituting the multipole expansion of ψ yields the following form for the magnetic field:

$$B_y + iB_x = \sum_{n=1}^{\infty} C_n (x + iy)^{n-1}, \tag{8}$$

where C_n are complex coefficients determined by the specific magnet configuration.

Each term in the expansion corresponds to a distinct type of magnetic field component:

- $n = 1$: Dipole field (uniform magnetic field)
- $n = 2$: Quadrupole field (linear focusing/defocusing)
- $n = 3$: Sextupole field (chromatic correction)
- Higher orders: Octupole, decapole, etc.

As an example, the following figure illustrates the scalar and vector potentials associated with the $n = 2$ (quadrupole) term in the multipole expansion.

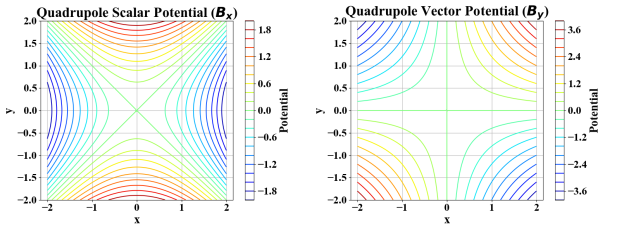

Figure 1: (Left) Quadrupole scalar potential in the xy plane.The contour lines represent the equipotential lines of the scalar potential. (Right) Quadrupole vector potential in the xy plane. The contours show the distribution of the vector potential.

In practical accelerator magnets, the field is ideally dominated by a single multipole component according to the magnet's function. For example, dipole magnets are designed to generate a uniform field for beam bending, while quadrupole magnets provide linear field gradients for beam focusing.

Measurement of the multipole content is crucial to ensure that undesired higher-order components, known as field errors, are minimized to maintain beam stability and quality.

MAGNETIC FIELD MEASUREMENTS

This section outlines practical methods for measuring magnetic fields in accelerator magnets, focusing on the rotating coil and Hall probe techniques. Their fundamental principles and laboratory applications at PAL are introduced, providing key data for evaluating magnet performance and optimizing field quality.

Rotating coil measurement

The rotating coil method is a widely used technique for measuring the multipole components of accelerator magnets. A coil is rotated at constant angular velocity within the magnet aperture. As the coil rotates, it intercepts the varying magnetic flux produced by different field components, inducing a voltage according to Faraday's law of electromagnetic induction. The induced voltage is recorded as a function of the coil's angular position. By performing a Fourier analysis of the acquired signal, the strengths of the individual multipole components—such as dipole, quadrupole, and sextupole fields—can be quantitatively determined. This approach enables precise characterization of the field quality and identification of unwanted higher-order multipole errors.

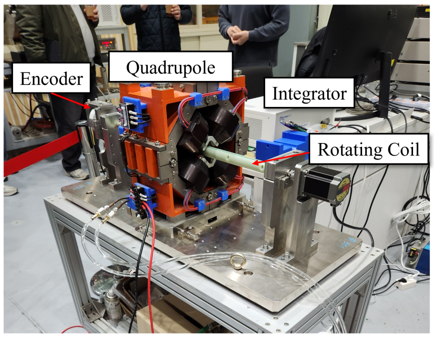

Figure 2: Measurement of field error using rotating coil method.

Hall probe measurement

The Hall probe measurement is a straightforward and versatile technique for mapping the local magnetic field in accelerator magnets. A Hall probe is a small sensor that generates a voltage proportional to the component of the magnetic field perpendicular to its active surface, based on the Hall effect.

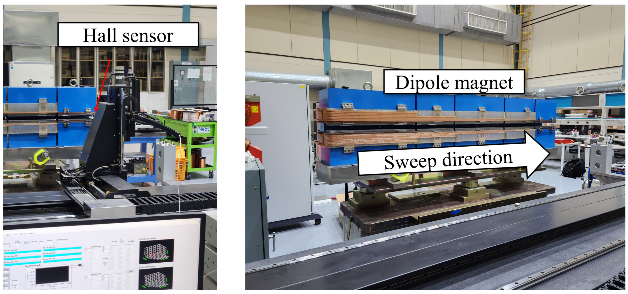

Figure 3: (Left) Hall sensor mounted on its holder for field mapping, and (Right) the target dipole magnet with the sweep direction indicated for the Hall sensor measurement.

In practice, the probe is mounted on a translation stage and systematically moved across the region of interest inside the magnet aperture (Fig. 3). By recording the Hall voltage at discrete positions, a spatial map of the magnetic field—typically the vertical component, B_y—can be constructed (Fig. 4). This method is particularly useful for visualizing the field uniformity and identifying regions where the field deviates from the ideal profile.

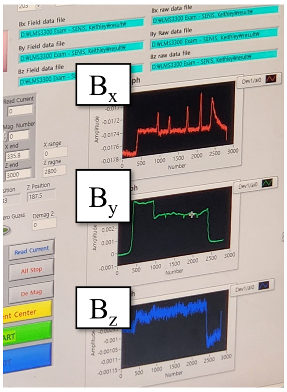

Figure 4: Measured magnetic field components (B_x, B_y, and B_z) as a function of position, obtained from the Hall sensor field mapping system.

SUPERCONDUCTING MAGNET FOR HIGH GRADIENTS

Superconducting magnets are essential for achieving high magnetic field gradients in modern accelerators. By using superconducting materials cooled below their critical temperature, these magnets can generate much stronger fields and gradients than conventional resistive magnets, enabling compact accelerator designs and improved beam focusing.

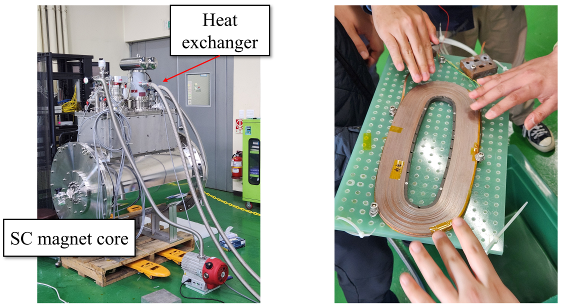

Figure 5: (Left) Superconducting magnet system with the core and heat exchanger. (Right) Close-up view of the superconducting coil structure.

In the laboratory, we briefly examined a superconducting magnet and its cryogenic system, as shown in Fig. 5. The system includes a superconducting magnet core and a heat exchanger, which are critical for maintaining the low temperatures required for superconductivity. A close-up view of the superconducting coil structure further illustrates the engineering complexity involved in these devices. Although detailed field measurements were not conducted, this hands-on experience highlighted the operational challenges and technological importance of superconducting magnets in advancing high-gradient accelerator applications.

Table 1 provides a comparison of the typical operating temperatures and maximum achievable fields for different types of accelerator magnets. From this comparison, it is evident that superconducting magnets (including high-temperature superconductors, HTS) can achieve much higher magnetic fields—exceeding 20 T in some cases—compared to normal-conducting (NC) magnets, which are generally limited to around 1–2 T. This clearly demonstrates that the use of superconducting technology is indispensable for realizing the high-gradient fields required in state-of-the-art accelerators.

Table 1: Comparison of magnet types used in accelerators

Type	Operating Temp.	Max. Field (Dipole)
NC	Room Temp.	~1 – 2 T
HTS	4 K to 77 K	**> 20 T**

SUMMARY

This report reviewed the basic principles and measurement methods for accelerator magnets, emphasizing their crucial role in beam control. Through laboratory exercises with rotating coil and Hall probe techniques, we gained practical insight into field characterization and quality assessment. The study also highlighted the importance of superconducting magnets for achieving high gradients, demonstrating that advanced accelerator applications depend on precise magnetic field control and measurement.

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